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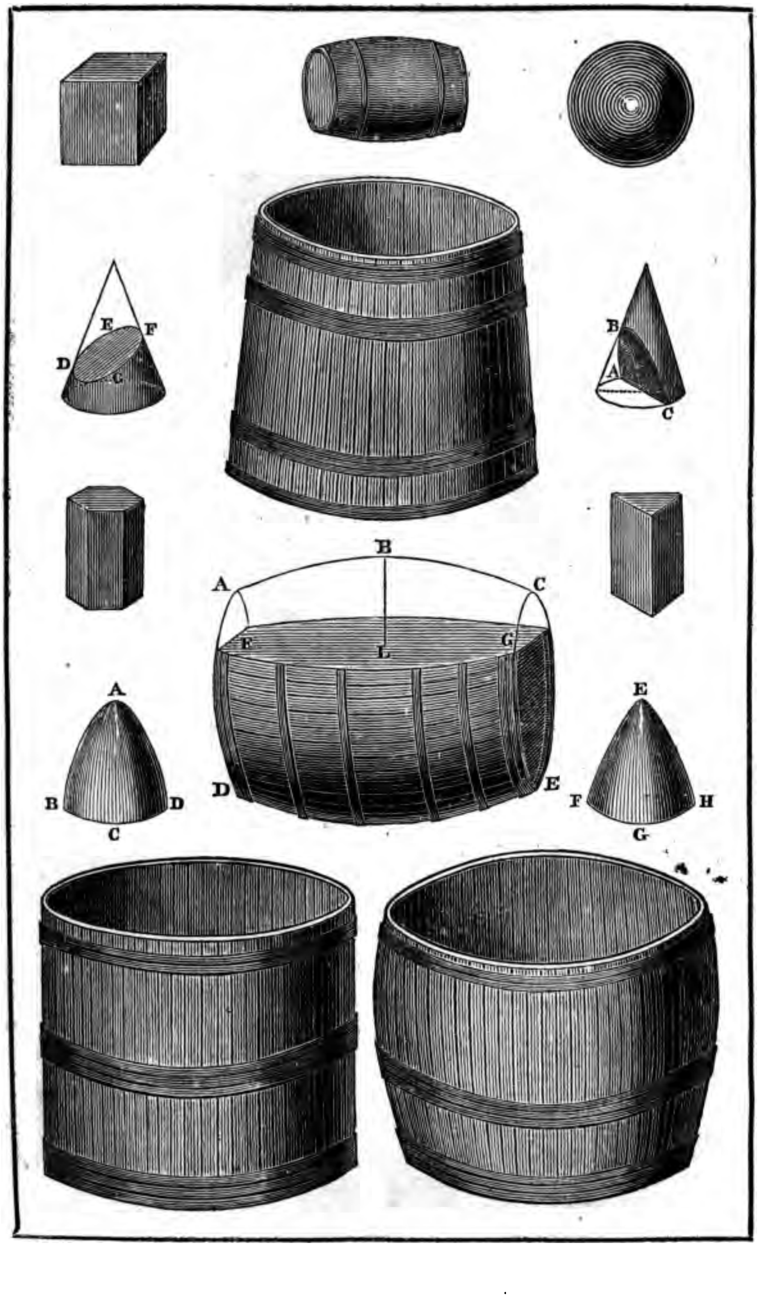
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THE

NEW PRACTICAL GAGER,

ON AN

Easy and Improved Plan;

FOUNDED ON

UNERRING PRINCIPLES, FAMILIARLY EXPLAINED:

AND ELUCIDATED BY PEN AND RULE.

CONTAINING EVERY THING NECESSARY TO BE KNOWN

IN THE

THEORY AND PRACTICE

OF THE

VALUABLE ART OF GAGING.

ENRICHED WITH

Modern Tables,

AND

ILLUSTRATED BY DIAGRAMS.

◆

EXPRESSLY INTENDED FOR

THE USE OF THE OFFICERS OF HIS MAJESTY'S REVENUE

OF

EXCISE AND CUSTOMS.

◆

BY

MATTHEW ILEY,

MANY YEARS A GAGER AND SURVEYOR ON THE LONDON ESTABLISHMENT.

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TO

COLONEL DOYLE,

OF

HIS MAJESTY'S HONORABLE BOARD

OF

EXCISE;

AND TO THE REST OF THE

Honorable the Commiſsioners

OF

THE REVENUE,

THIS NEW TREATISE ON GAGING,

PATRONIZED BY THEIR HONORS,

IS (WITH PERMISSION)

HUMBLY DEDICATED,

BY

THEIR HONORS' MUCH OBLIGED AND

MOST OBEDIENT SERVANT,

M. ILEY.

PREFACE.

NOTWITHSTANDING the many Treatises now extant on the ART OF GAGING, the want of a book in which the Theory and Practice of that useful branch of the Mathematics should be concisely, and at the same time more clearly and systematically explained than heretofore, and in which New Tables and better practical methods than are usually given, should be introduced, has been long felt, not only by young practitioners in the Excise, but even by officers of experience.

With the view of supplying this defect, the following sheets are presented to the public, no pains having been spared to render them worthy of general patronage.

The subject advances gradually through the several parts of knowledge upon which the

Page
MENSURATION OF SOLIDS PROPERLY SO CALLED 168
Content of a Cube . . . . ib.
Content of a Parallelopipedon 170
Content of a Cylinder 171
Content of a Prism 173
Content of Solids between Equal and Like Parallel Planes 176
Content of a Cone 179
Content of a Pyramid 181
Content of a Sphere 183
Spherical Divisors, and Spherical Gage Points . 185
Content of a Spheroid 187
Content of a Paraboloid 188
Content of a Parabolic Spindle 190
Parabolic-Spindle Divisors and Gage Points . 192
Content of the Frustum of a Cone 194
To determine Intermediate Diameters in a Cone . 197
Content of the Frustum of a Pyramid . . 199
Content of a Solid with Dissimilar Oval Ends . 202
Content of a Solid with Dissimilar Rectangular Ends 204
Content of the Ungula of a Cone 206
Content of the Ungula of a Square Pyramid . 210
Content of the Segment of a Sphere . . 212
Content of the Frustum of a Globe . . . 215
Content of the Frustum of a Paraboloid . . 216
Content of the Middle Frustum of a Spheroid . 219
Content of the Middle Frustum of a Parabolic Spindle 220

BREW-HOUSE.

To Gage and Inch a Copper 223
Of a Copper with a Rising Crown 224
TABLE OF CIRCULAR AREAS IN ALE . . 239
Use of the Table 252
Of a Copper with a Falling Crown . . . 257
To Gage and Inch a Copper Back 265
To Gage and Inch a Copper Pan 272

Page
To Gage and Inch a Round Mash-Tun . . 280
To Gage and Inch a Square Mash-Tun . . 286
To Gage and Inch an Oval Mash-Tun . . 292
To Gage and Inch a Square Jack-Back . . 293
To Gage and Inch a Round Hop-Tun . . 298
To Gage and Inch an Oval Hop-Back . . 305
To Gage and Tenth a Rectangular Cooler . . 306
To Gage and Tenth a Multilateral Cooler . . 311
To Gage and Tenth an Under-Back . . . 315
To Gage and Inch a Square Guile-Tun . . 316
To Gage and Inch a Round Guile-Tun . . 317
To Gage and Inch a Guile-Tun with Tapering Sides ib.
To Gage and Inch a Guile-Tun with Bending Sides 321
To Gage a By-Tub 323
To deduct for Heat in Warm Worts . . . 324

OF VICTUALLERS' UTENSILS.

General Observations 327

DISTILLERY.

To Gage and Inch a Still . . . 329
TABLE OF CIRCULAR AREAS IN WINE GALLONS 333
To Gage a Still Head 356
Distillery Utensils 360

MALT GAGING.

Of the Bushel Measure 361
To prove a Bushel Measure by Weight . . 362
Table of Dimensions for a Bushel Measure . 363
To Gage and Tabulate a Maltster's Cistern . 364
To Gage a Couch of Malt 369
To Gage a Floor of Malt 375

CASK GAGING.

To Gage a Cask Lying 379
To Gage a Cask Standing 384

Page
To Gage a Cask by Diagonals . . . 388
To Gage a Spheroidal Cask by a Table of Quotients - 389
TABLES OF QUOTIENTS FOR FOUR VARIETIES ib.
To determine the Content by a Table of Diameters 395
TABLE OF HEAD AND BUNG DIAMETERS - 396
To Ullage a Cask Lying - - - 401
To Ullage a Cask Standing - - - 404
To Inch a Spheroidal Cask Standing - 410
To Ullage a Cask Lying by the Pen . . 406
To Ullage a Cask Standing by the Pen . . 408
Method of Using the Callipers . . . 416
General Observations on Casks . . . 419

STARCH GAGING.

To Gage a Starch Maker's Vat . . . 424
To Gage and Tabulate a Water Frame . . 426
To Gage and Tabulate Boxes with Sliders . 428

OF SPECIFIC GRAVITY.

Definition 430
Table of Specific Gravities . . . 434
Examples in Specific Gravity . . . 435

SPIRITUOUS LIQUORS.

Of the Hydrometer 436
To ascertain the exact Strength of Spirits . . 437

TABLES.

Table of the Nett Duties on Beer . . 440
Table of the Nett Duty on Malt . . . 445
Table for Square Yards and Duty on Paper . . 450
General Cash Table for moneying the charges . 454
Table of the Duty on Sweets at £2 2*s*. . . . 459
Table of the Duty on Sweets at 7*s*. . . . 460
Table of the Duty on Cider 461
USE OF THE TABLES 462

THE
NEW PRACTICAL GAGER.

AS all instruments used in gaging are decimally divided, and as the dimensions of every utensil are taken in inches and tenths, the best introduction to Practical Gaging must evidently be a succinct account of decimal fractions.

OF DECIMALS.

In decimal arithmetic the unit is supposed to be divided into parts, which are first, tenths; then hundredths; then thousandths; &c. of the whole.

The half of any thing is therefore expressed as *five-tenths*, since, if in the whole there be ten equal parts, there must in the half be five. In like manner the quarter of any thing will be *twenty-five hundredth-parts*, since, as the whole is conceived to consist of 100 equal parts, the quarter must consist of 25 of those parts. So an eighth part is equivalent to 125 *thousandth-parts*, because, if in the whole there be 1000 equal parts, it follows that in an eighth there will be

125 such parts. To write these as vulgar fractions, it would be $\frac{5}{10}$, $\frac{25}{100}$, $\frac{125}{1000}$, which are respectively equal to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$.

The denominators of all decimal fractions being manifestly either 10, 100, 1000, 10000, &c., they are not written but understood: for five-tenths are denoted by the figure 5 preceded by a dot, thus, ·5, which dot is called the decimal point. The figure next to the decimal point always expresses *tenths;* the second figure, *hundredths;* the third, *thousandth-parts;* and so on.

Hence ·01 will represent the hundredth part of a unit, and ·001 the thousandth part: wherefore ·378 signifies three-tenths *plus* seven hundredths *plus* eight thousandth-parts; that is in all $\frac{378}{1000}$. From this it is apparent that any finite decimal fraction may be converted into a vulgar fraction by setting under the point the figure 1, annexing to it as many ciphers as there are places in the decimal, and then expunging the point. For example, ·087 will become $\frac{87}{1000}$, and ·0009 will become $\frac{9}{10000}$. Whereby it is evident that in decimal arithmetic, ciphers on the left of significant figures between them and the point decrease their value in the same ratio that they increase the value of integers when written after them: at the same time, it appears that a cipher annexed to a decimal cannot alter its value, inasmuch as ·50 or $\frac{50}{100}$ are precisely equal to ·5 or $\frac{5}{10}$.

When a number is partly integer and partly decimal, the integer is written, as in whole numbers, before the point, and the decimal after it. For example, $7\frac{1}{2}$ expressed decimally, will be 7·5, and $12\frac{1}{4}$ will be 12·25.

ADDITION OF DECIMALS.

Write the numbers to be added, in such manner that tenths fall under tenths, hundredths under hundredths, &c.—that is, let the decimal points of the several rows be perpendicular under one another; then draw a line and add as in addition of integers, placing a point in the sum directly under the other points.

As this will be better understood by an example or two, let it be required to find the sum of ·91 and ·773.

The work will appear as follows:

·91
·773

1·683 sum required.

The total of these two decimals is therefore $1\frac{683}{1000}$.

Next let it be proposed to collect into one sum the following mixed numbers and decimal parts, viz., 31·8, 2·047, 5684·29, 199·369, ·00265, and 12·42.

The several rows will stand thus:

31·8
2·047
5684·29
199·369
·00265
12·42

5929·92865 sum total.

In like manner if it were required to collect into one sum 3·02, ·124, 79·5, ·0008, 5296·43, and ·097, the total would be found to be 5379·1718.

SUBTRACTION OF DECIMALS.

When it is required to find the difference of two given decimals or mixed numbers, write the greater number, and place under it the less, taking care that units fall under units, tenths under tenths, &c.; that is, let the decimal point of the subtrahend be exactly under the point of the minuend. If there be fewer decimals in the upper than in the lower row, the deficiency may be supplied by annexing ciphers, or by supposing them annexed. Then draw a line, and subtract as in whole numbers, placing a point in the remainder perpendicularly under the other two points.

To illustrate this by examples, let it first be proposed to find the difference between 58·79215 and 416·76.

Here the latter number is the greater; wherefore it will be,

416·76000 minuend
58·79215 subtrahend
357·96785 difference.

Again, let 8·2321625 be given to be subtracted from 9·1, and the figures will stand in the manner following:

9·1——— minuend
8 2321625 subtrahend
·8678375 remainder.

In this example the remainder is entirely decimal, although both minuend and subtrahend consisted of mixed numbers.

As a further elucidation of the Rule of Subtraction of Decimals, let the difference of 1·903 and 1·90291 be sought, and it will be,

1·90300 greater number
1·90291 less number

·00009 difference required.

By which it appears that these two numbers differ only $\frac{9}{100000}$, or nine times the hundred-thousandth-part of a unit.

MULTIPLICATION OF DECIMALS.

To find the product of any two given decimal numbers, let the factor which consists of the greater quantity of figures be multiplied by the other number as if they were purely integers; then from the product point off, on the right hand, as many decimals as there are decimal places in both factors. When there are fewer figures in the product than decimals in both factors, the defect must be supplied by prefixing ciphers to the significant figures.

Two or three examples will make the Rule better understood, and first, let the product of 5·3 and ·8274 be required.

Here the latter number, though less than the other in value, consists of more figures, and for that reason may with more convenience be made the multiplicand.

The operation will therefore be as follows:

·8274 multiplicand
5·3 multiplier

24822
41370

4·38522 product.

In the product there are five figures on the right pointed off as decimals, because the number of decimal places in both factors amounts to five.

Next let it be proposed to multiply 462·09 by ·023, and it will be,

462·09 multiplicand
·023 multiplier

138627
92418

10·62807 product

In this product, for the same reason as in the last, the number of decimal places is five.

Again, let the two given numbers be ·302 and ·00176; and they will be placed in the manner following:

·00176 multiplicand
·302 multiplier

352
5280

·00053152 product.

In this example the number of decimals in the two factors amounts to eight, but in the product there are only five significant figures; consequently three ciphers must be prefixed, that the decimal places in the product may equal the sum of those in the factors.

DIVISION OF DECIMALS.

In division of decimals the method of operation is the same as in division of integers, the only difficulty being in valuing the quotient by pointing off the right number of decimal places.

Having placed the divisor and dividend exactly as in whole numbers, count the number of decimals in each, and if the dividend contain fewer decimal places than the divisor, annex to the dividend decimal ciphers, till there are at least as many decimals in the dividend as in the divisor. Then divide as in whole numbers till every figure in the dividend has been brought down. Lastly, point off as many decimals in the quotient as the number of decimal places in the dividend exceeds that in the divisor, prefixing ciphers to the figures of the quotient if necessary.

It is obvious that where there is a remainder after the dividend has been exhausted, the quotient may be extended at pleasure by annexing ciphers to the dividend, or conceiving them to be annexed. Although the quotient may by this means be carried to any length, yet the decimal point in it will be unalterable.

To illustrate the Rule by a few examples, we shall suppose it expedient to divide 152·3 by ·1523.

Here the dividend contains fewer decimal places than the divisor, by three; wherefore at least three decimal ciphers must be annexed to the dividend. Attending to this, and proceeding as in whole numbers, it will be,

·1523)152·3000(1000
1523
000

Now, as in this example the number of decimal places in the dividend has no excess over the number of decimals in the divisor, there will be no decimals in the quotient; hence the divisor is contained in the dividend 1000 times without a remainder, which is easily verified by multiplication.

For further illustration, let ·6285 be given to be divided by 2.7.

The operation will appear thus:

2·7)·6285(·232 quotient.
54
88
81
75
54
21 remainder.

In this example the decimal places in the dividend are three more than in the divisor, and consequently three places are pointed off in the quotient. The point being thus settled, the quotient may be carried to any number of places by conceiving ciphers annexed to the dividend.

One more example will suffice to set the Rule in the clearest light; and for this purpose let ·06 be given to be divided by 94·32.

Here it becomes necessary to annex ciphers to the dividend to extend it in order to subtract the figures that must unavoidably be placed under it. Annexing, therefore, five or six ciphers at pleasure, and proceeding as already directed, it will be,

94·32)·06000000(·000636 quotient.
56592
―――
34080
28296
―――
57840
56592
―――
1248 remainder.

In this dividend there is an excess of six decimals over the number of decimals in the divisor; but in the quotient there are only three significant figures. Now as the number of decimals in the quotient must equal the excess of the decimals in the dividend over those in the divisor, it follows that three ciphers prefixed must make up the deficiency; and thus the quotient becomes ·000636, which may be extended (as there is a remainder) to a degree of approximation still nearer, by imagining more ciphers in the dividend, and continuing the division.

REDUCTION OF VULGAR TO DECIMAL FRACTIONS.

Any vulgar fraction may be reduced to a decimal, by dividing the numerator by the denominator. In division of decimals, it is plain, that any number, however small, may be divided by any other number, however great, if a sufficient quantity of decimal ciphers be annexed to the dividend: in many instances, however, the quotient will not be finite, shewing that no decimal fraction whatever can equal exactly the vulgar fraction to be reduced.

In some cases the quotient will consist of the same figure in endless succession, and in others, several figures in regular order will arise and circulate for ever.

When the quotient, in reducing a vulgar to a decimal fraction, is likely to consist of several places, five or six decimals will in general be found to be accurate enough for common purposes, and where great precision is not requisite, three or four (or even fewer) will suffice.

When a vulgar fraction is given to be reduced to a decimal, annex (as already directed) a competent number of decimal ciphers to the numerator, and then divide by the denominator; pointing off the quotient as above instructed in division. The quotient will be the decimal fraction required;—if finite, precisely equal to the vulgar fraction—if not, nearly so.

To render these observations more intelligible, and to aid the learner in putting them in practice, we shall subjoin a few examples. And first, let it be proposed to reduce $\frac{3}{4}$ to a decimal fraction.

Setting 3, the numerator, with decimal ciphers annexed to it, for a dividend, and then dividing by 4, the denominator, it will be,

```
4)3·00(·75 decimal required.
  28
  ——
   20
   20
  ——
   ..
```

Hence it is manifest that $\frac{3}{4}$ of a unit are equivalent to the decimal ·75, that is, to $\frac{75}{100}$.

Next, let it be required to find the decimal equal to $\frac{1}{25}$, and the operation will be as follows:

```
25)1·00(·04 decimal sought.
   1 00
   ————
     ..
```

Again, let a decimal fraction equal to $\frac{7}{8}$ be demanded, and the work will be as under:

```
8)7·000(·875 decimal required.
  64
  ——
   60
   56
   ——
    40
    40
   ——
    ..
```

Whereby it is seen that the finite decimal ·875 is the equivalent of $\frac{7}{8}$.

If $\frac{2}{3}$ were given to be reduced to a decimal, the quotient would be infinite, as may easily be made apparent; for treating this fraction according to the Rule, we obtain the figure 6 in endless succession by the process following:

```
3)2·0000000(·6666666 quotient.
  18
  ——
   20
   18
   ——
    20
    18
    ——
     20
     18
     ——
      20
      18
      ——
       20
       18
       ——
        20
        18
        ——
         2 remainder.
```

So that $\frac{2}{3}$ are equivalent to 6 tenths *plus* 6 hundredths *plus* 6 thousandth-parts *plus* &c. to infinity.

A decimal of this description is termed a pure repeater, and is generally expressed by the first figure written with a dot over it. A pure repeating decimal is reduced to a vulgar fraction by expunging the point, and setting a 9 under each repeater; therefore the above decimal is equal to $\frac{66}{99}$ or $\frac{6}{9}$, manifestly equivalent to $\frac{2}{3}$, the fraction from which it was obtained.

If it were expedient to convert $\frac{1}{480}$ into a decimal fraction, the result would be ·002083, which not being terminate, is only an approximation to the value of the given vulgar fraction, and obtained as follows:

480)1·0000000000(·0020833333 quotient.
960
4000
3840
1600
1440
1600
1440
1600
1440
1600
1440
1600
1440
160 remainder.

In the above operation it would have been neater to cut off the cipher at the end of the divisor, placing the point, hereupon, before the 1 of the dividend; but those who wish to acquire an extensive knowledge of decimals must consult systems in which the subject is treated at large, since a few general principles with their application to the practice of gaging, are all that can be expected on this head, in a work like the present: yet nothing that is really useful shall be omitted.

TO REDUCE THE DECIMAL OF A HIGHER DENOMINATION TO INTEGERS OF A LOWER.

Multiply the given decimal by the number which, of the next lower denomination, makes a unit of the higher: then point off the decimal places of the product, and proceed with *them* in the same manner, always pointing off the same number of decimals in each product, till the lowest possible denomination is obtained. The integers on the left of the several products will be respectively of the denomination of the multipliers by which they were produced.

For example, let the value of ·8625 of a pound sterling be required in shillings and pence.

The operation, performed agreeably to the Rule, will be as below:

£·8625	given decimal
20	shillings in one pound
17·2500	shillings
12	pence in one shilling
3·0000	pence.

By which it appears that the value of the given decimal is precisely 17*s*. 3*d*.

If there had been any significant figures in the decimal of the pence line they would have been multiplied by 4 to reduce them to farthings, which denomination being the lowest in the table of money, is of course the least to which any decimal of British money accounts can be reduced.

In like manner the value of £·565625 will be found to be exactly 11*s*. 3¾*d*.

For the sake of further elucidation let the equivalent of ·621 of a cwt. be sought, and it will be,

Cwt. ·621	given decimal
4	quarters in 1 cwt.
2·484	quarters
28	pounds in 1 qr.
13·552	pounds
16	ounces in 1 lb.
8·832	ounces.

Hence the value of the given decimal is 2 qrs. 13 lb. 8 oz. and $\frac{832}{1000}$ of an ounce. The decimal of an ounce might, if deemed necessary, be reduced to drams, by multiplying it by 16, and the result would be 13·312 dr., which denomination is the lowest in the table of avoirdupois weight.

Lastly, let the value of ·049 of a tun of wine be demanded, and the process will be,

Tun ·049	given decimal
2	pipes in 1 tun
·098	pipes
2	hhds. in 1 pipe
·196	hogsheads
.63	gallons in 1 hhd.
12·348	gallons
4	quarts in 1 gallon
1·392	quarts
2	pints in 1 quart
·784	pints.

Consequently the value of the given decimal is 12 gal. 1 qrt. 0.784 pt.

TO REDUCE INTEGERS OF DIFFERENT DENOMINATIONS TO THE DECIMAL OF THE HIGHEST.

Reduce all the given integers to one denomination, namely that of the lowest, and their sum in this denomination will be the numerator of a vulgar fraction, of which the denominator will be the given integer reduced to the same denomination as the numerator. This vulgar fraction when changed to a decimal will be the number required.

To illustrate the Rule let it be proposed to express 17*s.* $8\frac{3}{4}$*d.* in the decimal of a pound sterling.

£0 . 17 . $8\frac{3}{4}$	given quantity
12	pence in 1 shilling
212	pence
4	farthings in 1 penny
851	farthings.

In the given quantity, then, are 851 farthings, and in one pound, the proposed integer, there are 960 farthings: the vulgar fraction is consequently $\frac{851}{960}$ of a pound sterling; and this reduced to a decimal becomes £·886458 &c., as underneath:

960)851·00000 (£·886458 &c.

The truth of this result is easily verified by means of the last Rule; but 17*s.* $8\frac{3}{4}$*d.* will not exactly, though *very nearly*, arise from the multiplication, because the decimal here given is not the true value of the vulgar fraction, since it is not determinate.

As additional illustration of this method, let 3qr. 12lb. be given to be reduced to the decimal of a cwt.

Cwt. 0 · 3 · 12 given quantity
28 pounds in 1 qr.
96 pounds.

There being 96 lb. in the given quantity, and 112lb. in the proposed integer, the equivalent vulgar fraction will be $\frac{96}{112}$, or, in its lowest terms, $\frac{6}{7}$. Now reducing $\frac{6}{7}$ to a decimal it will be,

7)6·000000
·857142 decimal required.

The following is a better mode of solving this problem. Reduce each of the quantities in the lower denominations, beginning with the lowest, into the decimal of the next higher denomination, and then the highest denomination into the decimal of the integer.

To do this expeditiously and with neatness, draw a perpendicular line, and on the right of it set the given denominations in order, beginning with the lowest, and ending with the highest: then opposite to each denomination write, for divisor, the number of units which of that denomination makes a unit of the next higher: proceed now to divide, placing the quotients as decimals of the next higher denominations respectively, as in the following examples:

1. Reduce 17*s*. $6\frac{3}{4}$*d*. to the decimal of a £.

4	*qr*. 3·
12	*d*: 6·75
20	*s*. 17·5625
	£ 0·878125 decimal required.

2. Reduce 4 qrs. 1 co. 1 bu. 3 pec. 0 gal. 2 pts. to the decimal of a last.

8	pts. 2·
2	gal. 0·25
4	pec. 3·125
4	bu. 1·78125
2	co. 1·4453125
5	qr. 4·72265625
2	w. 0·94453125
	lst. 0·472265625 decimal required.

3. Reduce 16 cwt. 2 qr. 20 lb. 12 oz. 10 dr. to the decimal of a ton.

16	dr. 10·
16	oz. 12·625
28	lb. 20·7890625
4	qr. 2·7424665 &c.
20	cwt. 16·6856166 &c.
	ton 0.8342808 &c. decimal sought.

EXTRACTION OF THE SQUARE ROOT.

The square root of every integral number consisting of either one or two figures, is a single integer: thus the square root of 4 is 2; the square root of 9 is 3; the square root of 64 is 8; and the square root of 99 is 9·94, &c. *in infinitum*. Again, the square root of an integral number consisting of either three or four figures, will contain *two* places of integers; and the square root of a number consisting of five or six integers, will contain *three* integral places. Wherefore, before we can extract the square root of a number, we must ascertain the number of places to be in the root. This is done by separating the given number into periods of two figures each, beginning at the units place in integers, and proceeding towards the left hand; but at the decimal point in decimals, and proceeding towards the right hand, annexing a cipher, if necessary, to make the number of decimals even. For example, if the square root of 38975 were required, we should form the three following periods:

3 | 89 | 75

From which we learn that the square root of 38975 will contain three integral places.

In like manner if the square root of ·08237956 were required, we should form the four periods following:

·08 | 23 | 79 | 56

And from this we know that the square root of ·08237956 must (if finite) be a decimal number of four places.

When the given number is partly integral and partly decimal, the process of separating it into periods is the same; for if the square root of the mixed number 137·92145 were required, the periods would appear as below:

1 | 37 | ·92 | 14 | 50

Now these periods indicate that the root will contain two integral places, and three decimals. When the operation, however, has been carried far enough to obtain these five figures, there will evidently be a remainder, since the last period of the given number is a cipher preceded by a significant figure, a termination which no unit multiplied by itself can produce; and consequently the decimal in the root must run on to infinity: that is, no number whatever multiplied by itself can exactly produce 137·92145.

Method of Extracting the Square Root.

Having separated the number, whereof the square root is required, into periods, as above directed, we begin at the left hand, and consider what whole number is the greatest square number in the first period.— This number we set under the period, to be subtracted, and write its root for the first figure of the root required. After subtracting the square number from the first period, we annex to the remainder the second period, and for a divisor we double the part found of the root, enquiring how often this double can be found in the tenth part of the augmented remainder. The quotient will in general be the second figure of the root, though sometimes too great. The figure last found is to be annexed to the double divisor, as well as to the part of the root previously found; the whole divisor is now to be multiplied by this figure, and the product subtracted from the proper dividend. Then to the remainder the third period must be annexed, from which the next figure of the root is to be found in the same manner as the second, by doubling the whole root yet found, for a divisor.

An example or two will render the method of proceeding more intelligible and easy: let, therefore, the square root of 15376 be required, and the work will appear as under:

```
     1|53|76(124 root required.
     1
    ---------
22)  53
     44
     --
 44) 976
     976
     ---
     ...
```

Again, let a number be sought which multiplied by itself shall produce 96410·25, and the operation will be,

```
     9|64|10|·25 (310·5 number required.
     9
    -----
61)  64
     61
     --
6205) 31025
      31025
      -----
      ....
```

In like manner the square root of 8108·822401 will be found to be 90·049.

Lastly, let the square root of the decimal ·072 be sought, and the process will be the following:

```
·07|20|00|00 (·2683 &c. in infinitum root required.
  4
 ---
46)320
   276
 -----
528)4400
    4224
  ------
5363)17600
     16089
   -------
      1511 remainder.
```

TO FIND A MEAN PROPORTIONAL BETWEEN TWO NUMBERS.

Multiply the two numbers together, and the square root of the product will be the mean proportional between them.

EXAMPLE 1.

Required a geometrical mean proportional between 36 and 64.

```
     64
     36
    ---
    384
   192
   ----
   23|04 (48 mean required.
   16
  ---
88)704
   704
   ---
   ...
```

EXAMPLE 2.

Required the geometrical mean between 2·5 and 57·6.

```
     57·6
      2·5
    ─────
     2880
    1152
   ──────
   1|44|·00 (12 mean sought.
   1
   ──
22)44
   44
   ──
   ..
```

EXTRACTION OF THE CUBE ROOT.

Separate the number, whereof the cube root is required, into periods of three figures each, beginning at the units place and proceeding towards the left hand in integers, but at the decimal point, and proceeding towards the right hand in decimals, making up, if necessary, the last period, by the adjunction of ciphers.

This process will determine the number of figures in the root, and their denomination.

For example, if the cube root of 1404·928 were demanded, the periods would be the three following:

1 | 404 | ·928

Hence the cube root of the number proposed will consist of three places, of which the first two will be integral, and the last decimal.

Having separated the given number into periods, we begin to extract the root by setting the greatest whole cube number contained in the first period, under that period, for subtraction; writing its root as the first figure of the root required.

After subtraction we annex to the remainder the second period, and the remainder thus augmented we term the *Resolvend.*

Then for a divisor we add to triple the part of the root found, thirty times the square of that part, taking for a dividend the tenth part of the resolvend. The quotient will in general be the next figure of the root, though sometimes too great by one or more units.

Lastly, for a subtrahend, we cube the last figure of the root yet found, and to that cube we add thirty times the square of the last figure, multiplied by the part before it of the root; together with three hundred times the square of that part, multiplied by the last figure. The sum of these three are to be subtracted from the resolvend, and the next period annexed to the remainder for a *New Resolvend.*

In this manner we go on till every period has been exhausted; after which, if there be a remainder, and we wish to extend the root, new decimal periods of three ciphers each may be adjoined at pleasure.

As example surpasses precept, we shall illustrate the Rule by two operations at length: and first let the cube root of 3581·577 be demanded.

The work will be as follows;

	3|581|·577	(15·3 root required.
	1	cube of 1.
	2581	*resolvend.*
	3	triple of 1.
	30	30 times 1 × 1.
	33	divisor. 33)258(7*
	125	cube of 5.
	750	30 times 5 × 5 × 1.
	1500	300 times 1 × 1 × 5.
Sum	2375	subtrahend.
	206577	*new resolvend.*
	45	triple of 15.
	6750	30 times 15 × 15.
	6795	divisor. 6795)20657(3
	27	cube of 3.
	4050	30 times 3 × 3 × 15.
	202500	300 times 15 × 15 × 3.
Sum	206577	subtrahend.
	……	

---

* The figure 7, is on trial, found to be too great; as is also 6, wherefore recourse is next had to 5, and this figure answers.

EXAMPLE 2.

The cube root of 29631·5 is required.

29|631|·500 (30·9 &c. root sought.

27	cube of 3.
2631	*resolvend.*
9	triple of 3.
270	30 times 3 × 3.
279	divisor. 279)263(0
2631500	*new resolvend.*
900	triple of 30.
27000	30 times 30 × 30.
27900	divisor. 27900)263150(9
729	cube of 9.
72900	30 times 9 × 9 × 30.
2430000	300 times 30 × 30 × 9.
Sum 2503629	subtrahend.
127871	remainder.

It is evident that, by annexing decimal periods of three ciphers each, or supposing them annexed to the given number, the root might be extended at pleasure to any proposed degree of approximation.

As the above Rules and Examples contain every thing in decimals, and the extraction of roots, necessary to our purpose, we shall take our leave of this branch, and next introduce the learner to a valuable instrument called THE SLIDING RULE.

DESCRIPTION AND USE
OF THE
SLIDING RULE.

THIS INSTRUMENT is commonly made of box-wood, of various lengths, from six inches to two feet; but these are all in principle the same, and therefore we shall confine ourselves to a twelve-inch Rule with four slides, as being upon the whole one of the most convenient and useful.

The Lines on the First Face of the Rule.

The first line is marked with the letter A, and is called Gunter's line, being numbered from left to right with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, at distances in geometrical proportion. In this line are two brass pins, the first at 2150·42, and marked MB, signifying that there are 2150·42 cubic inches in a bushel of malt: the second pin is placed at 282, and marked AG, signifying that 282 cubic inches make an ale-gallon.

The second line on this face is the line on the slide, noted by the letter B, and differing nothing from the line A; for when unity at the end of the one is applied to unity at the end of the other, all the divisions and figures of the one correspond with those of the other; so that the one seems a duplicate of the other. On this line there is a brass pin at 231, the number of cubic inches in a wine-gallon, and marked WG.

The third line is on the Rule and characterised by the letters MD, signifying malt-depth, for which reason it has the name of the malt line. This is also a line of numbers beginning at 2150·42, and proceeding from left to right in reverted order with the figures 2, 10, 9, 8, 7, 6, 5, 4, 3, ending at the right hand, as it began at the left, with the number 2150·42.

The Lines on the Second Face of the Rule.

The first line on this face is known by the letter D, being a line of numbers beginning with 1 and ending with 10. On this line are marked with brass pins the following gage points.

1. WG at 17·15 the circular gage point for wine gallons.
2. AG at 18·95 the circular gage point for ale gallons.
3. MS at 46·37 the square gage point for malt bushels.
4. MR at 52·32 the circular gage point for malt bushels.

The second line on this face is the line C on the slide, which is a line of numbers differing nothing from the lines A and B.

NOTE.—On all Rules on which the lines A, B, and C are of single radius, the line D is broken at 3·162, the square root of 10; the first part of the line, viz., from 1 to 3·162 being placed on one side of C, and the other part of it, that is, from 3·162 to 10, being situate on the opposite side of C.

Lines on the Third and Fourth Faces of the Rule.

On these faces are two lines of segments used in finding the ullage of a cask, that is, the quantity of liquor which a cask wants of being full, or the actual quantity of liquor in it when not quite full. These lines are both on the body of the Rule, and alike numbered from left to right with the figures 1, 2, 3, &c. to 10; and from 10 with the figures 20, 30, 40, &c. to 100. One of the lines is marked SL, signifying segment lying; and the other SS, for segment standing. On the first of these faces the line of segments is broken at 4, and on the second at 8. The slide on the third face is called B, and on the fourth C. They are similarly divided, and marked from left to right with the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, at distances in geometrical proportion like the lines A and B.

Underneath the slides are,

1. The gage points, and divisors; as well as the necessary factors for the reduction of liquors of one sort to their equivalent value in another.

2. The malt divisor 2150·42, doubled, trebled, &c. to 9 times, for readily estimating the contents of a floor of malt.

3. A line of inches decimally divided, and numbered from 1 to 12.

4. A line marked *Spheroid*, being a line of differences for reducing casks of the first variety to a mean diameter; and numbered from 1 to 7.

5. A line numbered from 1 to 6, and marked *2nd*

Variety, being a line of differences for reducing casks of that description to a mean diameter.

Besides a Rule of four slides, there are other Rules in use with one, two, or three slides; but these commonly contain only the more useful lines on a Rule of four slides.

Of Estimating the Divisions on the Sliding Rule.

The figures 1, 2, 3, 4, &c. on the lines A and B are all arbitrary, and may be conceived to represent either units, tens, hundreds, thousands, &c., or tenths, hundredths, thousandth-parts, &c. But whatever value is assigned to the first 1 towards the left, the following integral numbers 2, 3, 4, 5, &c. must be taken as twice, thrice, four times, five times, &c. as much: that is, the proportion must always be kept up. For example, if the figure 1 at the beginning of the line represent 10, then 2 will represent 20; or if the figure 1 be considered as $\frac{1}{10}$, then the figure 2 must be considered as $\frac{2}{10}$; and so on for all other values.

The value of the integral divisions being thus estimated, the intermediate divisions will be easily understood; for it is evident that, whatever value is assigned to the difference of any two integral adjoining numbers, each of the sub-divisions between them must be the quotient arising from that difference divided by the number of parts between the two numbers. If, for example, 1 at the beginning of the line A, stand for 1, every subdivision must stand for $\frac{1}{10}$, since the distance between 1 and 2 is on the scale divided into 50 parts; or if 1 at the beginning of the line be assumed as 100, then each subdivision will count for two units, two being the fiftieth part of 100, that is, of the difference between 1 and 2. Lastly, if 1 be supposed to express ten, each of the longer subdivisions will express 1, and each of the others will count for $\frac{2}{10}$.

Enough, it is presumed, has been said to enable the learner to find, on any of the lines, the point corresponding to a given number. For the sake of practice, then, let it be proposed to find on the line D (where the divisions between 1 and 2 are all decimal) the point answering to 18·95.

If 1 at the beginning of the scale stand for 10, then it is manifest that 18·95 will be found between 1 and 2, that is, between 10 and 20. For the first figure of the given number, therefore, count 1 at the beginning of the line, and for 8, the second figure, count 8 of the larger divisions: again for 9 tenths count *nine* of the intermediate or smaller divisions; and lastly, for $\frac{5}{100}$ take the half of the next small division. At this point is fixed a brass pin marked AG, being the circular gage point for ale-gallons.

If it were required to find 189·5, 1·895, ·1895, and 1895, they would all appear at the same point; only in the first instance the value of 1 at the beginning of the line, would be 100; in the second it would be 1; the third, ·1; and in the last, 1000.

In like manner is the point that represents any other number, found upon D, or any of the other lines.

Multiplication by the Lines A *and* B.

The product of two given numbers may be found by those two lines on the Sliding Rule, as follows: To either of the given numbers on A set 1 on B, and against the other number on B is the product on A. This will be sufficiently illustrated by two examples.

EXAMPLE 1.

Let the product of 7 by 9 be required.

Set 1 on B to 7 on A, and opposite to 9 on B you have 63 on A: or set 1 on B to 9 on A, and against 7 on B you again have 63 on A.

Here it may be proper to observe that it frequently happens that, when 1 on B is set to the given number on A, the other number cannot, according to the true numeration of the Rule, be expressed on the line B; or, if it can be expressed, it falls beyond the line A.

In such event it is most convenient, after setting 1 on B, to either of the given numbers on A, to take some decimal quotient of the other number; such as one tenth, one hundredth, one thousandth-part, &c., till such quotient can be found opposite one of the divisions on A. Then the product that arises must be taken 10, 100, 1000, &c. times, according as the given number was divided.

The next example will render this plainer.

EXAMPLE 2.

Required the product of 8·5 by 1400.

Setting 1 on B to 1400 on A, it is found that 8·5 on B, falling beyond the line A, will give no product. Again, setting 1 on B to 8·5 on A, the other number 1400 cannot be found on B, the Rule being of single radius.

Next, setting 1 on B to 1400 on A, and looking for ·85 (the tenth part of the other number) on B, we in like manner cannot find it since its situation is on the left of unity at the beginning of the slide. To remedy this defect the slide must be moved on to the left till 10 on B stands opposite to the number on A at which 1 ought to stand.

This having been done, 10 on B now counting for 1, we readily discover opposite to ·85 on B, 1190 on A, for the tenth part of the product; and consequently the whole product is 11900.

In like manner may any two numbers whatever be multiplied by the Sliding Rule.

Division by the Lines A *and* B.

When one number is given to be divided by another, the quotient may be found by setting 1 on B to the divisor on A, and taking the number on B which stands opposite to the dividend on A.

For example, let 7·6 be given to be divided by 4, and the quotient will be 1·9, as follows:

We set 1 on B to 4 on A, and looking opposite to 7·6 on A, we find 1·9 on B.

Again, let 8·8 be given to be divided by 1·6, and the quotient will be 5·5; for 1 on B being set to 1·6 on A, we have 5·5 on B against 8·8 on A.

The divisor or dividend may, if necessary, be first divided by 10, 100, 1000, &c., provided the quotient found on the slide be afterwards divided or multiplied (as the case may be) by the same number: that is, divided where the divisor was divided, but multiplied for the dividend.

To illustrate this, suppose 8500 given to be divided by 5, and it will be expedient after setting 1 on B, to the divisor on A to look opposite the thousandth part of 8500 on A. Now on B, opposite to 8·5 on A, we find 1·7 as the thousandth part of the quotient, and consequently the whole quotient is 1700.

If 5 were given to be divided by 8500, we should set 1 on B to the thousandth part of the divisor, and then *divide* the quotient found on the slide by 1000.

In working this example it would be necessary to shift the slide as directed in the last example of multiplication.

The Rule of Three Direct on the Lines A *and* B.

When three numbers are given to find a fourth in direct proportion, the method is the following: Set the first term on B to the second on A, then against the third on B is the fourth on A.

We shall illustrate this by the solution of three questions.

QUESTION 1.

The price of 18 gallons of rum is demanded, when 8 gallons sell for 98*s*.

Setting 8 on B to 98 on A, and looking against 18 on B, we find 220½, which being shillings, are equal to £11. 0*s*. 6*d*. the price required.

It matters not on which of the two lines A and B, the first term of the proportion is taken, provided the third term be always on the same line as the first. To prove this, let 8 and 18 in the last example be both taken on A, that is, let 8 on A be set to 98 on B, and opposite to 18 on A, we shall have 220½ on B, as before.

QUESTION 2.

If the duty on 6 bushels of malt be 21 shillings, what is the duty on 28 bushels?

SOLUTION.

on B. on A. on B. on A.
As 6 : 21 : : 28 : 98 = £4. 18*s*. Ans.
bu. *sh.* *bu.* *sh.*

QUESTION 3.

If 144lb. avoirdupois be equal to 175lb. troy, how many lb. troy are equivalent to 1 cwt. avoirdupois?

SOLUTION.

lb. Av. lb. T. lb. Av. lb. T.
As 144 : 175 : : 112 : 136·111 *number required.*
on A. on B. on A. on B.

Many exercises might be proposed for the practice of the learner, but the above will be sufficient, as he will be able to extract abundance of questions in the Rule of Three Direct from any Treatise on Arithmetic, or to set himself, at pleasure, a variety of examples which he can prove by the pen.

The Rule of Three Inverse on the Lines A *and* B.

Proportion is Inverse when three numbers are given to find a fourth which shall have the same ratio to the second as the first has to the third : hence any question in the Rule of Three is inverse when more requires less, or less requires more. For example, if the following exercise be proposed, the proportion is inverse.

If 12 *men can fit up a brewery in* 21 *days, in what time ought* 8 *men to do it ?*

It is evident that, since the first term must be to the third, as the fourth to the second, *directly*, the third will be to the first as the second to the fourth, in direct proportion. Therefore, the question has only to be stated as if direct, and afterwards the third term transplanted; for if the third term be removed and placed before the first, the four terms will be truly in direct proportion.

First, then, it will be,

As 12 : 21 : : 8 : — inversely.

But, transposing the third,

As 8 : 12 : : 21 : — directly.

Wherefore setting 8 on A to 12 on B, we have, opposite to 21 on A, 31½ on B. The result is the same if 8 on A be set to 21 on B, for opposite to 12 on A we find 31½ on B.

In both these proportions the line A may be used for B, and B for A. But in whatever way the problem is solved, we have 31 days and a half for the answer, which may easily be verified with the pen.

In this manner are all questions in the Rule of Three Inverse to be stated, of which any further example here would be superfluous.

Extraction of the Square Root by the Lines C *and* D.

When the lines C and D are applied to one another so that unity at the beginning of the one corresponds to unity at the beginning of the other, the two lines form a table of squares and their roots: for opposite to any number on D stands its square on C, and consequently opposite to any number on C will be found its square root on D.

To extract the square root of any number by these lines, observe the following directions:

DIRECTION I.

For numbers containing an odd *number of integers.*

If the line C be of single radius seek the number on the further edge of it, and opposite on D will be the root: but if C be of double radius, let the number be sought on the first radius, and the root will be against it on D as before.

DIRECTION II.

For numbers containing an even *number of integers.*

If the line C have two radiuses the given number is to be sought on the second, but if C be of one radius the number is to be taken on the hither edge, and the root will in either case stand opposite on D.

EXAMPLES.

$$\text{Against}\left\{\begin{array}{l}4\\121\\342{\cdot}25\\90000\end{array}\right\}\begin{array}{c}\text{on the first ra-}\\\text{dius or further}\\\text{side of C, are,}\\\text{respectively,}\end{array}\left\{\begin{array}{l}2\\11\\18{\cdot}5\\300\end{array}\right\}\begin{array}{c}\text{the roots}\\\text{on D.}\end{array}$$

And,

$$\text{Against}\left\{\begin{array}{l}49\\14{\cdot}44\\2150{\cdot}42\\360000\end{array}\right\}\begin{array}{c}\text{on the second}\\\text{radius or hither}\\\text{side of C, are,}\\\text{respectively,}\end{array}\left\{\begin{array}{l}7\\3{\cdot}8\\46{\cdot}37\\600\end{array}\right\}\begin{array}{c}\text{the roots}\\\text{on D.}\end{array}$$

OF SLIDING RULES HAVING THE LINE E.

On the second face of some Rules the line D slides between two lines, the one marked C, and the other marked E. In this case the line C is of double radius, the line D of single radius, and the line E of triple radius. The numbers on E are respectively the cubes of the numbers on D when unity on the one is set to unity on the other: hence if the beginning of the line on the slide D be set even with the beginning of the lines C and E, the whole forms a table of cubes, roots and squares; that is, on E are the cubes, on C the squares, and on D the roots.

Extraction of the Cube Root by the Lines D *and* E.

If unity at the beginning of D be set to unity at the beginning of E, the cube root of any number on E may be found by inspection on D. Yet there is some difficulty in knowing on which of the three radiuses on E the given cube should be taken.

To obviate this difficulty observe the following directions:

DIRECTION I.

If the given number consists of 1, 4, or 7 places of integers, find it on the first radius of E, and opposite to it will be its cube root on D.

DIRECTION II.

If the given number consists of 2, 5, or 8 places of integers, look for it on the second radius of E, and you have the cube root against it on D.

DIRECTION III.

If the given number consists of 3, 6, or 9 places of integers, it must be found on the third radius of E, and directly against it on D, will, as in the other cases, be seen the root required.

EXAMPLES.

	Cubes.		Roots.	
Against	6·859 1728 9261000	on the first radius of E, are, respectively,	1·9 12 210	on D.

And,

	Cubes.		Roots.	
Against	27 15625 27000000	on the second radius of E, are, respectively,	3 25 300	on D.

Also,

	Cubes.		Roots.	
Against	166·375	on the third	5·5	on D.
	838375	radius of E, are,	95	
	729000000	respectively,	900	

After the manner of these examples may the square or cube roots of any number be extracted with sufficient accuracy on the lines C, D, and E; but it is obvious that, to arrive at expedition and certainty, practice is necessary.

Reduction of a Vulgar to a Decimal Fraction, by the Sliding Rule.

Set 1 on B to the denominator on A, and opposite to the numerator on A will be the decimal required on B.

The reason of this will be apparent if we reflect on what has been said under the head of Division, and on the method of reducing a vulgar fraction to a decimal by the pen.

But it is evident that in reducing any proper fraction to a decimal, the quotient will not be found upon B, if 1 at the beginning of the line be set to the denominator on A: hence it is necessary to set 10 on B to the denominator on A, taking $\frac{1}{10}$th part of the quotient for the decimal required.

In illustration let the following examples be proposed:

EXAMPLE 1.

Reduce $\frac{3}{4}$ to a decimal fraction.

Setting 10 on B to 4 on A, we may observe opposite to 3 on A, 7·5 on B; now 7·5 divided by 10 becomes ·75 *the decimal required.*

EXAMPLE 2.

Reduce $\frac{1}{25}$ to a decimal fraction.

Setting 10 on B to 25 on A, and looking against 1 on A, we see ·4 on B; but ·4 divided by 10 becomes ·04 *the decimal required.*

EXAMPLE 3.

Reduce $\frac{9}{14}$ to a decimal fraction.

Here if we set 10 on B to 14 on A, we cannot find 9 on A, unless the Rule be of double radius. To obviate this inconvenience, let us multiply both numerator and denominator by 2, and the fraction will become $\frac{18}{28}$. Then setting 10 on B to 28 on A, and looking opposite 18 on A, we have 6·43 on B. Now the *tenth* part of 6·43 is ·643 *the decimal equivalent to* $\frac{9}{14}$.

In like manner may any other vulgar fraction be reduced to its equivalent decimal by the Sliding Rule.

OF FACTORS.

When a divisor and dividend are given, it is sometimes convenient to perform the operation by means of a multiplier that shall by multiplication give the same result, that the given divisor would give by division. For instance, if 525 were given to be divided by 25, the result will be the same whether 525 be divided by 25 or multiplied by the decimal corresponding to $\frac{1}{25}$, which has been found in the second example above to be ·04.

A multiplier of this description is in Gaging technically called a FACTOR.

To find a FACTOR *by the Lines* A *and* B.

From the definition we have given of a FACTOR, it is manifest that the proportion will be, *As the given divisor is to unity, so is unity to the factor sought.* Wherefore to ascertain the fourth term of this proportion by the Sliding Rule, we must set the given divisor on A to unity on B, and opposite to unity on A will be the factor on B.

In treating of Division and Reduction of Vulgar to Decimal Fractions, enough has been said to preclude the necessity of further explanation of the particular cases that may occur, since if sufficient attention has been paid to the preceding examples, every difficulty will vanish. The exercises following will, however, serve for practice.

EXAMPLE 1.

Required the factor corresponding to 125.

The proportion is,

on A. on B. on A. on B.
As 125 : 1 : : 1 : ·008 *the factor required.*

In solving this example we count 1 at the beginning of A for 100; then 125 are two of the large, and two and a half of the small divisions on from the 1; which 125 being set to 10 on B, we observe that the lowest number on A is 100; and opposite thereto we see 8 on B: wherefore dividing 8 by 100 and also by 10, that is in all by 1000, the quotient is ·008 *the factor required.*

To prove the truth of the result let any dividend whatever be named, for instance, 2150·42, and it will be,

By Division.	By Multiplication.
125)2150·42000(17·20336	2150·42
125	·008
900	17·20336
875	
254	
250	
420	
375	
450	
375	
750	
750	
...	

EXAMPLE 2.

Required the factor corresponding to 183·34.

The proportion is,

on A. on B. on A. on B.
As 183·34 : 1 : : 1 : ·005454 *the factor sought.*

To Find by the Lines A *and* B, *the Divisor Corresponding to a Given Factor.*

This being the converse of the last problem, the proportion will be, *As the given factor is to unity, so is unity to the divisor required.*

Let, for the sake of illustration, the divisor corresponding to ·008 be demanded, and the proportion will be as follows:

on A. on B. on A. on B.
As ·008 : 1 : : 1 : 125 *the divisor required.*

In solving this example we multiply the given factor ·008 by 1000, and it becomes 8. We then set 8 on A to 1000 on B, and look opposite to 1 on A, for 125 on B.

In a similar way may be found the divisor that answers to any given factor whatever.

As the art of Gaging is a branch of the Mathematics as much depending on Geometry as on Number, we deem it proper as the next introductory step, to discuss briefly such parts of Geometry as are more fundamentally connected with the subject.

GEOMETRY.

GEOMETRY treats of points, lines, superficies, and solids; to have a clear conception of which, it is necessary first to define the sense applied to the terms.

GEOMETRICAL DEFINITIONS.

1.

A point is an inconceivably small portion of space, being position without any magnitude.

2.

A line is length only, and generated by the motion of a point.

3.

An area or superficies has length and breadth, but no thickness.

4.

A solid has length, breadth, and thickness.

5.

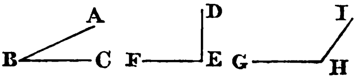
A line joining any two points is said to be straight when it is the shortest possible.

6.

A plane is a superficies in which no two points can be taken such that, the straight line joining them does not coincide entirely with that superficies.

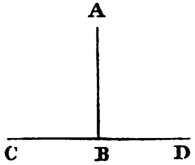
7.

A rectilineal angle is the meeting of two straight lines in a point, with any degree of inclination to one another; as ABC, or DEF, or GHI.



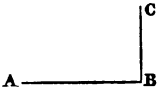
8.

A line is said to be perpendicular to another when it stands on it in such manner as to make the angle on one side equal to that on the other; as AB on CD.



9.

A right angle is formed by two lines standing perpendicular to one another; as ABC.

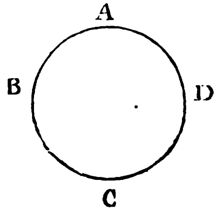


10.

A geometrical figure is any space inclosed by one or more boundaries, and is either superficial or solid—superficial when the boundary consists of a line or lines, and solid when the figure has thickness.

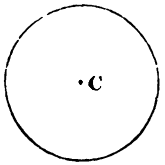
11.

A circle is a figure bounded by one uniformly curve line called the circumference, lying wholly in the same plane; as ABCD.



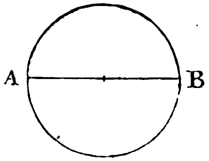
12.

The centre of a circle is the point which, in the plane of the circumference, is equidistant from every part of the curve; as C.



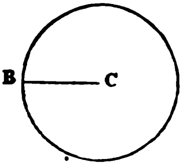
13.

The diameter of a circle is a line drawn through the centre, and terminated both ways by the circumference; as AB.



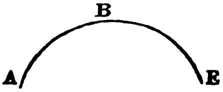
14.

The radius of a circle is half the diameter; as BC.



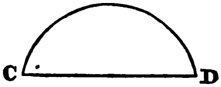
15.

Any part of the circumference of a circle is called an arc; as ABE.



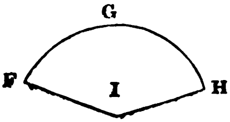
16.

A chord is a straight line joining the extremities of an arc; as CD.



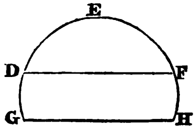
17.

A sector is the figure inclosed by two radii and the part of the circumference between them; as FGHI.



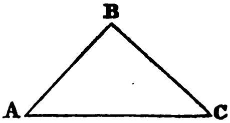
18.

A segment of a circle is the figure inclosed by an arc and its chord; as DEF, or GEH.



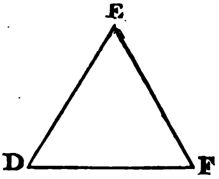
19.

A plane triangle is bounded by three straight lines, two of which are called sides, and the other the base: sometimes, however, all three are termed sides; as ABC.



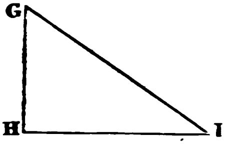
20.

An equilateral triangle has its sides and base equal; as DEF.



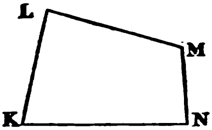
21.

A right-angled triangle has one of its sides perpendicular to the other; as GHI.



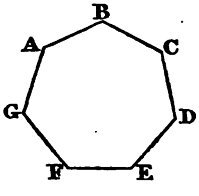
22.

Any figure wholly in the same plane, and bounded by four straight lines, is called a quadrilateral; as KLMN.



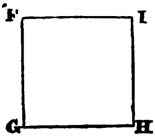
23.

Polygons are figures bounded by more than four straight lines all in the same plane, and are termed regular when their angles are equal; as ABCDEFG.



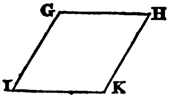
24.

A square is a four-sided figure whereof all the angles are right angles, and all the sides equal, in the same plane; as FGHI.



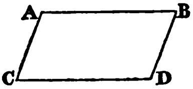
25.

A rhombus is a four-sided figure bounded by equal straight lines, but its angles are not right angles; as GHIK.



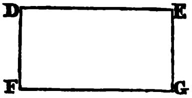
26.

A parallelogram is a quadrilateral whereof the opposite sides are equal; as ABCD.



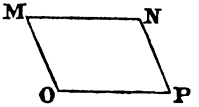
27.

A right-angled parallelogram is termed a rectangle; as DEFG.



28.

Every parallelogram which is neither a rectangle nor rhombus is called a rhomboid; as MNOP.



29.

By the area of a figure is meant the quantity of plane space actually inclosed by the linear boundary of the figure.

30.

By superficial content is understood the quantity of surface without reference to depth.

31.

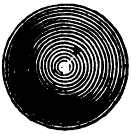
By solidity is meant the quantity of actual space inclosed by the superficies bounding a solid figure.

32.

The measuring unit of surface is the square inch; and of solidity, the cubic inch.

33.

A sphere or globe is the solid figure generated by the revolution of a semicircle about its diameter.

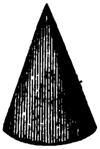


34.

A cylinder is the solid figure generated by the revolution of a rectangle about one of its sides.

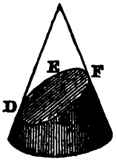
35.

A cone is the solid figure generated by the revolution of a right-angled triangle, about either of the sides containing the right angle.



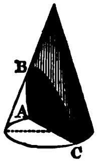
36.

An ellipsis is the plane figure formed by the section of a cone when cut obliquely through its axis by a plane not meeting the base; as DEFG.



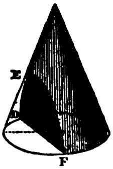
37.

A parabola is the plane figure formed by the section of a cone when cut by a plane parallel to the side; as ABC.



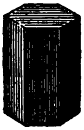
38.

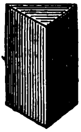
An hyperbola is the plane figure formed by the section of a cone when cut by a plane parallel to the axis ; as DEF.



39.

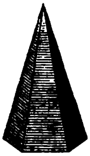
Prisms are solid figures having rectangular planes for their sides, and regular, that is, equilateral and equiangular planes for their ends.





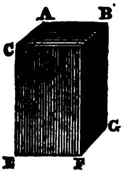
40.

A pyramid has for its sides equal and similar triangles similarly situated, and for its base any equilateral and equiangular plane.



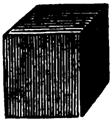
41.

A parallelopipedon is a solid figure contained under six parallelograms, the opposite sides being equal, and all its angles right-angles; as ABCDEFG.



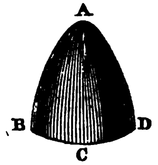
42.

A cube is a parallelopipedon with six equal sides.



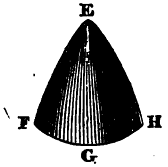
43.

A parabolic conoid is the solid generated by the revolution of a semi-parabola about its axis; as ABCD.



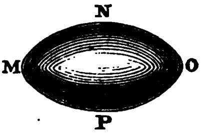
44.

A hyperbolic conoid is the solid generated by the rotation of a semi-hyperbola about its axis; as EFGH.



45.

A parabolic spindle is the solid generated by the revolution of a semi-parabola about its ordinate or base; as MNOP.



46.

A spheroid is the solid generated by the rotation of a semi-ellipsis, about either diameter. In the following figure the rotation is about the transverse diameter.

Before we treat of the mensuration of surfaces and solids, it will be useful to give a few problems in Practical Geometry; and these we shall divide into Five Sections.

PRACTICAL GEOMETRY.

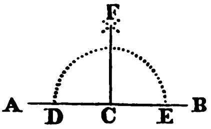
SECTION I.

OF DRAWING AND DIVIDING LINES.

PROBLEM I.

To erect a perpendicular from a given point in a given straight line.

Let AB be the given line, and C the given point in it, from which a perpendicular is to be erected.

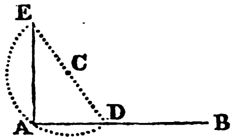


From C as a centre, with any convenient opening of the compasses, describe the semicircle DE; then from D and E as centres, with a radius greater than DC, describe two arcs intersecting each other in the point F; and draw CF the perpendicular required.

PROBLEM II.

To erect a perpendicular on the extremity of a given right line.

Let AB be the given right line, and A the extreme point from which a perpendicular is to be erected.

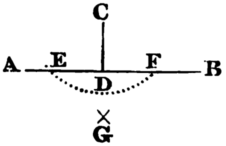


Take at pleasure between A and B, but without the line, and on the side of it on which the perpendicular is to be drawn, the point C, nearer to A than to B. Next, from C as a centre, with the opening CA, describe an arc cutting AB in D, and carried so far beyond A, as that EAD may be a semicircle. Through the points D and C draw the straight line DCE, and then from A to E let the required perpendicular be drawn.

PROBLEM III.

To let fall a perpendicular upon a given right line from a point without it.

Let AB be the given line, and C the point without it, from which the perpendicular is to fall.

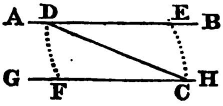


Take any point D on the other side of AB, and with the opening CD, describe from C the arc EF, cutting AB in the two points E and F. Then from E and F as centres, with any radius greater than the half of EF, describe two arcs intersecting each other in G. Lastly, lay a ruler from C to G, and draw the perpendicular CD.

PROBLEM IV.

To draw a straight line parallel to a given straight line, through a given point without it.

Let AB be the given straight line, and C the given point without it.



In AB take any point D, and draw the straight line

CD. Then from C and D as centres with the opening CD, describe CE, DF; making the arc DF equal to the arc CE. Lastly, through C and F draw the straight line GH, and it will be parallel to AB.

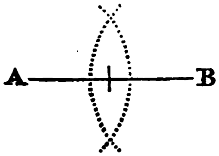
Note on the first four Problems.

Perpendicular lines are more expeditiously drawn in practice by means of a scale of box-wood or of ivory, with a square end: and parallel lines are best made with a parallel ruler.

PROBLEM V.

To bisect a given finite straight line, that is, to divide it into two equal parts.

Let AB be a given finite straight line to be divided into two equal parts.

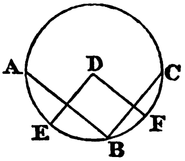


From A as a centre, with any opening greater than the half of AB, describe an arc; and from B as a centre, with the same radius, describe a similar arc intersecting the former in two points. Then draw a straight line from one of the points of intersection to the other, and that line will bisect AB.

PROBLEM VI.

To find the centre of a given circle or circular arc.

Let ABC be the arc of a circle, whereof the centre is required.

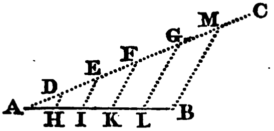


Draw in the arc any two chords making an angle with one another as AB, BC: then bisect these chords by the perpendiculars DE, DF. The point D where the perpendiculars intersect one another is the centre required.

PROBLEM VII.

To divide a given right line into any proposed number of equal parts.

Let AB be a right line given to be divided into five equal parts.

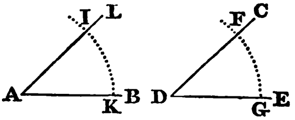


From the point A draw an indefinite straight line AC making an angle with AB. Then from A lay off any convenient distance, as AD, five times towards C, which distances are represented by D, E, F, G, and M. From M to B draw the straight line MB, and parallel to MB through the points D, E, F, and G, draw the straight lines DH, EI, FK, and GL, dividing AB into five equal parts.

PROBLEM VIII.

To draw from the extremity of a given straight line, a straight line making an angle with the former, equal to a given rectilineal angle.

Let AB be the given straight line, and CDE the given rectilineal angle; and let A be the extremity from which the line is to be drawn.



From D as a centre with any convenient opening of the compasses, describe the arc FG; and from A as a centre with the same radius describe the arc IK. Make the arc IK equal to the arc FG, and through the points A and I, draw the straight line AL, which will be the line required.

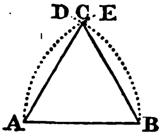
SECTION II.

CONSTRUCTION OF PLANE FIGURES.

PROBLEM I.

To describe an equilateral triangle upon a given finite straight line.

Let AB be the given line upon which it is required to describe an equilateral triangle.

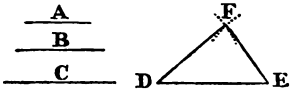


From A as a centre, with the opening AB, describe the arc BD; and from B as a centre, with the opening BA, describe the arc AE cutting the former in C. Lastly draw the straight lines CA and CB.

PROBLEM II.

To construct a triangle whereof the three sides shall be respectively equal to three given straight lines, provided every two of them be greater than the third.

Let the three given straight lines be A, B, and C, whereof A and B are greater than C; B and C, greater than A; and A and C greater than B.

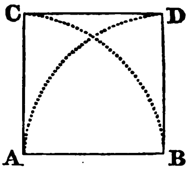


Draw a straight line DE, equal to C the longest of the given lines, and from D as a centre, with a radius equal to the line B describe an arc; then from E as a centre with a radius equal to the line A, describe another arc cutting the former in F. Finally draw the straight lines FD and FE.

PROBLEM III.

To describe a square upon a given finite right line.

Let AB be the given right line upon which it is required to describe a square.

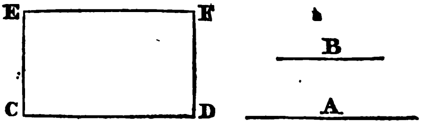


From A as a centre, with the opening AB, describe an arc, and from B as a centre, with the distance BA, describe another arc. Then from the point A raise the perpendicular AC meeting the first arc in C, and from B draw BD parallel to AC, and meeting the second arc in D. Lastly draw the straight line CD.

PROBLEM IV.

To describe a rectangle whereof the length and breadth shall be respectively equal to two given right lines.

Let the proposed length of the figure be A, and the breadth B.

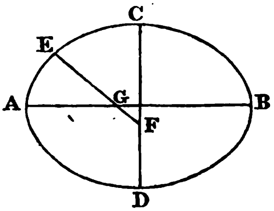


Draw the straight line CD equal to A the given length, and from C raise the perpendicular CE equal to the proposed breadth B. Then draw DF equal and parallel to CE, and join EF.

PROBLEM V.

To describe an oval, having given the transverse and conjugate diameters.

Let it be required to describe an oval or ellipsis of which the transverse diameter shall be to the conjugate as 117 to 88.

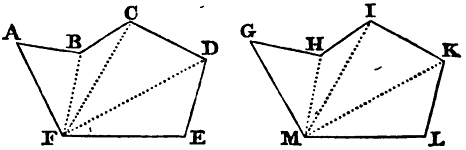


Draw a straight line AB equal to 117 on any convenient scale of equal parts, and having bisected it, draw through the point of bisection another right line CD at right angles to the former ; and let its point of bisection coincide with the point of bisection of AB, the whole line CD being equal to 88 on the scale employed for the first line. Then procure a ruler in length equal to the half of AB, which ruler let be EF ; and from E towards F mark EG equal to the half of CD. Now to describe the oval apply the ruler to the lines AB and CD, so that the point G may be always in AB, and the extremity F in CD, till E has made a complete revolution: the path of E will be the curve required.

PROBLEM VI.

To describe a rectilineal figure equal and similar to a given rectilineal figure.

Let ABCDEF be the given figure to which an equal and similar figure is to be constructed.



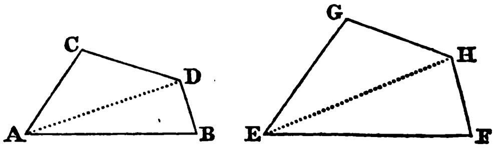
Divide the given figure into triangles by drawing from the angle F the straight lines FB, FC, and FD. Then construct a triangle MKL having its three sides respectively equal to the sides of the triangle FDE. Next, from the point M, with the opening FC, describe an arc, and cut it from K with the opening DC. I is the point of intersection of these two arcs. Again, from M as a centre, with the opening FB, describe an arc, and cut it in H from I as a centre with the opening CB. Likewise from M as a centre, with the opening FA, describe an arc, and cut it in the point G from H as a centre with the opening AB. Lastly draw the straight lines MG, GH, HI, IK, and the thing required is done.

In like manner may any other rectilineal figure be exactly represented.

PROBLEM VII.

To construct upon a given base a rectilineal figure similar to a given rectilineal figure.

Let the given rectilineal figure be ABCD, and the given base EF.



From A draw the straight line AD dividing the given figure into triangles, and from E draw the right line EH making with EF the angle HEF equal to the angle DAB. Also from F draw FH making with EF the angle HFE equal to the angle DBA. The triangle HEF is similar to the triangle DAB. Again, draw EG making the angle GEH equal to the angle CAD, and draw HG making the angle GHE equal to the angle CDA. The figure EGHF has its angles respectively equal to the angles of the figure ACDB, and is consequently similar to it.

In the same way, upon a given base may be constructed a rectilineal figure, that shall be similar to any other rectilineal figure whatever, by dividing the given figure into triangles.

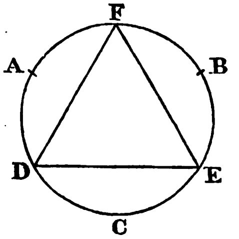
SECTION III.

OF INSCRIBING ONE FIGURE IN ANOTHER.

PROBLEM I.

To inscribe an equilateral triangle in a given circle.

Let ABC be the circle in which an equilateral triangle is to be inscribed.

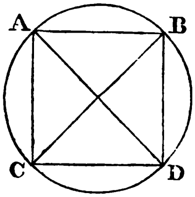


From any point in the circumference, as C, with the radius of the given circle, describe an arc cutting the circumference of the given circle in two points, D and E. From D to E draw the straight line DE, and it will be the side of the triangle required. If, therefore, from D as a centre with the opening DE, an arc be described cutting the given circumference in F, we have only to draw a straight line from D to F, and another from E to F, to complete the triangle.

PROBLEM II.

To inscribe a square in a given circle.

Let ABCD be a circle in which it is required to inscribe a square.

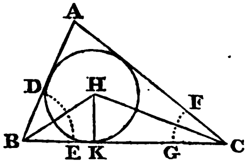


Draw the two diameters AD and BC, intersecting each other at right angles. Then join AB, BD, DC, and CA ; the figure ABCD is the square required.

PROBLEM III.

To inscribe a circle in a given triangle.

Let ABC be the triangle in which it is required to inscribe a circle.



About the angular points B and C, with any convenient radius, describe the arcs DE and FG. Then draw the straight lines BH and CH, respectively bisecting the arcs through which they pass, and meeting in H. From H let fall a perpendicular, as HK, upon either of the sides of the triangle: and lastly, from H as a centre, with the opening HK, describe the required circle.

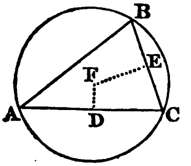
SECTION IV.

OF CIRCUMSCRIBING ONE FIGURE ABOUT ANOTHER.

PROBLEM I.

To circumscribe a circle about a given triangle.

Let ABC be the triangle about which it is required to circumscribe a circle.

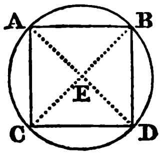


Bisect any two of the sides, as AC and CB; then from D and E, the points of bisection, raise the perpendiculars DF and EF, intersecting one another in the point F. Lastly, form F, with the opening FA, describe a circle about the given triangle.

PROBLEM II.

To circumscribe a circle about a given square.

Let ABCD be the given square.

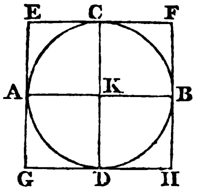


Draw the diagonals AD, BC, intersecting each other in E, and from E as a centre, with EA for radius, describe the circle required.

PROBLEM III.

To circumscribe a square about a given circle.

Let the circle to be circumscribed with a square be ABCD.



Draw the diameters AB, CD, intersecting each other at right angles; and from the points A, C, B, D, as centres, with AK for radius, describe semi-circles meeting one another in the points E, F, G, and H. Lastly, draw the straight lines EF, FH, HG, and GE.

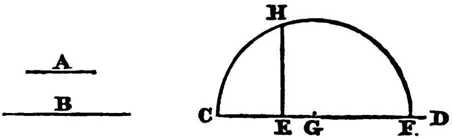
SECTION V.

OF DRAWING PROPORTIONAL LINES.

PROBLEM I.

To find a mean proportional between two given lines.

Let A and B be the lines between which a mean proportional is to be found.

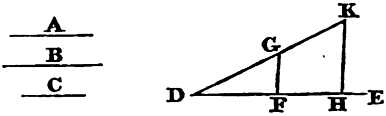


Draw the indefinite straight line CD, and from C set off CE equal to A. Then from E set off EF equal to B, and bisect CF in G. From G as a centre, with the opening GC, describe a semicircle as CHF; and lastly, raise the perpendicular EH, which will be the mean sought.

PROBLEM II.

To find a fourth proportional to three given lines.

Let the three lines to which a fourth proportional is to be found be A, B, and C.

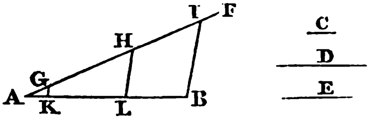


Make at pleasure the rectilineal angle KDE, and from D towards E lay off DF equal to A. Also from D towards K lay off DG equal to B. Again, from F towards E lay off FH equal to C. Join FG, and through the point H draw HK parallel to FG. The line GK is the fourth proportional required.

PROBLEM III.

To divide a given right line in any proposed ratio.

Let AB be the given right line to be divided in the ratios C, D, E.



Draw any line AF making an angle with AB at the point A; and from A towards F set off AG equal to C, GH equal to D, and HI equal to E. Join IB, and parallel to IB, draw through the points G and H, the straight lines GK, HL, dividing AB as required.

Having now offered such problems in Practical Geometry as seem more intimately connected with our subject, we shall proceed to shew how the several varieties of Superficies and Solids are measured, and how the measures are more particularly applied to GAGING.

OF MENSURATION.

MENSURATION is a branch of the Mathematics, shewing how the CONTENT of every kind of Superficies, and of Solid, is accurately determined.

Of superficial measure the lowest denomination is the square inch; and of solid measure the least integer is the cubic inch.

Square inches are reducible to feet, yards, poles, roods, acres, miles, and leagues; which have severally surface without thickness.

Cubic inches may likewise be reduced to cubic feet, cubic yards, cubic miles, &c.; but in Gaging they are more frequently converted into gallons or bushels.

Measure barely superficial forms no part of the business of a Gager, except in as far as he is concerned with the duty on paper, therefore, in speaking of areas, he invariably pre-supposes the depth to be uniformly an inch, thus blending surface with solidity.

To render the subject of MENSURATION as complete as possible, we shall, in order not to confound superficial with solid measure, divide it into Three Sections; whereof the *First* will treat of surface only, that is, of area without thickness, and throughout this section the square inch will constitute the measuring unit.

The *Second Section* will be a sort of recapitulation of the First, but in which all areas will be considered as they are understood in Gaging, namely, an inch deep. Here the measuring unit will be the solid or cubic inch, always reduced to one or more of the Excise measures or weights, such as the ale-gallon, the wine-gallon, the malt bushel, the dry-starch pound, &c.

The *Third Section* will be exclusively on Solids.

SECTION I.

OF THE AREAS OF PLANE FIGURES.

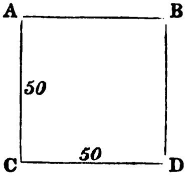
PROBLEM I.

To find the Area of a Square.

RULE.

Multiply the side by itself, and the product will be the area.

Let ABCD be a square, of which the side is 50 inches; the area is required.



SOLUTION.

50 side of the square
50 side

Product 2500 square inches, *area sought.*

BY THE SLIDING RULE.

Set 1 on B to 50 on A, and opposite to 50 on B will be found 2500 on A, for the area of the square.

In like manner if the side were 109·2 inches, it would be by the pen,

109·2 side of the square.
109·2 side

2184
9828
10920

Product 11924·64 square inches, *area required.*

BY THE SLIDING RULE.

If 1 on B be set to 109·2 on A, we have for the area 11924·6 on A, against 109·2 on B.

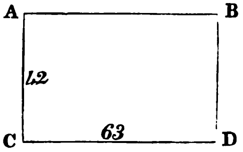
PROBLEM II.

To find the Area of a Rectangle.

RULE.

Multiply the length by the breadth, and the product will be the area.

Let ABCD be a rectangle whereof the length is 63 inches, and the breadth 42 inches; the area is required.



SOLUTION.

63 length
42 breadth

126
252

Product 2646 square inches, *area of the rectangle.*

BY THE SLIDING RULE.

Set 1 on B to 63 on A, and against 42 on B you will find 2646 on A, for the area of the parallelogram.

In the same way if the length were 30·5 and the breadth 20·9, it would be by the pen,

30·5 length
20·9 breadth
2745
6100
Product 637·45 *area required.*

BY THE SLIDING RULE.

Let 1 on B be set to 30·5 on A, and opposite to 20·9 on B, there will be found 637·45 on A, for the area.

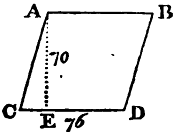
PROBLEM III.

To find the Area of a Parallelogram not Rectangular.

RULE.

Multiply the base by the perpendicular let fall upon it from the opposite side, and the product will be the area.

Let A BCD be either a rhombus or rhomboid, whereof the base measures 76 inches, and the perpendicular from the base to the opposite side 70 inches; the area is required.



SOLUTION.

76 base of the parallelogram
70 perpendicular from the opposite side

Product 5320 square inches, *area required.*

BY THE SLIDING RULE.

Set 1 on B to 76 on A, and opposite to 70 on B you will have 5320 on A.

If the base had been 83·2, and perpendicular 30, it in like manner would have been by the pen,

83·2 base.
30 perpendicular

Product 2496·0 *area sought.*

BY THE SLIDING RULE.

When 1 on B is set to 83·2 on A, the number 2496 stands on A against 30 on B.

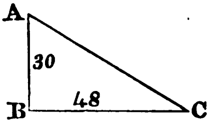
PROBLEM IV.

To find the Area of a Right-angled Triangle.

RULE.

Half the product of the two sides containing the right angle, will be the area. To obtain which, either multiply half the one by the whole of the other; or multiply the one by the other, and divide by 2.

Let ABC be a triangle right-angled at B, whereof the sides containing the right angle are respectively 30 inches, and 48 inches; the area is required.



SOLUTION.

First way.	Second way.	Third way.
48 one side	24 half the base	48 base
15 half the other	30 perpendicular	30 perpendicular
720 product.	720 product.	2) 1440 product
		720 half product.

Wherefore the area of the given triangle is 720 square inches.

BY THE SLIDING RULE.

Set 1 on B to 48 on A, and against 15 on B you will find 720 on A, for the area of the triangle.

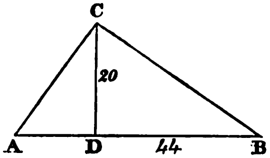
PROBLEM V.

To find the Area of any Triangle by a Perpendicular let fall upon the longest Side from the opposite Angle.

RULE.

Half the product of the base and perpendicular will be the area; to obtain which proceed as in the last problem.

Let ABC be a triangle, whereof the base AB measures 44 inches, and a perpendicular CD let fall upon it from the opposite angle measures 20 inches: the area is required.



SOLUTION.

44 base
10 half the perpendicular
440 square inches, *area.*

BY THE SLIDING RULE.

Unity on B being set to 44 on A, we have 440 for the area on A, against 10 on B.

As an additional example suppose the base 148·6, and the perpendicular 51; the work will be as under—

74·3 half the base
51 perpendicular

74·3
51

743
3715

3789·3 product and *area.*

BY THE SLIDING RULE.

Set 1 on B to 74·3 on A, and against 51 on B you have 3789·3 on A, for the area.

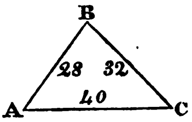
PROBLEM VI.

To find the Area of a Triangle without a Perpendicular, having the three Sides given.

RULE.

Add the three sides together, and from half the sum subtract each side separately, noting the remainders. Then multiply into one continued product the half sum and three remainders. The square root of this product will be the area.

Let ABC be a triangle, whereof the side AB measures 28 inches; the side BC, 32 inches; and the base AC, 40 inches: the area is required.



SOLUTION.

28 first side
32 second side
40 third side

2) 100 sum

50 half sum.

50 — 28 = 22 first remainder
50 — 32 = 18 second remainder
50 — 40 = 10 third remainder.

And $50 \times 22 \times 18 \times 10 = 198000$, whereof the square root is 445 square inches NEARLY, *the area of the given triangle.*

In like manner if the three sides were respectively 30 inches, 40 inches, and 50 inches, it would be,

$$\begin{array}{rrl} & 30 & \text{first side} \\ & 40 & \text{second side} \\ & 50 & \text{third side} \\ \hline 2) & 120 & \text{sum} \\ \hline & 60 & \text{half sum.} \\ \hline \end{array}$$

$60 - 30 = 30$ first remainder
$60 - 40 = 20$ second remainder
$60 - 50 = 10$ third remainder.

And $60 \times 30 \times 20 \times 10 = 360000$, whereof the square root is 600 square inches for the *area of the triangle.*

This last example admits of a different mode of solution, and consequently the result may be verified; for it will be found, on constructing the figure, that the sides 30 and 40 contain a right angle.

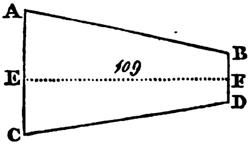
PROBLEM VII

To find the Area of a Quadrilateral wherein two unequal Sides are Parallel to one another.

RULE.

Multiply half the sum of the parallel sides by the perpendicular distance between them, and the product will be the area.

Let ABCD be a quadrilateral, wherein AC and BD are parallel but unequal; and let EF, the perpendicular distance between them, be 109 inches, AC being 67 inches, and BD, 27 inches; the area is required.



SOLUTION.

67 side AC.
27 side BD.
2) 94 sum
47 half sum or *mean* breadth
109 perpendicular EF.
423
470
Product 5123 square inches, *area sought*.

BY THE SLIDING RULE.

Set 1 on B to 109 on A, and opposite to 47 on B, is the area 5123 on A.

PROBLEM VIII.

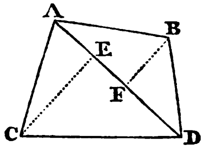
To find the Area of any Right-lined Multilateral Figure.

RULE.

Divide the given figure into triangles by diagonal lines, and find the area of the several triangles; for their sum will be the area of the given figure.

When the method of perpendiculars is employed, and both perpendiculars can be made to fall on the same diagonal line for base, the half-sum of the perpendiculars, multiplied by the diagonal, will give the area of the trapezium.

Let ABCD be a trapezium or four-sided figure, whereof the area is required.



Draw the diagonal AD, which let be 50 inches, and upon it, from the angles C and B, demit the perpendiculars CE, BF, whereof the former, we shall suppose, measures 32 inches, and the latter 20 inches.

SOLUTION.

32 perpendicular CE.
20 perpendicular BF.

2) 52 sum

26 half sum
50 diagonal AD.

Product 1300 square inches, *area required.*

BY THE SLIDING RULE.

Set 1 on B to 50 on A, and opposite to 26 on B will be found 1300, *the area*, on A.

If the triangles be worked separately, the result will be the same; for,

$16 \times 50 =$ 800, area of ACD; and
$10 \times 50 =$ 500, area of ABD.

Sum 1300 square inches, as before.

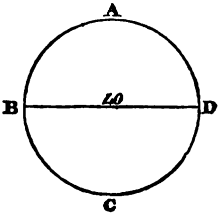
PROBLEM IX.

To find the Area of a Circle.

RULE.

Multiply the square of the diameter by the decimal ·7854, and the product will be the area.

Let ABCD be a circle, whereof the diameter is 40 inches; the area is required.



SOLUTION.

·7854 constant decimal
$40 \times 40 = 1600$ square of the diameter

4712400
7854

1256·6400 square inches, *area required.*

BY THE SLIDING RULE.

Set ·7854 on C, to unity on D, then against 40, the diameter, on D, you will have 1256·6 on C, for the area.

When the Rule has been thus set, we have a Table of areas and diameters, for opposite any diameter on D, appears the area on C, and opposite any area on C, stands the diameter on D. The young practitioner must here bear in mind that the numbers on the hither edge of C are always 10 times the same number on the further edge.

In working the example before us it is necessary to shift the slide by moving 1 on C to the mark at which 10 should properly stand.

NOTE. When any number on C, is multiplied or divided by 10, the corresponding number on D, must be multiplied or divided by 100, the square of 10.

Again, if the diameter of a circle were 100, and the area were required, it would be by the pen,

100	·7854 constant decimal
100	10000 square of the diameter
10000	7854·0 *area.*

BY THE SLIDING RULE.

Set ·7854 on C, to 1 on D, and you will have 7854 on C, against 100 on D, but it will be first necessary to shift the slide.

The area found by this Rule, although sufficiently correct for practice, is only an approximation to the true area of the circle. If instead of the decimal

·7854, we make use of the decimal ·7853981633974483, the product will be scrupulously near the truth, but still not exact; for although it can easily be demonstrated that the area of every circle is *precisely* equal to half the diameter multiplying half the circumference, yet, as no finite number can express the ratio that the diameter and circumference bear to one another, the true quadrature of the circle is impossible.

If the diameter of a circle be 1, the circumference is 3·1415926535897932384626433832795 &c. *ad infinitum.*

Since circles are similar plane figures, they are to one another as the squares of their diameters, each being equal to a parallelogram having half its own circumference for length, and its own radius for breadth.

Hence when the diameter is unity or 1, the area is ·78539 &c., and when the diameter is 2, the area is 3·14159 &c., being four times as great, as may be shewn in the manner following:

The diameter being unity,

Half the circumference is 1·570796 &c.
Half the diameter is ·5

Product and *area* ·7853980

Again,

The diameter being 2,

1 × 1 = 1, square of the first diameter
2 × 2 = 4, square of the second diameter.

But,

As 1 : 4 : : 785398 : 3·141592, the area of the second circle.

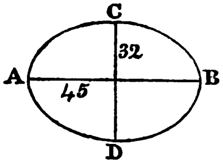
PROBLEM X.

To find the Area of an Ellipsis.

RULE.

Multiply the transverse by the conjugate diameter, and the product by ·7854 for the area.

Let ABCD be an ellipsis, whereof the transverse diameter AB is 45 inches, and the conjugate diameter CD, 32 inches; the area is required.



SOLUTION.

45	transverse diameter
32	conjugate diameter
90	
135	
1440	product of the diameters
·7854	constant decimal
5760	
7200	
11520	
10080	
1130·9760	square inches, *area of the oval.*

If the square of the mean proportional between the diameters be multiplied by the constant decimal ·7854, the product will in like manner be the area; for every ellipsis is a geometrical mean between its inscribed and circumscribing circles.

BY THE SLIDING RULE.

Set 45 on C, to the same number on D, and against 32 on C, will be found 38 on D, for the mean proportional between the diameters.

Then set ·7854 on C, to unity on D, and you will have, for the area, 1130·98 on C, opposite to 38 on D.

PROBLEM XI.

To find the Area of a regular Polygon.

RULE.

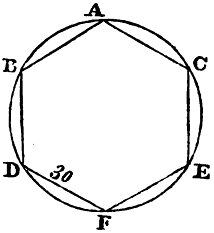
Multiply the square of the side by the number corresponding to the figure in the following Table.

TABLE OF REGULAR POLYGONS.

Number of Sides.	Names of the Polygons.	Area, the side being 1.
5	Pentagon	1·720
6	Hexagon	2·598
7	Heptagon	3·634
8	Octagon	4·828
9	Nonagon	6·182
10	Decagon	7·694
11	Undecagon	9·365
12	Duodecagon	11·196

EXAMPLE.

Let ABCDEF be a hexagon, whereof the side is 30 inches; the area is required.



SOLUTION.

30	side
30	side
900	square of the side
2·598	tabular number for hexagons
2338·2	square inches, *area required.*

BY THE SLIDING RULE.

Set the tabular number 2·598 on C, to 1 on D, and opposite to 30 on D, look for the area on C, which you will find to be 2338-2 square inches, as before.

It is evident that the area of any polygon may be found by dividing the figure into triangles, and then letting fall perpendiculars, as already directed for right-lined multilateral plane figures.

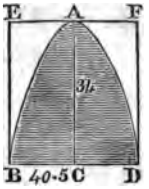
PROBLEM XII.

To find the Area of a Parabola.

RULE.

Multiply the base or ordinate by the perpendicular or axis, and *two-thirds* of the product will be the area: for every parabola is equal to *two-thirds* of its circumscribing rectangle, and on this property is the Rule founded.

Let ABCD be a parabola, whereof the base BD measures 40·5 inches, and the axis AC, 34 inches; the area is required.



SOLUTION.

40·5 base or ordinate
34 altitude

1620
1215

1377·0 product
2

3) 27540

918·0 square inches, *area required.*

BY THE SLIDING RULE.

Set I on B to 27 *(two-thirds of the base)* on A, and opposite to 34 *(the altitude)* on B, may be seen 918 for the *area* on A.

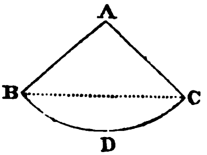
PROBLEM XIII.

To find the Area of a Sector.

RULE.

Multiply the radius by half the length of the arc, and the product will be the area.

Let ABDC be a sector, whereof the radius AB measures 17 inches, and the arc BDC, 26 inches ; the area is required.



SOLUTION.

17 radius
13 half the arc

51
17

221 square inches, *area of the sector.*

When the chord of the arc, and chord of half the arc are given, the length of the arc may be found *nearly*, though *not exactly*, as follows:

Multiply the chord of half the arc by 8, and from the product subtract the chord of the whole arc ; then divide the remainder by 3, and the quotient will be the arc *nearly.*

EXAMPLE.

What is the length of a circular arc when the chord is 30 inches, and the chord of half the arc, 20 inches?

20 chord of half the arc
Multiply by 8

160 product
Subtract 30 chord of the arc

3) 130 remainder

Quotient 43·3 *length of the arc* NEARLY.

When the radius and number of degrees in the angle at the centre are given, the length of the arc is easily found, by first determining the whole circumference: for, if the circumference be multiplied by the degrees in the given angle, the product divided by 360 will be the length of the arc.

EXAMPLE.

The radius of a sector is 50 inches, and the angle at the centre 60 degrees, what is the length of the arc?

50 × 2 = 100 diameter of the circle
3·1416 circumference to unity

Product 314·16 circumference of the given circle
Multiply by 60 degrees in the arc

360) 18849·6 product

Quotient 52·36 *length of the arc.*

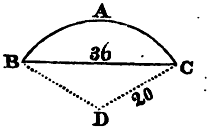
PROBLEM XIV.

To find the Area of the Segment of a Circle.

RULE.

Find the area of the sector, and also of the triangle formed by the two radii and chord. Then from the area of the sector subtract the area of the triangle, and the remainder will be the area of the segment.

Let BAC be a segment of a circle, whereof the altitude is 11·28 inches, the radius BD, of the circle, being 20 inches, and the chord BC, 36 inches; the area is required.



SOLUTION.

To find the chord of half the arc, add the square of the altitude to the square of half the base of the segment, and the square root of the product will, on the principle of a right-angled triangle, be the chord of half the arc. Wherefore it is,

$11{\cdot}28 \times 11{\cdot}28 = 127{\cdot}24$ square of the altitude

$18 \times 18 = 324{\cdot}00$ square of half the base

Sum 4|51|·24(21·2 *chord of half the arc.*

```
  4
 ----
41) 51
    41
   ----
422) 1024
      844
     ----
      180 remainder.
```

Again, to determine the length of the arc, it will be,

21·2 chord of half the arc
Multiply by 8
169·6 product
Subtract 36 chord of the arc
3) 133·6 remainder
Quotient 44·5 *length of the arc.*

(arc) ($\frac{1}{2}$ rad.) (prod.)
Now 44·5 × 10 = 445 square inches, *area of the sector* ABDC.

Next for the triangle, we shall have,

36 + 20 + 20 = 76, of which the half is 38.

Also,

38 — 36 = 2 first remainder
38 — 20 = 18 second remainder
38 — 20 18 third remainder.

And,

38 × 2 × 18 × 18 = 24624, of which the square root is 157 nearly, *area of the triangle* BCD.

But 445 — 157 = 288 square inches, *the area of the given segment.*

The following method, being short, is better adapted for practice, and, though *far* less correct than the former, will yet be found sufficient for common purposes, particularly in segments small in comparison of their circles.

RULE.

Multiply the segment's base by *two-thirds* of the altitude, and the product will be the area *nearly*, but always too little.

EXAMPLE 1.

The base of a segment of a circle is 36 inches, and the altitude is 11·28 inches; the area is demanded.

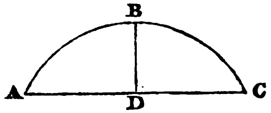
```
 11·28 altitude
    24 two-thirds of the base
 -----
  4512
 2256
 -----
270·72 product, and area NEARLY.
 -----
```

Here the area differs $17\frac{1}{4}$ square inches from the area found by the first method, the latter area being truly that of a parabola, and not of a circular segment.

In multiplying it is evident that *two-thirds* of the altitude multiplied by the base, and *two-thirds* of the base multiplied by the altitude, will be the same product.

EXAMPLE 2.

Let ABC be the segment of a circle, whereof the chord or base AC is 90 inches, and the altitude DB 30 inches; the area is demanded.



SOLUTION.

90 base or chord
20 two-thirds of the altitude

1800 square inches, *area of the segment* NEARLY.

The best practical method, however, of finding the area accurately is by the following Rule, and Table of Circular Segments.

RULE.

Divide the square of half the chord or base by the altitude of the segment, and to the quotient add the altitude; the sum will be the diameter of the circle whereof the given segment is a part.

Next divide the altitude of the segment by the diameter of its circle, and the quotient will be the *versed sine* of a similar segment when the diameter is UNITY.

With this *versed sine* enter the Table of Circular Segments, and take out the area corresponding to it. The area taken from the table is the area of a segment, similar to the given segment, in a circle of which the diameter is *an inch.* Consequently this area multiplied by the square of the diameter of the circle whereof the given segment is a part, in inches, will be the area of the given segment in *Square Inches.*

TABLE

OF THE AREAS OF CIRCULAR SEGMENTS IN DECIMAL PARTS OF THE INTEGER,

FOR EVERY ALTITUDE FROM ·001 TO ·5, THE DIAMETER OF THE CIRCLE BEING 1.

Note.—*In a circle whereof the diameter is one inch, the areas in the Table are decimal parts of a square inch. If the diameter be one foot, then the areas are decimal parts of a square foot; and so on.*

DIAMETER BEING UNITY.					
Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area
·001	·000042	·021	·004031	·041	·010931
·002	·000119	·022	·004322	·042	·011330
·003	·000219	·023	·004618	·043	·011734
·004	·000337	·024	·004921	·044	·012142
·005	·000470	·025	·005230	·045	·012554
·006	·000618	·026	·005546	·046	·012971
·007	·000779	·027	·005867	·047	·013392
·008	·000951	·028	·006194	·048	·013818
·009	·001135	·029	·006527	·049	·014247
·010	·001329	·030	·006865	·050	·014681
·011	·001533	·031	·007209	·051	·015119
·012	·001746	·032	·007558	·052	·015561
·013	·001968	·033	·007913	·053	·016007
·014	·002199	·034	·008273	·054	·016457
·015	·002438	·035	·008638	·055	·016911
·016	·002685	·036	·009008	·056	·017369
·017	·002940	·037	·009383	·057	·017831
·018	·003202	·038	·009763	·058	·018296
·019	·003471	·039	·010148	·059	·018766
·020	·003748	·040	·010537	·060	·019239

DIAMETER BEING UNITY.

Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area
·061	·019716	·096	·038496	·131	·060672
·062	·020196	·097	·039087	·132	·061348
·063	·020680	·098	·039680	·133	·062026
·064	·021168	·099	·040276	·134	·062707
·065	·021659	·100	·040875	·135	·063389
·066	·022154	·101	·041476	·136	·064074
·067	·022652	·102	·042080	·137	·064760
·068	·023154	·103	·042687	·138	·065449
·069	·023659	·104	·043296	·139	·066140
·070	·024168	·105	·043908	·140	·066833
·071	·024680	·106	·044522	·141	·067528
·072	·025195	·107	·045139	·142	·068225
·073	·025714	·108	·045759	·143	·068924
·074	·026236	·109	·046381	·144	·069625
·075	·026761	·110	·047005	·145	·070328
·076	·027289	·111	·047632	·146	·071033
·077	·027821	·112	·048262	·147	·071741
·078	·028356	·113	·048894	·148	·072450
·079	·028894	·114	·049528	·149	·073161
·080	·029435	·115	·050165	·150	·073874
·081	·029979	·116	·050804	·151	·074589
·082	·030526	·117	·051446	·152	·075306
·083	·031076	·118	·052090	·153	·076026
·084	·031629	·119	·052736	·154	·076747
·085	·032186	·120	·053385	·155	·077469
·086	·032745	·121	·054036	·156	·078194
·087	·033307	·122	·054689	·157	·078921
·088	·033872	·123	·055345	·158	·079649
·089	·034441	·124	·056003	·159	·080380
·090	·035011	·125	·056663	·160	·081112
·091	·035585	·126	·057326	·161	·081846
·092	·036162	·127	·057991	·162	·082582
·093	·036741	·128	·058658	·163	·083320
·094	·037323	·129	·059327	·164	·084059
·095	·037909	·130	·059999	·165	·084801

DIAMETER BEING UNITY.					
Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area
·166	·085544	·201	·112624	·236	·141537
·167	·086289	·202	·113426	·237	·142387
·168	·087036	·203	·114230	·238	·143238
·169	·087785	·204	·115035	·239	·144091
·170	·088535	·205	·115842	·240	·144944
·171	·089287	·206	·116650	·241	·145799
·172	·090041	·207	·117460	·242	·146655
·173	·090797	·208	·118271	·243	·147512
·174	·091554	·209	·119083	·244	·148371
·175	·092313	·210	·119897	·245	·149230
·176	·093074	·211	·120712	·246	·150091
·177	·093836	·212	·121529	·247	·150953
·178	·094601	·213	·122347	·248	·151816
·179	·095366	·214	·123167	·249	·152680
·180	·096134	·215	·123988	·250	·153546
·181	·096903	·216	·124810	·251	·154412
·182	·097674	·217	·125634	·252	·155280
·183	·098447	·218	·126459	·253	·156149
·184	·099221	·219	·127285	·254	·157019
·185	·099997	·220	·128113	·255	·157890
·186	·100774	·221	·128942	·256	·158762
·187	·101553	·222	·129773	·257	·159636
·188	·102334	·223	·130605	·258	·160510
·189	·103116	·224	·131438	·259	·161386
·190	·103900	·225	·132272	·260	·162263
·191	·104685	·226	·133108	·261	·163140
·192	·105472	·227	·133945	·262	·164019
·193	·106261	·228	·134784	·263	·164899
·194	·107051	·229	·135624	·264	·165780
·195	·107842	·230	·136465	·265	·166663
·196	·108636	·231	·137307	·266	·167546
·197	·109430	·232	·138150	·267	·168430
·198	·110226	·233	·138995	·268	·169315
·199	·111024	·234	·139841	·269	·170202
·200	·111823	·235	·140688	·270	·171089

DIAMETER BEING UNITY.

Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area
·271	·171978	·306	·203683	·341	·236421
·272	·172867	·307	·204605	·342	·237369
·273	·173758	·308	·205527	·343	·238318
·274	·174649	·309	·206451	·344	·239268
·275	·175542	·310	·207376	·345	·240218
·276	·176435	·311	·208301	·346	·241169
·277	·177330	·312	·209227	·347	·242121
·278	·178225	·313	·210154	·348	·243074
·279	·179122	·314	·211082	·349	·244026
·280	·180019	·315	·212011	·350	·244980
·281	·180918	·316	·212940	·351	·245934
·282	·181817	·317	·213871	·352	·246889
·283	·182718	·318	·214802	·353	·247845
·284	·183619	·319	·215733	·354	·248801
·285	·184521	·320	·216666	·355	·249757
·286	·185425	·321	·217599	·356	·250715
·287	·186329	·322	·218533	·357	·251673
·288	·187234	·323	·219468	·358	·252631
·289	·188140	·324	·220404	·359	·253590
·290	·189047	·325	·221340	·360	·254550
·291	·189955	·326	·222277	·361	·255510
·292	·190864	·327	·223215	·362	·256471
·293	·191775	·328	·224154	·363	·257433
·294	·192684	·329	·225093	·364	·258395
·295	·193596	·330	·226033	·365	·259357
·296	·194509	·331	·226974	·366	·260320
·297	·195422	·332	·227915	·367	·261284
·298	·196337	·333	·228858	·368	·262248
·299	·197252	·334	·229801	·369	·263213
·300	·198168	·335	·230745	·370	·264178
·301	·199085	·336	·231689	·371	·265144
·302	·200003	·337	·232634	·372	·266111
·303	·200922	·338	·233580	·373	·267078
·304	·201841	·339	·234526	·374	·268045
·305	·202761	·340	·235473	·375	·269013

DIAMETER BEING UNITY.					
Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area
·376	·269982	·411	·304171	·446	·338804
·377	·270951	·412	·305155	·447	·339798
·378	·271920	·413	·306140	·448	·340793
·379	·272890	·414	·307125	·449	·341787
·380	·273861	·415	·308110	·450	·342782
·381	·274832	·416	·309095	·451	·343777
·382	.275803	·417	·310081	·452	·344772
·383	·276775	·418	·311068	·453	·345768
·384	·277748	·419	·312054	·454	·346764
·385	·278721	·420	·313041	·455	·347759
·386	·279694	·421	·314029	·456	·348755
·387	·280668	·422	·315016	·457	·349752
·388	·281642	·423	·316004	·458	·350748
·389	·282617	·424	·316992	·459	·351745
·390	·283592	·425	·317981	·460	·352742
·391	·284568	·426	·318970	·461	·353739
·392	·285544	·427	·319959	·462	·354736
·393	·286521	·428	·320948	·463	.355732
·394	·287498	·429	·321938	·464	.356730
·395	·288476	·430	·322928	·465	.357727
·396	·289453	·431	·323918	·466	.358725
·397	·290432	·432	·324909	·467	.359723
·398	·291411	·433	·325900	·468	.360721
·399	·292390	·434	·326892	·469	.361719
·400	·293369	·435	·327882	·470	.362717
·401	·294349	·436	·328874	·471	.363715
·402	·295330	·437	·329866	·472	·364713
·403	·296311	·438	·330858	·473	·365712
·404	·297292	·439	·331850	·474	·366710
·405	·298273	·440	·332843	·475	·367709
·406	·299255	·441	·333836	·476	·368708
·407	·300238	·442	·334829	·477	·369707
·408	·301220	·443	·335822	·478	·370706
·409	·302203	·444	·336816	·479	·371705
·410	·303187	·445	·337810	·480	·372704

DIAMETER BEING UNITY.					
Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area	Altitude or Versed Sine	Segm. Area
·481	·373703	·488	·380700	·495	·387699
·482	·374702	·489	·381699	·496	·388699
·483	·375702	·490	·382699	·497	·389699
·484	·376702	·491	·383699	·498	·390699
·485	·377701	·492	·384699	·499	·391699
·486	·378701	·493	·385699	·500	·392699
·487	·379700	·494	·386699	1·000	·785398

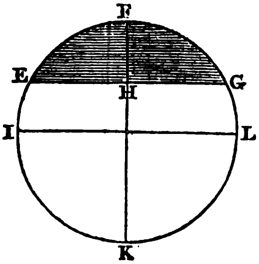
In the Table above, the greatest segment inserted is the semicircle, of which the area is evidently half the area of the circle. If the area of a segment greater than a semicircle be required, the altitude of the segment has only to be subtracted from the diameter of its circle, and the remainder will be the altitude of the opposite segment, that is, of the segment by which the given segment is less than a complete circle.

The area of this opposite segment, when the diameter is 1, being found in the Table, and subtracted from the area of Unity, namely, from ·785398, the remainder multiplied by the square of the diameter of the given circle, will be truly the area of the greater segment.

To render this subject as clear as possible, we shall subjoin an example of a segment less than a semicircle, and also of a segment greater.

EXAMPLE 1.

Let EFG be a segment of a circle, whereof the altitude FH is 30 inches, and the base or chord EG, 102 inches; the area is required.



This segment is manifestly less than a semicircle, because the altitude is less than the semi-chord.

SOLUTION.

2) 102 chord or base

51 semichord
51 semichord

51
255

Altitude 30) 2601 square of half the base

86·7 quotient
Add 30·0 the altitude

Sum 116·7 *diameter of the circle.*

Again,

Diameter. Altitude. Quotient.
116·7) 30·00 (·257 *tabular versed sine.*

Wherefore entering the Table with ·257, we find, opposite to this altitude, the number ·159636 as the area when the diameter of the circle is unity.

Consequently it will be,

$$\begin{array}{rcrl}
 & & \cdot159636 & \text{tabular area} \\
116\cdot7 \times 116\cdot7 & = & \underline{13618\cdot89} & \text{square of the diameter} \\
 & & 1436724 & \\
 & & 1277088 & \\
 & & 1277088 & \\
 & & 159636 & \\
 & & 957816 & \\
 & & 478908 & \\
 & & \underline{159636} & \\
\text{Product} & & \underline{2174\cdot06512404} & \textit{area required.}
\end{array}$$

In finding this area we have retained more decimal places than were absolutely necessary, but we were unwilling to curtail the operation, being desirous of exhibiting the area to the greatest nicety.

EXAMPLE 2.

Let EKG be a circular segment, whereof the base EG is 102 inches, and the altitude *(see the last figure)* HK, 86·7 inches; the area is required.

This segment is manifestly greater than a semicircle, because the altitude is greater than the semi-chord.

SOLUTION.

2) 102 chord or base

51 semichord

51 semichord

51

255

Altitude 86·7) 2601 square of the half base

Quotient 30 *altitude of the opposite segm.*

Now the area of a segment similar to a segment of which the base is 102 inches, and altitude 30 inches, has, in the last Example, been found to be ·159636 when the diameter of the circle is unity.

Consequently,

·785398 whole area

Subtract ·159636 segment area

·625762 remainder

116·7 × 116·7 = 13618·89 square of the diameter

5631858

5006096

5006096

625762

3754572

1877286

625762

Product 8522·18384418 *area required.*

PROOF.

	13618·89	square of the diameter
Multiply by	·785398	constant decimal
	10895112	
	12257001	
	4085667	
	6809445	
	10895112	
	9533223	
Product	10696·24896822	*area of the whole circle.*

	2174·06512404	area of EFG.
	8522·18384418	area of EKG.
Sum	10696·24896822	*area of the whole circle.*

Here the sum of the two areas, and the area of the whole circle agree EXACTLY.

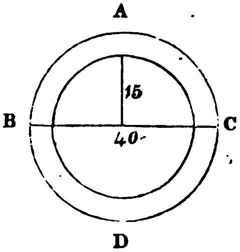
PROBLEM XV.

To find the Area of an Annular Space.

RULE.

Find the area of both circles, and their difference will be the area of the space between the circumferences.

Let ABDC be an annular space, of which the outer diameter BC is 40 inches, and the inner diameter 30 inches; the area is required.



SOLUTION.

40 × 40 × ·7854 = 1256·64 greater circle
30 × 30 × ·7854 = 706·86 less circle

Difference 549·78 *area required.*

The work may be shortened by omitting to multiply by ·7854 till the difference of the squares of the diameters is found, as below.

40 × 40 = 1600 square of the greater diameter
30 × 30 = 900 square of the less diameter

Remainder 700 difference of the squares
Multiply by ·7854

Product 549·78 square inches *as before.*

BY THE SLIDING RULE.

Set ·7854 on A, to 1 on B, then against 700 *(the difference of the squares of the diameters)* on B, will be found 549·78 *for the area of the ring*, on A.

PROBLEM XVI.

To find the Area of the Segment of an Ellipsis cut by a right Line parallel to either Axis; the Altitude of the Segment and both Diameters of the Ellipsis being given.

CASE 1.

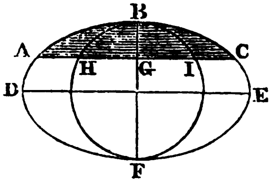
When the section is by a right line parallel to the transverse diameter.

RULE.

Find the area of a circular segment of the same altitude, the diameter of the circle being equal to the conjugate diameter of the ellipsis. Then multiply this area by the transverse axis, and divide the product by the conjugate, for the area of the elliptical segment.

EXAMPLE.

Let ABC be a segment of an ellipsis cut by a right line parallel to the longer axis DE; and let BG (the altitude of the segment) be 6 inches, the conjugate diameter BF of the ellipsis being 30 inches, and the transverse diameter DE, 50 inches; the area of the segment ABC is demanded.



SOLUTION.

The altitude 6 divided by the diameter 30 of the inscribed circle, quotes ·2 for the *versed sine* with which to enter the Table of Circular Segments; and the area corresponding is

·111823 tabular number
Multiply by 900 square of BF.
Product 100·6407|00 area of HBI.
Multiply by 50 transverse diameter
Conjugate 30) 5032·0350 product
Quotient 167·7345 *area required.*

CASE II.

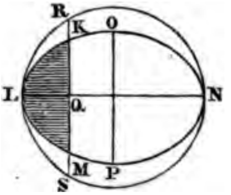
When the section is by a right line parallel to the conjugate diameter.

RULE.

Find the area of a circular segment of the same altitude, the diameter of the circle being equal to the transverse diameter of the ellipsis. Then multiply this area by the conjugate, and divide the product by the transverse axis of the ellipsis, for the area of the elliptical segment.

EXAMPLE.

Let KLM be the segment of an ellipsis cut by a right line parallel to the shorter diameter, the altitude LQ being 10 inches, and the two diameters OP and LN, being respectively 27 inches, and 40 inches; the area of the segment KLM is required.



SOLUTION.

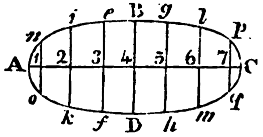
The altitude 10 divided by the diameter 40 of the circumscribing circle, quotes ·25 for the *versed sine* with which to enter the Table of Circular Segments; and the area corresponding is

·153546 tabular number
Multiply by 1600 square of LN.
92127600
153546
Product 245·6736|00 area of RLS.
Multiply by 27 conjugate diameter
17197152
4913472
Transv. 40) 6633·1872 product
Quotient 165·82968 *area required.*

PROBLEM XVII.

To find, by equidistant Ordinates, the Area of an Elliptical plane Figure differing from a true Oval.

Let ABCD be an elliptical plane figure differing from a true oval; the area is required by the method of equidistant ordinates.



Measure the longest diameter AC, which let be 54·9 inches, and through the middle of AC draw the straight line BD at right angles to it. Then parallel to BD, at equal distances, draw the lines *ef*, *gh*, *ik*, *lm*, *no*, *pq*, intersecting AC in the points 1, 2, 3, 4, 5, 6, 7; and let the distances be such that A1, and 7C, may be comparatively small. This may easily be effected by dividing the longest diameter AC (here supposed to be 54·9 inches) by the proposed number of spaces, which will always be even, since the number of ordinates will evidently be *odd.*

In the instance before us the number of spaces we have chosen is 6; and if 54·9 inches be divided by 6, the quotient in whole numbers will be 9 inches for the distance between any two ordinates, and $\frac{9}{10}$ of an inch will remain for the sum of A1 and 7C.

The ordinates being all drawn, let them be severally measured, and let their lengths be as under:

1. *no*...... 16·2 inches
2. *ik*...... 21·8 inches
3. *ef*...... 24·4 inches
4. BD.... 26·0 inches
5. *gh*...... 24·6 inches
6. *lm*...... 22·4 inches
7. *pq*...... 16·8 inches

RULE

FOR PROCEEDING AFTER THE DIMENSIONS ARE TAKEN.

To the sum of the first and last ordinates add four times the sum of all the *even* ordinates, with double the sum of all the *odd* ones, except the first and last. Then multiply the total by one third part the equi-distance assigned, and the product will be the area between the extreme ordinates.

Lastly, for the area of the segments at the ends, multiply the sum of the first and last ordinates, by one third part the sum of the altitudes of the two segments, and the product will be the area of both, nearly, which being added to the former area, will give the superficial content of the whole figure.

THE AREA OF THE ABOVE CURVILINEAL FIGURE DETERMINED.

First ordinate	16·2 inches			
Last ———	16·8			
	——— sum................			33·0
2nd ordinate	21·8			
4th ———	26·0			
6th ———	22·4			
	——— sum	70·2		
	Multiply by	4	prod.	280·8
3rd ordinate	24·4			
5th ———	24·6			
	——— sum	49·0		
	Multiply by	2	prod.	98·0
			Total	411·8

Total brought over 411·8
Multiply by $\frac{1}{3}$ the equidistance 3

Area between the extreme ordinates 1235·4

First ordinate 16·2
Last ——— 16·8
——— sum.................. 33·0
Multiply by $\frac{1}{3}$ the altitudes ·3

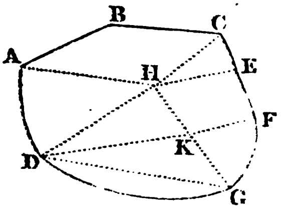
Area of both segments 9·9

WHOLE AREA in square inches 1245·3

PROBLEM XVII.

To find the Area of an Irregular plane Figure.

Let ABCDEFG be a figure so irregular that its area cannot be found by any of the foregoing Rules.



If two straight lines not parallel be drawn in the curvature DG, and perpendiculars bisecting them be raised, those perpendiculars will meet in a point without the figure, which is the centre of the arc DG. Wherefore, if a chord be drawn from D to G, that part of the given figure is a segment of a circle.

In like manner it will be found that ADH, FKG, and CHE are sectors.

Also EHKF, and ABCH are quadrilaterals; and DHK is a triangle.

Evidently then, the given figure has been divided into figures that can be measured by the preceding problems, and the sum of the areas of those figures will be the area of the given figure.

In like manner any plane space whatever may be divided into portions, each of which shall fall under some one of the shapes treated of, in the preceding problems. No general Rule, however, can be given for the boundless variety of irregular figures that will occur in practice, and consequently much must be left to the discretion of the practitioner, which will always be great in proportion to his acquaintance with the manner of constructing the more common geometrical figures, and the correctness of his theory on measuring them.

The main object to be kept in view is to attain a MAXIMUM of accuracy with a MINIMUM of labour, and hence no more lines than are *absolutely necessary*, ought ever to be drawn.

SECTION II.

OF AREAS AN INCH IN DEPTH.

It is evident by the First Section that the number of square inches in the area of the plane figure constituting the base, or surface, will be the number of cubic inches in the solid space between the base and surface; hence further Rules on the subject would be needless, were it not that those cubic inches are to be converted into exciseable measures, and that this may sometimes be done without finding the number of square inches in the area of the surface or base. Besides, as this species of Mensuration is one of the most useful parts of Gaging, since all areas in every figure whatever are considered as an inch in depth, and not, like in other branches of the Mathematics, as merely superficial, we shall, with this clearly understood, enter more fully into the spirit of the work before us.

Exciseable liquors are measured either by the Table of WINE MEASURE, or by the Table of ALE AND BEER MEASURE, the gallon being in both instances the lowest integer. For which reason, as a preliminary step, we shall subjoin these Tables, specifying the number of cubic inches in the several denominations, with the times any one lower denomination has to be taken to make a unit of a higher.

TABLE OF WINE MEASURE.

Gallons	Rundlets	Barrels	Tierces	Hogsheads	Puncheons	Butts	Tuns	Cubic Inches
1								231
18	1							4158
31½	1¾	1						7276·5
42	2⅓	1⅓	1					9702
63	3½	2	1½	1				14553
84	4⅔	2⅔	2	1⅓	1			19404
126	7	4	3	2	1½	1		29106
252	14	8	6	4	3	2	1	58212

By Wine measure all sorts of wines and spirits, mead, cider, perry, verjuice, oil, and other liquids are bought and sold.

The fourth part of a gallon is called a *quart*, and contains 57·75 cubic inches.

The half of a quart is termed a *pint*, and is equal to 28·875 cubic inches.

TABLE OF ALE AND BEER MEASURE.

Gallons	Firkins	Kilderkins	Barrels	Hogsheads	Butts	Tuns	Cubic Inches
1							282
9	1						2538
18	2	1					5076
36	4	2	1				10152
54	6	3	1½	1			15228
108	12	6	3	2	1		30456
216	24	12	6	4	2	1	60912

This measure is exclusively for Ale and Beer.

Exciseable goods not falling under the head of liquors are generally reducible to some one of the following weights or measures.

	Cubic Inches
Malt or corn bushel....................	2150·42
Malt gallon..............................	268·8
Mash-tun gallon........................	227·
Pound of hard soap, cold............	27·14

	Cubic Inches
Pound of hot soap	28·
Pound of green soap	25·67
Pound of white soft soap	25·56
Pound of tallow nett	31·4
Pound of green starch	34·8
Pound of dry starch	40·3
Pound of flint glass	10·56
Pound of white glass	14·06
Pound of green glass	12·18

Of all these measures of solidity the three principal integers used in Gaging are the *Wine-gallon*, the *Ale-gallon*, and the *Malt-bushel*,

		Cubic Inches		
The wine gallon	containing	231	marked	W.G.
The ale gallon	containing	282	marked	A. G.
The malt bushel	containing	2150·42	marked	M. B.

The square roots of these numbers are called the Square Gage-points of the respective measures.

Thus,

The square root of	231	is	15·19	square gage-point for	Wine-gallons.
The square root of	282	is	16·79	square gage-point for	Ale-gallons.
The square root of	2150·42	is	46·37	square gage-point for	Malt-bushels.

The numbers themselves are called simply Gage-points, and are used on the lines A and B, but their square roots, that is, the square gage-points, are used on the lines C and D.

PROBLEM I.

To determine the Content of a Square Area an Inch in Depth.

RULE.

By the Pen.

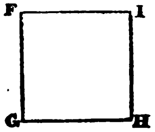
Multiply the side by itself, and divide the product by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the square gage-point, for the proposed integer, on D, to 1 on C, and opposite to the given side on D, will be found the content on C.

EXAMPLE 1.

Let FGHI be a square, whereof the side is 90 inches; the area, for an inch in depth, is required in wine gallons, ale-gallons, and malt-bushels.



SOLUTION BY THE PEN.

90 side of the square
90 side

W.G. divisor 231) 8100 (35·06 wine gallons.

A. G. divisor 282) 8100 (28·72 ale gallons.

M.B. divisor 2150) 8100 (3·76 malt bushels.

NOTE. The decimal in the malt-bushel divisor is generally omitted, as comparatively of little value.

BY THE SLIDING RULE.

on D. on C. on D. on C.

$$\text{Sq. G.P.}\left\{\begin{matrix}15{\cdot}19\\16{\cdot}79\\46{\cdot}37\end{matrix}\right\}\text{to}\left\{\begin{matrix}\\1\\\end{matrix}\right\}\text{against}\left\{\begin{matrix}\\90\\\end{matrix}\right\}\text{are}\left\{\begin{matrix}35{\cdot}06\ \text{W.G.}\\28{\cdot}72\ \text{A. G.}\\3{\cdot}76\ \text{M.B.}\end{matrix}\right\}$$

EXAMPLE 2.

The side of a square being 76·4 inches, what is the area, at an inch deep, in wine-gallons, ale-gallons, and malt-bushels?

SOLUTION BY THE PEN.

```
 76·4 side of the square
 76·4 side
 ────
 3056
4584
5348
────
```

W.G. divisor 231) 5836·96 (25·26 wine gallons.

A. G. divisor 282) 5836·96 (20·69 ale gallons.

M.B. divisor 2150) 5836·96 (2·71 malt bushels.

BY THE SLIDING RULE.

on D. on C. on D. on C.

$$\text{Sq. G.P.}\left\{\begin{matrix}15{\cdot}19\\16{\cdot}79\\46{\cdot}37\end{matrix}\right\}\text{to}\left\{\begin{matrix}\\1\\\end{matrix}\right\}\text{against}\left\{\begin{matrix}\\76{\cdot}4\\\end{matrix}\right\}\text{are}\left\{\begin{matrix}25{\cdot}26\ \text{W.G.}\\20{\cdot}69\ \text{A. G.}\\2{\cdot}71\ \text{M. B.}\end{matrix}\right\}$$

PROBLEM II.

To determine the Content of a Rectangular Area an Inch in Depth.

RULE.

By the Pen.

Multiply the length by the breadth, and divide the product by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the gage point for the proposed integer, on A, to the breadth on B, and opposite to the length on A, will be the content on B.

EXAMPLE.

Let DEFG be a rectangle, whereof the length FG is 80 inches, and the breadth FD, 35 inches; the area for an inch in depth is required in wine-gallons, ale-gallons, and malt-bushels.

SOLUTION BY THE PEN.

35 breadth of the rectangle
80 length

W.G. divisor 231) 2800 (12·12 wine gallons.

A. G. divisor 282) 2800 (9·93 ale gallons.

M.B. divisor 2150) 2800 (1·30 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{cccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} \\ \text{G. P.} & \left\{\begin{array}{c}231\\282\\2150\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\35\\ \\ \end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\80\\ \\ \end{array}\right\} & \text{are} & \left\{\begin{array}{r}12{\cdot}12 \text{ W.G.}\\ 9{\cdot}93 \text{ A.G.}\\ 1{\cdot}30 \text{ M.B.}\end{array}\right\}\end{array}$$

PROBLEM III.

To determine the Content of an Inch-deep Area in Form of a Parallelogram not Rectangular.

RULE.

By the Pen.

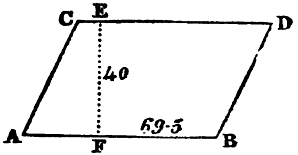
Upon one of the longest sides demit a perpendicular from the opposite side, and multiply the length of the base, on which the perpendicular falls, by the perpendicular. Then the product divided by the number of cubic inches in the proposed integer will be the content.

By the Sliding Rule.

Set the gage-point, for the proposed integer, on A, to the perpendicular breadth on B, and opposite to the length on A, will stand the content on B.

EXAMPLE.

Let ABCD be a rhomboid, whereof the base is 69·5 inches, and the perpendicular breadth EF, 40 inches; the number of wine gallons, ale gallons, and malt bushels the figure will contain, the depth being an inch, is required.



SOLUTION BY THE PEN.

69·5 length
Multiply by 40 breadth

W.G. divisor 231) 2780·0 (12·03 wine gallons.

A. G. divisor 282) 2780·0 (9·86 ale gallons.

M.B. divisor 2150) 2780·0 (1·29 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{lccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} \\ \text{G. P.} & \begin{Bmatrix}231\\282\\2150\end{Bmatrix} & \text{to} & \begin{Bmatrix}40\end{Bmatrix} & \text{against} & \begin{Bmatrix}69{\cdot}5\end{Bmatrix} & \text{are} & \begin{Bmatrix}12{\cdot}03\ \text{W.G.}\\9{\cdot}86\ \text{A. G.}\\1{\cdot}29\ \text{M. B.}\end{Bmatrix}\end{array}$$

PROBLEM IV.

To determine the Content of a Right-angled Triangle an Inch in Depth.

RULE.

By the Pen.

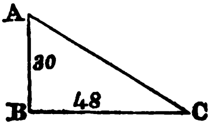
Divide half the product of the sides containing the right angle by the number of cubic inches in the proposed integer, and the quotient will be the content.

By the Sliding Rule.

Set the gage point, for the proposed integer, on A, to half the base on B, then opposite to the perpendicular on A, will be the content on B.

EXAMPLE.

The base BC, of a right-angled triangle ABC, is 48 inches, and the perpendicular BA, 30 inches; the content for an inch deep is required in wine-gallons, ale-gallons, and malt-bushels.



SOLUTION BY THE PEN.

24 half the base
30 perpendicular

W.G. divisor 231) 720 (3·11 wine gallons.

A. G. divisor 282) 720 (2·55 ale gallons.

M.B. divisor 2150) 720 (0·33 malt bushels.

BY THE SLIDING RULE.

on A. on B. on A. on B.

$$\text{G. P.}\left\{\begin{matrix}231\\282\\2150\end{matrix}\right\}\text{to}\left\{24\right\}\text{against}\left\{30\right\}\text{are}\left\{\begin{matrix}3\cdot11\ \text{W. G.}\\2\cdot55\ \text{A. G.}\\0\cdot33\ \text{M. B.}\end{matrix}\right\}$$

PROBLEM V.

To determine the Content of a Triangle an Inch in Depth; the Base, and Perpendicular let fall on it from the opposite Angle, being given.

RULE.

By the Pen.

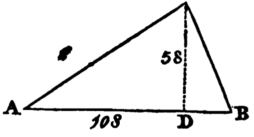
Divide half the product of the base and perpendicular by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the gage point, for the proposed integer, on A, to half the base on B, and against the perpendicular on A, will be the content on B.

EXAMPLE.

ABC is a triangle, whereof the base AB measures 108 inches, and the perpendicular CD, let fall on it from the opposite angle measures 58 inches; the content is required, for an inch deep, in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

$$\begin{array}{rl}2)\ \underline{108} & \text{base}\\ 54 & \text{half base}\\ \underline{58} & \text{perpendicular}\\ 432 & \\ \underline{270} & \\ \end{array}$$

W.G. divisor 231) 3132 (13·56 wine gallons.

A. G. divisor 282) 3132 (11·10 ale gallons.

M.B. divisor 2150) 3132 (1·45 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{ccccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} \\ \text{G. P.} & \left\{\begin{array}{r}231\\282\\2150\end{array}\right\} & \text{to} & \left\{54\right\} & \text{against} & \left\{58\right\} & \text{are} & \left\{\begin{array}{r}13{\cdot}56\text{ W. G.}\\11{\cdot}10\text{ A. G.}\\1{\cdot}45\text{ M. B.}\end{array}\right\}\end{array}$$

PROBLEM VI.

To determine the Content of a Triangle an Inch Deep, having the three Sides given.

RULE.

By the Pen.

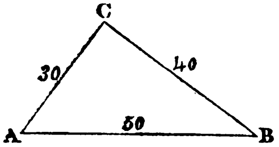
Add the three sides together, and from half the sum subtract each side separately, noting the remainders. Then multiply into one continued product the half sum and three remainders. The square root of this product, divided by the number of cubic inches in the proposed integer, will be the content.

By the Sliding Rule.

Find the number of cubic inches by the pen, and set the gage point, for the proposed integer on A, to 1 on B, then look opposite to the number of cubic inches on A, for the content on B.

EXAMPLE.

Let ABC be a triangle, whereof the side AB measures 50 inches, the side BC, 40 inches, and the side AC, 30 inches; the content, at an inch deep, is demanded in wine-gallons, ale-gallons, and malt-bushels.



SOLUTION BY THE PEN.

	50	first side
	40	second side
	30	third side
2)	120	sum
	60	half sum.

And

60 — 50 = 10 first remainder
60 — 40 = 20 second remainder
60 — 30 = 30 third remainder.

Again,

$60 \times 10 \times 20 \times 30 = 360000$, whereof the square root is 600 *cubic inches.*

Lastly,

W.G. divisor 231) 600 (2·59 wine gallons.

A. G. divisor 282) 600 (2·12 ale gallons.

M. B. divisor 2150) 600 (0·27 malt bushels.

BY THE SLIDING RULE.

on A. on B. on A. on B.

$$\text{G. P.}\left\{\begin{matrix}231\\282\\2150\end{matrix}\right\}\text{to}\left\{\begin{matrix}\\1\\ \end{matrix}\right\}\text{against}\left\{\begin{matrix}\\600\\ \end{matrix}\right\}\text{are}\left\{\begin{matrix}2.59\text{ W. G.}\\2\cdot12\text{ A. G.}\\0\cdot27\text{ M. B.}\end{matrix}\right\}.$$

PROBLEM VII.

To determine the Content, for an Inch in Depth, of a Quadrilateral having two Parallel unequal Sides.

RULE.

By the Pen.

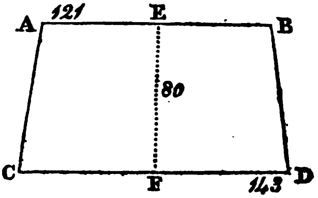
Multiply half the sum of the parallel sides by the perpendicular distance between them, and divide the product by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the gage-point, for the proposed integer, on A, to half the sum of the parallel sides on B, and against the perpendicular distance of the sides on A, will be the content on B.

EXAMPLE.

Let ABCD be a quadrilateral wherein two unequal sides are parallel, namely, AB and CD, the former being 121 inches, the latter 143 inches; and let EF, the perpendicular distance of the parallel sides, be 80 inches: the content, for one inch in depth, is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

121 side AB.
143 side CD.

2) 264 sum

132 half sum
80 perpendicular EF.

W.G. divisor 231) 10560 (45·71 wine gallons.

A. G. divisor 282) 10560 (37·44 ale gallons

M. B. divisor 2150) 10560 (4·91 malt bushels.

BY THE SLIDING RULE.

on A. on B. on A. on B.

$$\text{G. P.}\left\{\begin{matrix}231\\282\\2150\end{matrix}\right\}\text{to}\left\{80\right\}\text{against}\left\{132\right\}\text{are}\left\{\begin{matrix}45{\cdot}71\text{ W.G.}\\37{\cdot}44\text{ A.G.}\\4{\cdot}91\text{ M.B.}\end{matrix}\right\}$$

Since the lines A and B are perfectly alike, and may be used for one another indiscriminately, as it may best suit particular numbers, we have here set the gage-points on A to the *perpendicular distance* on B, and taken the *half sum* of the parallel sides on A, because by this mode of operation it is not necessary to shift the slide, which, in the case before us, it otherwise would be.

PROBLEM VIII.

To determine the Content of any Right-lined Multilateral plane Figure, taken an Inch in Depth.

RULE.

By the Pen.

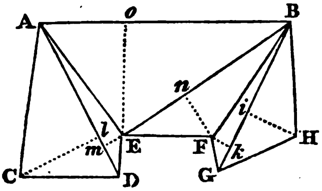
Divide the given plane figure into triangles by diagonal lines, and find the area of the several triangles. Then divide the sum of the areas by the number of cubic inches in the proposed integer, and the quotient will be the content.

By the Sliding Rule.

Find the number of cubic inches in the given figure by the pen, and set the gage-point for the proposed integer, on A, to 1 on B; then look opposite to the number of cubic inches in the area, on A, and you will have the content on B.

EXAMPLE.

Let ACDEFGHB be a multilateral plane figure, whereof the area is required for an inch deep, in wine-gallons, ale-gallons, and malt-bushels.



Draw the diagonals AD, AE, BE, BF, and BG, dividing the given figure into triangles. Then demit the perpendiculars C*l*, E*m*, E*o*, F*n*, F*k*, H*i*; and take the lengths of the lines required in the calculation, which let be the following:

AB = 133·4 inches
E*o* = 60·7 inches
AD = 92·2 inches
C*l* = 45·8 inches
E*m* = 13·4 inches
BE = 105·0 inches
F*n* = 26·4 inches
BG = 86·2 inches
H*i* = 29·1 inches
F*k* = 10·0 inches

SOLUTION BY THE PEN.

66·7 half of AB.

60·7 perpendicular E*o*.

4669

40020

Product 4048·69 *area of* AEB.

46·1 half of AD.

$45{\cdot}8+13{\cdot}4 = 59{\cdot}2$ sum of C*l* and E*m*.

922

4149

2305

Product 2729·12 *area of* ACDE.

13·2 half of F*n*.

105 base BE.

660

1320

Product 1386·0 *area of* EFB.

43·1 half of BG.

$29{\cdot}1+10 = 39{\cdot}1$ sum of H*i* and F*k*.

431

3879

1293

Product 1685·21 *area of* BFGH.

Having now found the several areas constituting the whole figure, we are to add them together, and it will be,

	4048·69	area of AEB.
	2729·12	area of ACDE.
	1386·00	area of EFB.
	1685·21	area of BFGH.
Sum	9849·02	WHOLE AREA.

Wherefore,

W.G. divisor 231) 9849·02 (42·63 wine gallons.

A. G. divisor 282) 9849·02 (34·92 ale gallons.

M. B. divisor 2150) 9849·02 (4·58 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{cccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} \\ \text{G. P.} & \left\{\begin{array}{r}231\\282\\2150\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\1\\ \end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\9849\\ \end{array}\right\} & \text{are} & \left\{\begin{array}{r}42{\cdot}63\ \text{W.G.}\\34{\cdot}92\ \text{A. G.}\\4{\cdot}58\ \text{M. B.}\end{array}\right\}\end{array}$$

PROBLEM IX.

To determine the Content of a Circular Area an Inch in Depth.

Before we give directions for solving this problem, we think it right to shew the manner of obtaining the *Circular Divisors*, and the *Circular Gage-points*. And to enable the learner to understand this subject clearly, it will be necessary to enlarge a little more on the circle.

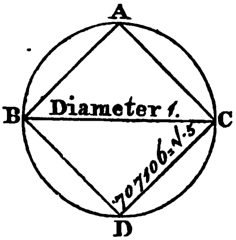
It has already been stated, in the First Section of Mensuration, that no rectilineal figure whatever can be constructed precisely equal to the circle, because, although a parallelogram in length equal to half the circumference, and in breadth equal to the radius of the circle, is in area equal to the circle, yet as no finite number will express the breadth of the parallelogram when the length is finite, nor the length when the breadth is finite, the parallelogram, reduced to numbers, can exist only in idea.

Notwithstanding the impossibility, however, of squaring the circle, the side of a square that shall differ in area extremely little from the circle, is easily found, and for every practical purpose this is sufficient.

It is evident that the square root of the number of square inches in any area, regular or irregular, will be the side of a square of equal area; since the side multiplied by itself must produce the area: hence, if the area of a circle be 1, the side of the equivalent square will be 1. Or if the diameter of the circle be 1, the area will, as before mentioned, be ·785398, and the side of the equal square will be the square root of that decimal, or ·886226.

This being premised, we shall point out the relation between the diameter of a circle and the side of its *Inscribed Square*, which thoroughly to comprehend is essential in the theory as well as practice of Gaging.

CIRCLE AND ITS INSCRIBED SQUARE.



Let ABCD be a circle, of which BC is the diameter; and let ACDB be a square inscribed in the circle, having for its diagonal BC. Then it is plain that the diameter of the circle is equal to the diagonal of the inscribed square.

Again the triangles BAC, and BDC are both right-angled. But Euclid, in the 47th proposition of his First Book, has demonstrated that the square described upon the side opposite to the right angle, is, in every right-angled triangle, equal to the sum of the squares described upon the sides containing the right angle.

Now, in either of the triangles BAC and BDC, the sides about the right angle are equal; hence the square of the diagonal is double the square of the side. That is, if the diameter of the circle be 1, the side of the inscribed square will be the square root of the half of 1, or ·70716 &c.

In other words the square root of twice the square of the radius of the circle is invariably equal to the side of the inscribed square.

As it is exceedingly convenient to know all the proportions of the circumference, equal square, inscribed trigon, inscribed square, and area, when the diameter is unity (since circles (and consequently similar figures described in them) are always to one another in the direct ratio of the squares of their diameters) the numbers representing them are here stated in order.

If the diameter of a circle be 1,

Then	The circumference	is	3·1416
	Side of the square equal		·8862
	Side of the inscribed trigon		·8660
	Side of the inscribed square		·7071
	Area		·7854

The numbers thus carried to four decimal places only, are sufficiently correct for ordinary purposes, and should be carefully committed to memory.

That the young practitioner may exercise himself in these proportions, and thus become better acquainted with them, we subjoin, solely for this end, the lengths of the several lines, and the area, when each is *unity* in succession.

1. *When the Circumference is* 1.

The diameter is	·3183
The side of the square equal	·2821
The side of the inscribed trigon	·2756
The side of the inscribed square	·2251
The area	·0796

2. *When the Side of the equal Square is* 1.

The circumference is	3·5449
The diameter	1·1284
Side of the inscribed trigon	·9772
Side of the inscribed square	·7979
Area	1·0000

3. *When the Side of the inscribed Trigon is* 1.

The circumference is	3·6276
The diameter	1·1547
The side of the square equal	1·0232
Side of the inscribed square	·8165
Area	1·0481

4. *When the Area of the inscribed Square is* 1.

The circumference is	4·4429
The diameter	1·4142
Side of the square equal	1·2533
Side of the inscribed trigon	1·2247
Area	1·5708

5. *When the Area of the Circle is* 1.

The circumference is	3·5449
The diameter	1·1284
Side of the square equal	·10000
Side of the inscribed trigon	·9772
Side of the inscribed square	·7979

Since the area of a circle is ·7854 when the diameter is 1, it is manifest, that if the number of cubic inches, in any measure whatever, be divided by ·7854, the quotient will be the square of the diameter of a circle, which, being *an inch deep*, will contain that measure; for the square of the diameter of the circle multiplied by ·7854 gives the area in square inches, and these, by conceiving them to be an inch in depth, become cubic inches.

Hence the number 231 divided by ·7854 will be the square of the diameter of the least circle, which, being *an inch deep*, will contain a wine gallon. So 282 divided by ·7854 will quote the square of the least circle that will hold, at an inch deep, an ale gallon. In like manner 2150·42 divided by ·7854 will be the square of the diameter of the least circle, which, being *an inch deep*, will contain a malt bushel; and so for any other measure whatever.

The quotient arising from the division of the cubic inches in any proposed measure by ·7854 is termed the *Circular Divisor* of that measure, being truly the square of the diameter of a circle, which being *an inch deep*, shall contain that measure; and the square root of the quotient has the name of the *Circular Gage-point*, being actually the DIAMETER of the circle, containing, for every inch in depth, the given measure.

Hence the *Circular Divisors* and *Circular Gage-points* for wine gallons, ale gallons, and malt bushels are obtained as follows:

CIRCULAR DIVISORS AND CIRCULAR GAGE-POINTS.

Divisor	Dividend	Quotient	Square Roots
·7854)	231·0000	(294·12	(17·15 wine gallons.
·7854)	282·0000	(359·05	(18·95 ale gallons.
·7854)	2150·4200	(2737·99	(52·32 malt bushels.
Divisor	*Dividend*	*Cir. Divisors*	*Circular Gage-points*

Precisely in the same manner may the circular divisor and circular gage-point corresponding to any other measure or weight be ascertained, if the number of cubic inches in the proposed integer, be exactly known.

The *Circular Divisors* and *Circular Gage-points* being now understood, we shall proceed with the Rule for calculating circular areas at an inch in depth.

RULE.

By the Pen.

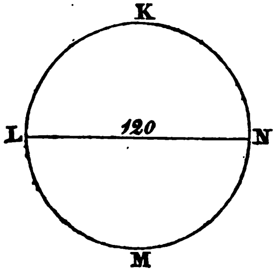
Divide the square of the diameter by the circular divisor for the proposed integer, and the quotient will be the content.

By the Sliding Rule.

Set the circular gage-point, for the proposed integer, on D, to 1 on C, and opposite to the given diameter on D, will be the content on C.

EXAMPLE.

Let KLMN be a circle, whereof the diameter LN is 120 inches; the content is required, for an inch deep, in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

diameter 120
diameter 120

W.G. cir. divisor 294·12) 14400 (48·95 wine gallons.

A. G. cir. divisor 359) 14400 (40·11 ale gallons.

M. B. cir. divisor 2738) 14400 (5·25 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{lcccc} & \text{on D.} & \text{on C.} & \text{on D.} & \text{on C.} \\ \text{C. G. P.} & \left\{\begin{matrix}17\cdot15\\18\cdot95\\52\cdot32\end{matrix}\right\}\text{to} & \left\{\begin{matrix}\\1\\ \end{matrix}\right\}\text{against} & \left\{\begin{matrix}\\120\\ \end{matrix}\right\}\text{are} & \left\{\begin{matrix}48\cdot95\ \text{W.G.}\\40\cdot11\ \text{A. G.}\\5\cdot25\ \text{M.B.}\end{matrix}\right\} \end{array}$$

In treating of the Sliding Rule we defined, in page 42, the sense applied in Gaging to the word FACTOR, and shewed, in page 43, by an appropriate example, that a quotient in *Division* may sometimes be more expeditiously obtained by *Multiplication*.

For this purpose it is necessary only to employ, as multiplier, the factor corresponding to the divisor.

Now a factor is no more than the quotient that arises when 1 is divided by any divisor.

Hence a Set of Factors for all the divisors in common use in Gaging is easily made, and should always be at hand, if not well fixed in the memory.

The factors for the three circular divisors above, may be obtained either by dividing 1 by the proposed divisor, or by dividing ·7854 (the area when the diameter is 1) by the number of cubic inches in the measure constituting the integer.

Thus,

Divisor	Dividend	Quotient
231)	·7854	(·00340 C. factor for wine gallons.
282)	·7854	(·00278 C. factor for ale gallons.
2150·42)	·7854	(·00036 C. factor for malt bushels.

In like manner may the circular factor for any other solid measure be determined.

The result will evidently be the same, if unity be divided by the circular divisor, as below:

Divisor	Dividend	Quotient
294·12)	1	(·00340 C. factor for wine gallons.
359·05)	1	(·00278 C. factor for ale gallons.
2737·99)	1	(·00036 C. factor for malt bushels.

All this being clearly understood, and with reference to page 139, there will occur no difficulty in constructing the following useful Table.

TABLE

OF FACTORS, DIVISORS, AND GAGE-POINTS.

EXCISEABLE MEASURES AND WEIGHTS	FACTORS FOR		DIVISORS FOR		GAGE-POINTS FOR	
	Squares	Circles	Squares	Circles	Sqrs.	Circles
Wine gallon	·004329	003399	231·	294·12	15·19	17.15
Ale gallon	·003546	·002785	282·	359·05	16·79	18.95
Malt or corn bushel	·000465	·000365	2150·42	2737·99	46·37	52·32
Malt gallon	·003720	·002922	268·8	342·24	16·39	18·5
Mash tun gallon	·004405	·003465	227·	289·	15·10	17·07
Cold hard soap lb.	·036845	·028939	27·14	34·56	5·21	5·88
Hot soap lb.	·035714	·028050	28·	35·65	5·29	5·97
Green soap lb.	·038956	·030600	25·67	32·68	5·06	5·72
White soft soap lb.	·039123	·030731	25·56	32·54	5·05	5·7
Nett tallow lb.	·031844	·025101	31·4	39·98	5·6	6·32
Green starch lb.	·028736	·022565	34·8	44·32	5·9	6·66
Dry starch lb.	·024813	·019491	40·3	51·3	6·35	7·16
Flint glass lb.	·094697	·074405	10.56	13·44	3·25	3·69
White glass lb.	·071123	·055860	14·06	17·9	3·74	4·32
Green glass lb.	·082102	·064516	12·18	15·5	3·48	3·94

By what has been said the reader will readily comprehend that the factors and divisors for squares are factors and divisors for areas of every shape reduced to square inches uniformly an inch in depth, that is, to cubic inches.

The circular divisors and circular gage-points are used for circles only, and for ovals, as will be explained in the next problem.

To solve the example last given, *(see page* 139*)* by the aid of the circular factor, it would be,

For Wine Gallons.

14400 square of the diameter
·0034 circular factor for wine gallons

57600
43200

Product 48·9600 *content* in wine gallons.

For Ale Gallons.

14400 square of the diameter
·00278 circular factor for ale gallons

115200
100800
28800

Product 40·03200 *content* in ale gallons.

For Malt Bushels.

14400 square of the diameter
·00036

86400
43200

Product 5·18400 *content* in malt bushels.

We have in this example, for the sake of brevity, curtailed the factors, and therefore the results differ a little from those found by the divisors and gage-points.

If, however, the factors in the Table be employed without abridgement, there will be no difference, that is, the contents will be the same by all the methods.

As every problem should be worked in two ways, that the one may check the other, the factors will be found useful in the absence of the Sliding Rule.

PROBLEM X.

To determine the Content of an Oval or Ellipsis, an Inch Deep.

RULE.

By the Pen.

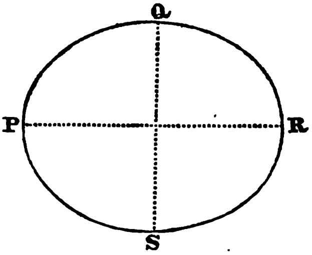
Divide the product of the transverse and conjugate diameters by the circular divisor for the proposed integer, and the quotient will be the content.

By the Sliding Rule.

Set the circular factor, for the proposed integer, on A, to the conjugate diameter on B, and opposite to the transverse diameter on A, will be found the content on B.

EXAMPLE.

Let PQRS be an ellipsis, whereof the transverse diameter PR is 60 inches, and the conjugate diameter QS, 48 inches; the content, for an inch deep, is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

48 conjugate axis

Multiply by 60 transverse diameter

W.G. cir. divisor 294·12) 2880·00 (9·79 wine gallons.

A. G. cir. divisor 359) 2880·00 (8·02 ale gallons.

M.B. cir. divisor 2738) 2880·00 (1·05 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{lccccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} \\ \text{Cir. Divis.} & \left\{\begin{matrix}294\\359\\2738\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}48\end{matrix}\right\} & \text{against} & \left\{\begin{matrix}60\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}9{\cdot}79 \text{ W.G.}\\8{\cdot}02 \text{ A. G.}\\1{\cdot}05 \text{ M.B.}\end{matrix}\right\} \end{array}$$

SOLUTION BY THE FACTORS.

For Wine Gallons.

2880 product of the diameters

·0034 circular factor

11520

8640

Product 9·7920 *content* in wine gallons.

For Ale Gallons.

2880 product of the diameters
·0028 circular factor
23040
5760
Product 8·0640 *content* in ale gallons, *nearly.*

For Malt Bushels.

2880 product of the diameters
·00036 circular factor
17280
8640
Product 1·03680 *content* in malt bushels.

Here the respective contents by the factors differ a little in the decimal from the contents by the divisors, but the scanty number of places to which both the divisors and factors have been carried, is incapable of giving very accurate results.

PROBLEM XI.

To determine the Content, for an Inch deep, of a Regular Polygon.

RULE.

By the Pen.

Multiply the inch-area, corresponding to the proposed integer and given figure in the following Table, by the square of the side of the figure, and the product will be the content.

TABLE

OF THE CONTENT OF REGULAR POLYGONS, IN WINE GALLONS, ALE GALLONS, AND MALT BUSHELS.

NOTE.—*All similar plane figures are to one another as the squares of their homologous parts or boundaries; and all similar solids are as the cubes of their corresponding dimensions.*

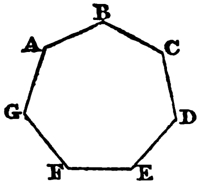
THE SIDE BEING AN INCH, AND DEPTH AN INCH.			
Names	Content in Wine Gallons	Content in Ale Gallons	Content in Malt Bushels
Pentagon	·007445	·006099	·000780
Hexagon	·011246	·009212	·001208
Heptagon	·015727	·012883	·001689
Octagon	·020900	·017120	·002245
Nonagon	·026726	.021925	·002875
Decagon	·033311	.027287	·003579
Undecagon	·040544	.033211	·004356
Dodecagon	·048467	.039702	·005207

By the Sliding Rule.

Set 1 on D, to the number, corresponding in the Table to the proposed integer and given figure, on C, then opposite to the given side on D, will be found the content on C.

EXAMPLE.

Let ABCDEFG be a regular heptagon, whereof the side is 80 inches; the content is required, for an inch deep, in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

For Wine Gallons.

·015727 tabular number
80 × 80 = 6400 square of the side

62908
94362

Product 100·6528 *content* in wine gallons.

For Ale Gallons.

·012883 tabular number
80 × 80 = 6400 square of the side

51532
77298

Product 82·4512 *content* in ale gallons.

For Malt Bushels.

·001689 tabular number
80 × 80 = 6400 square of the side

6756
10134

Product 10·8096 *content* in malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{lllllllll} & \text{on C.} & & \text{on D.} & & \text{on D.} & & \text{on C.} & \\ \text{Tab. num.} & \left\{\begin{matrix}\cdot01573\\ \cdot01288\\ \cdot00169\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\ 1\\ \\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\ 80\\ \\ \end{matrix}\right\} & \text{are} & \left\{\begin{matrix}100\cdot6\,\text{W.G.}\\ 82\cdot4\,\text{A. G}\\ 10\cdot8\,\text{M.B.}\end{matrix}\right\} & \end{array}$$

PROBLEM XII.

To determine the Content of a Parabola an Inch in Depth.

RULE.

By the Pen.

Divide two-thirds of the product of the base and perpendicular altitude, by the number of cubic inches in the proposed integer.

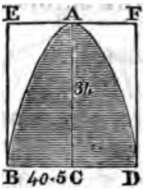
By the Sliding Rule.

Set the gage-point, for the proposed integer, on A, to two-thirds of the altitude on B, then against the base on A, will be found the content of the parabola on B.

The Rule for determining the content of a parabola, as has been already stated, is founded on this fact, that every parabola is equal to *two-thirds* of its circumscribing parallelogram.

EXAMPLE.

Let ABCD be a parabola, whereof the base BD, is 40·5 inches, and the altitude AC, 34 inches; the content is required for an inch in depth.



SOLUTION BY THE PEN.

27 two-thirds of the base
34 altitude or axis AC.

108
81

W.G. divisor 231) 918 (3·97 wine gallons.

A. G. divisor 282) 918 (3·25 ale gallons.

M. B. divisor 2150) 918 (0·42 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{ccccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} & \\ \text{G. P.} & \left\{\begin{array}{r}231\\282\\2150\end{array}\right\} & \text{to} & \left\{22\cdot6\right\} & \text{against} & \left\{40\cdot5\right\} & \text{are} & \left\{\begin{array}{l}3\cdot97\ \text{W.G.}\\3\cdot25\ \text{A. G.}\\0\cdot42\ \text{M. B.}\end{array}\right\} & \end{array}$$

BY THE FACTORS.

For Wine Gallons.

918 cubic inches in the parabola
·0043 W.G. factor for squares

2754
3672

Product 3·9474 *content* in wine gallons.

For Ale Gallons.

918 cubic inches in the parabola
·0035 A.G. factor for squares

4590
2754

Product 3·2130 *content* in ale gallons.

For Malt Bushels.

918 cubic inches in the parabola
·00046 M.B. factor for squares

5508
3672

Product 0·42228 *content* in malt bushels.

PROBLEM XIII.

To determine the Content of a Sector an Inch in Depth.

RULE.

By the Pen.

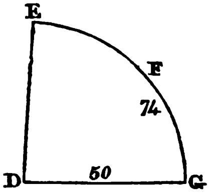
Multiply the radius by half the length of the arc, and divide the product by the number of cubic inches in the proposed integer, for the content.

By the Sliding Rule.

Set the gage-point for the proposed integer, on A, to the radius on B, and against half the length of the arc on A, will be the content on B.

EXAMPLE.

Let DEFG be a sector, whereof the radius DG is 50 inches, and the arc EFG, 74 inches; the content for an inch deep is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

2) 74 length of the arc

Quotient 37 half the arc
Multiply by 50 radius

W.G. divisor 231) 1850 (8·00 wine gallons.

A. G. divisor 282) 1850 (6·56 ale gallons.

M.B. divisor 2150) 1850 (0·86 malt bushels.

BY THE SLIDING RULE.

on A. on B. on A. on B.

$$\text{G. P.}\begin{Bmatrix}231\\282\\2150\end{Bmatrix}\text{to}\begin{Bmatrix}50\end{Bmatrix}\text{against}\begin{Bmatrix}37\end{Bmatrix}\text{are}\begin{Bmatrix}8\cdot00\ \text{W.G.}\\6\cdot56\ \text{A. G.}\\0\cdot86\ \text{M.B.}\end{Bmatrix}$$

PROBLEM XIV.

To determine the Content of a Segment of a Circle an Inch in Depth.

RULE.

By the Pen.

If the segment be small in comparison of its circle, and if *great* accuracy be not required, multiply the base by two-thirds of the altitude, and divide the product by the number of cubic inches in the proposed integer; the quotient will be the content *nearly.*

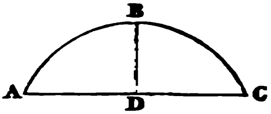
But if great accuracy be required, find the area of the segment by the Rule and Table given in PROB. XIV. SECT. I. OF MENSURATION, and when found, divide it by the number of cubic inches in the proposed integer, for the content.

By the Sliding Rule.

Set the gage-point for the proposed integer, on A, to two-thirds of the altitude on B, and opposite to the base on A, will be the content *nearly*, on B, provided the segment be less than a semicircle.

EXAMPLE.

Let ABC be the segment of a circle, whereof the base AC is 120 inches, and the altitude DB, 40 inches; the content in wine gallons, ale gallons, and malt bushels is required, for an inch in depth.



SOLUTION.

Less accurately.

80 two-thirds the base
40 altitude

W.G. divisor 231) 3200 (13·85 wine gallons.

A. G. divisor 282) 3200 (11·34 ale gallons.

M.B. divisor 2150) 3200 (1·49 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{cccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} \\ \text{G. P.} & \left\{\begin{array}{r}231\\282\\2150\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\80\\ \\ \end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\40\\ \\ \end{array}\right\} & \text{are} & \left\{\begin{array}{rl}13{\cdot}85 & \text{W.G.}\\11{\cdot}34 & \text{A.G.}\\1{\cdot}49 & \text{M.B.}\end{array}\right\}\end{array}$$

More correctly.

60 half the chord or base
60 half the base

Altitude 40) 3600 product

Quotient 90 altitude of the opposite segment
Add 40 altitude of the given segment

Sum 130 *diameter of the circle.*

Again,

Diameter	Altitude	Quotient
130)	40	(·333 *tabular versed sine.*

Now entering the Table of Circular Segments with ·333, we find the corresponding area to be ·228858.

Wherefore it will be,

·228858 tabular area
130 × 130 = 16900 square of the diam.

205972200
1373148
228858

W.G. divisor 231) 3867·700200 (16·74 wine gallons.

A. G. divisor 282) 3867·700200 (13·71 ale gallons.

M. B. divisor 2150) 3867·700200 (1·79 malt bushels.

By the first of these methods (which at best is only an approximation) the result is *considerably* less than the true content of the segment; the given altitude, however, in this instance was not diminutive in comparison of the diameter of the circle.

PROBLEM XV.

To determine the Content of an Annular Space an Inch in Depth.

RULE.

By the Pen.

Multiply the sum of the outer and inner diameters, by their difference, and divide the product by the circular divisor for the proposed integer.

By the Sliding Rule.

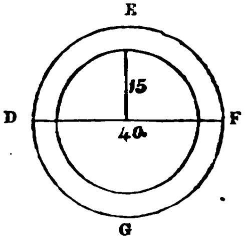
Set the circular gage-point, for the proposed integer, on D, to the *sum* of the diameters, on C, and opposite to the *difference* of the diameters on D, will be the content on C.

Or,

Set the circular divisor on A, to the difference of the diameters on B, and against the sum of the diameters on A, will be found the content on B.

EXAMPLE.

Let DEFG be a ring or annular space, whereof the outer diameter DF is 40 inches, and the inner diameter, 30 inches ; the content, at an inch deep, is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

40 outer diameter
30 inner diameter
70 sum
40 — 30 = 10 difference
700 product

Wherefore it will be,

W.G. cir. divisor 294·12) 700 (2·38 wine gallons.

A. G. cir. divisor 359) 700 (1·95 ale gallons.

M. B. cir. divisor 2738) 700 (0·25 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{lcccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{Cir. G. P.} & \left\{\begin{array}{r}17{\cdot}15\\18{\cdot}95\\52{\cdot}32\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\70\\ \\\end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\10\\ \\\end{array}\right\} & \text{are} & \left\{\begin{array}{l}2{\cdot}38\text{ W.G.}\\1{\cdot}95\text{ A. G.}\\0{\cdot}25\text{ M. B.}\end{array}\right\}\end{array}$$

Or,

$$\begin{array}{lcccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} \\ \text{Cir. Divis.} & \left\{\begin{array}{r}294\\359\\2738\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\10\\ \\\end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\70\\ \\\end{array}\right\} & \text{are} & \left\{\begin{array}{l}2{\cdot}38\text{ W. G.}\\1{\cdot}95\text{ A. G.}\\0{\cdot}25\text{ M. B.}\end{array}\right\}\end{array}$$

PROBLEM XVI.

To determine the Content, for an Inch in Depth, of the Segment of an Ellipsis, the Base of the Segment being parallel either to the transverse or to the conjugate Diameter, both of which must be given.

CASE 1.

When the segment's base is parallel to the transverse axis.

RULE.

Find the content of a circular segment of the same altitude, for an inch in thickness, the diameter of the circle being equal to the conjugate diameter of the ellipsis. Then multiply this content by the transverse axis, and divide the product by the conjugate, for the content of the elliptical segment.

CASE II.

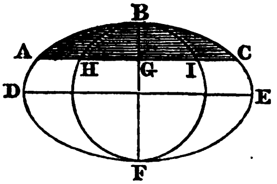
When the segment's base is parallel to the conjugate axis.

RULE.

Find the content of a circular segment of the same altitude, for an inch in thickness, the diameter of the circle being equal to the transverse diameter of the ellipsis. Then multiply the content by the conjugate, and divide by the transverse axis, for the content required.

EXAMPLE 1.

Let ABC be the segment of an ellipsis cut by a right line parallel to the transverse axis DE, and let the altitude BG, of the segment, be 9 inches; the transverse diameter DE, being 75 inches; and the conjugate diameter BF, 45 inches: the content, for an inch deep, is required in wine gallons, ale gallons, and malt bushels.



SOLUTION.

The altitude 9, divided by 45, the diameter of the inscribed circle, quotes ·2 for the *versed sine* with which to enter the Table of Circular Segments, and the area corresponding is

·111823 tabular number
45 × 45 = 2025 square of the diameter
559115
223646
2236460

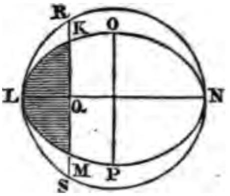
W.G. divisor 231)226·441575(0·98 quotient
75 longer axis
Conjugate 45) 73·50 (1·63 wine gallons.

A.G. divisor 282)226·441575(0·80 quotient
75 longer axis
Conjugate 45) 60·00 (1·33 ale gallons.

M.B. divisor 2150)226·441575(0·10 quotient
75 longer axis
Conjugate 45) 7·50 (0·16 malt bushels.

EXAMPLE 2.

Let KLM be the segment of an ellipsis, cut by a right line parallel to the shorter axis, the altitude QL, of the segment being 20 inches, the conjugate axis OP, 54 inches, and the transverse LN, 80 inches; the content, for an inch deep, is required in wine gallons, ale gallons, and malt bushels.



SOLUTION.

The altitude 20, divided by 80, the diameter of the circumscribing circle, quotes ·25 for the *versed sine* with which to enter the Table of Circular Segments; and the area corresponding is

·153546 tabular number
80 × 80 = 6400 square of the diameter
614184
921276

W.G. divisor 231) 982·6944(4·25 quotient
54 conjugate axis
Transverse 80)229·5(2·87 wine gallons.

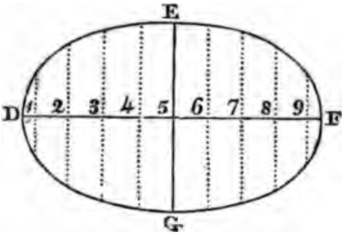
A. G. divisor 282) 982·6944(3·48 quotient
54 conjugate axis
Transverse 80)187·92(2·35 ale gallons.

M. B. divisor 2150) 982·6944(0·45 quotient
54 conjugate axis
Transverse 80) 24·30(0·30 malt bushels.

PROBLEM XVII.

To determine, by the Method of Equidistant Ordinates, the Content of any Curvilineal plane Space an Inch deep.

Let DEFG be an elliptical plane figure, not a true oval; the content for an inch deep, is required in wine gallons, ale gallons, and malt bushels, by the method of equidistant ordinates.



Draw two diameters DF, EG, intersecting each other at right angles in the point 5 (the middle of the figure) and such that DF shall be the longest possible, EG being the shortest.

Then parallel to EG, at equal distances on each side of it, draw any number of ordinates, cutting DF in the points 1, 2, 3, 4, 6, 7, 8, and 9. The total of ordinates in this instance is 9, an *odd* number; but the number of ordinates will always be *odd*, since equal distances are set off on both sides of the middle ordinate.

Let the length of the transverse axis, and ordinates, the equidistance, and altitude of the segments, be as follows:

Transverse axis DF = 154 inches
1st ordinate = 44 inches
2nd ——— = 76 inches
3rd. ——— = 90 inches
4th ——— = 96 inches
5th ——— = 100 inches
6th ——— = 96 inches
7th ——— = 88 inches
8th ——— = 72 inches
9th ——— = 40 inches
Equidistance = 18 inches
Altitude of both Segments 10 inches

RULE.

To the sum of the first and last ordinates add four times the sum of all the *even* ordinates, with double the sum of all the *odd* ones, except the first and last. Then multiply the total by one-third part the equidistance, and the product will be the number of cubic inches between the extreme ordinates, for an inch in depth.

Again, for the two segments, multiply the sum of the first and last ordinates by one-third part the sum of the altitudes, and the product will be the content of both segments in cubic inches, for an inch in depth.

Lastly, add these two contents together, and divide the sum by the number of cubic inches in the proposed integer.

SOLUTION.

			Inches
First ordinate	44 inches		
Last ———	40		
	— sum		84
2nd ordinate	76		
4th ———	96		
6th ———	96		
8th ———	72		
	— sum	340	
Multiply by		4	
		—— product	1360
3rd ordinate	90		
5th ———	100		
7th ———	88		
	— sum	278	
Multiply by		2	
		—— product	556
		Total	2000
Multiply by one-third the equidistance			6
Content, in cubic inches, between the extreme ordinates			12000
First ordinate	44		
Last ———	40		
	— sum		84
Multiply by one-third the altitudes = $\frac{10}{3}$ = 3·3			
Content of the segments in cubic inches			280
WHOLE CONTENT in cubic inches			12280

Wherefore it will be,

W.G. divisor 231) 12280 (53·16 wine gallons.

A. G. divisor 282) 12280 (43·54 ale gallons.

M.B. divisor 2150) 12280 (5·71 malt bushels.

Various other methods of determining the area of a curvilineal space have been proposed, and put in practice; but none of them has been found to apply so generally to every kind of curve, or with equal facility to give so accurate results, as the above. To enlarge on them, therefore, would be foreign to our purpose.

PROBLEM XVIII.

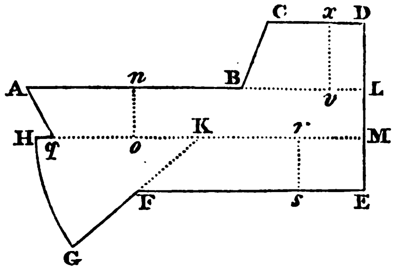
To determine the Content, for an Inch Deep, of an Irregular plane Figure.

RULE.

Divide the given figure into figures that can be measured by the foregoing problems, and having determined each of the figures, add the several contents in cubic inches together, and divide the sum by the number of cubic inches in the proposed integer.

EXAMPLE.

Let ABCDEFGH be an irregular plane figure, of which the content, for an inch in depth, is required in wine gallons, ale gallons, and malt bushels.



After a little inspection, a very rational way of dividing the given figure, appears to be as follows:

Find the centre K of the circular curve HG, and complete the sector GHKF. HK seems to be parallel to AB; let, therefore, these two lines be prolonged to L and M. It now appears that BL is parallel to CD, and that KM is parallel to FE. Wherefore take the perpendicular breadths xv, no, rs; and the three figures BCDL, AqML, FKME, come all under the description of figures in PROB. VII.

The dimensions necessary to be known, and which we shall suppose known, are the following:

CD = 51 inches
BL = 61 inches
xv = 36 inches
AL = 173 inches
qM = 163 inches
no = 26 inches
KM = 83 inches
FE = 117 inches

rs = 28 inches
GK = 90 inches
The arc GH = 66 inches

SOLUTION.

51 side CD.
61 line BL.
2) 112 sum
56 half sum
Multiply by 36 perpendicular *xv*.
336
168
Product 2016 *cubic inches in* BCDL.

173 line AL.
163 line *q*M.
2)336 sum
168 half sum
Multiply by 26 perpendicular *no*.
1008
336
Product 4368 *cubic inches in* A*q*ML.

83 line KM.
117 side FE.
200 sum

2) 200 sum brought over
100 half sum
Multiply by 28 perpendicular *rs.*
Product 2800 *cubic inches in* KFEM.

2) 66 arc GH.
33 half the arc
Multiply by 90 radius GH.
Product 2970 *cubic inches in the sector.*

These four figures constituting the whole of the given figure, it will be,

2016 content of BCDL.
4368 content of AqML.
2800 content of KFEM.
2970 content of the sector GHK.

W.G. divisor 231)12154 (52·61 wine gallons.

A.G. divisor 282)12154 (43·10 ale gallons.

M.B. divisor 2150)12154 (5·65 malt bushels.

It has been stated in the First Section that no general Rule can be given for the division of irregular figures into compartments. The results, however, will in every way, be the same, or nearly the same; and to obtain these in the shortest and easiest way is the only DESIDERATUM.

SECTION III.

OF SOLIDS PROPERLY SO CALLED.

PROBLEM I.

To determine the Content of a Cube.

RULE.

By the Pen.

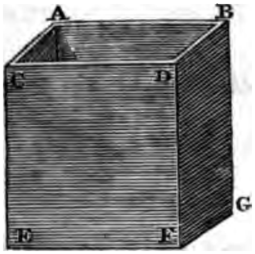
Multiply the side of the cube by itself, and the product again by the side. Then divide the last product by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the square gage point, for the proposed integer on D, to the number expressing the side of the cube on C, then against the same number on D, will be found the content on C.

EXAMPLE.

Let ABCDEFG be a cistern in the form of a cube, whereof the side is 100 inches; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

100 side of the cube
Multiply by 100 side
Product 10000 square of the side
Multiply by 100 side
Product 1000000 *cubic inches in the cube.*

And,

W.G. divisor 231)1000000 (4329·00 wine gallons.

A. G. divisor 282)1000000 (3546·10 ale gallons.

M. B. divisor 2150)1000000 (465·11 malt bushels.

BY THE SLIDING RULE.

	on D.		on C.		on D.		on C.
Sq.G.P.	15·19 16·79 46·37	to	100	against	100	are	4329·0 W.G. 3546·1 A. G. 465·1 M.B.

PROBLEM II.

To determine the Content of a Parallelopipedon.

RULE.

By the Pen.

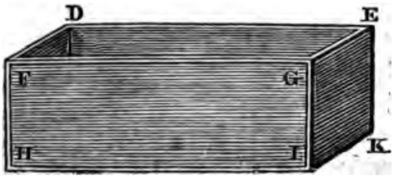
Multiply the length by the breadth, and the product by the depth: then divide the last product by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Find a mean proportional between the length and breadth; then set the square gage point on D, to the depth on C, and opposite to the mean proportional on D, will be the content on C.

EXAMPLE.

Let DEFGHIK be a box in the form of a parallelopipedon, the length DE being 80 inches; the breadth, EG, 20 inches; and the depth, EK, 30 inches: the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

80 length
Multiply by 20 breadth

Product 1600 *area of the base.*
Multiply by 30 breadth

W.G. divisor 231) 48000 (207·79 wine gallons.

A. G. divisor 282) 48000 (170·21 ale gallons.

M. B. divisor 2150) 48000 (22·32 malt bushels.

BY THE SLIDING RULE.

Set 80 on C, to 80 on D, and against 20 on C, are 40 on D, for *the mean proportional between the length and breadth of the box.*

Then,

$$\begin{array}{lccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{Sq. G.P.} & \left\{\begin{matrix}15{\cdot}19\\16{\cdot}79\\46{\cdot}37\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\30\\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\40\\ \end{matrix}\right\} & \text{are} & \left\{\begin{matrix}207{\cdot}8 & \text{W.G.}\\170{\cdot}2 & \text{A. G.}\\22{\cdot}3 & \text{M.B.}\end{matrix}\right\}\end{array}$$

PROBLEM III.

To determine the Content of a Cylinder.

RULE.

By the Pen.

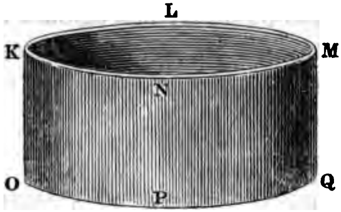
Find the area of the base, and multiply it by the depth: then divide the product (if in cubic inches) by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the circular gage point on D, to the altitude or depth on C, and against the diameter on D, will be the content on C.

EXAMPLE.

Let KLMNOPQ be a cylindrical vessel, the diameter KM, being 38 inches, and the depth NP, 15·16 inches; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

38 diameter
Multiply by 38 diameter
304
114
Product 1444 square of the diameter
Multiply by ·7854 constant decimal
Product 1134·117

```
Product     1134·117  area of the base.
Multiply by    15·16  depth of the vessel
            ________
             6804702
            1134117
           5670585
          1134117
         ___________
Product  17193·21372  content in cubic inches.
         ___________
```

Lastly,

W.G. divisor 231) 17193·21 (74·43 wine gallons.

A. G. divisor 282) 17193·21 (60·97 ale gallons.

M.B. divisor 2150) 17193·21 (7·99 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{ccccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C.G.P.} & \left\{\begin{matrix}17\cdot15\\18\cdot95\\52\cdot32\end{matrix}\right\} & \text{to} & \left\{15\cdot16\right\} & \text{against} & \left\{38\right\} & \text{are} & \left\{\begin{matrix}74\cdot43\text{ W.G.}\\60\cdot97\text{ A. G.}\\8\cdot00\text{ M. B.}\end{matrix}\right\}\end{array}$$

PROBLEM IV.

To determine the Content of a Prism.

RULE.

By the Pen.

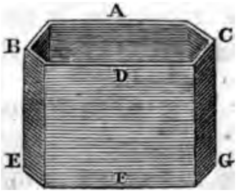
Find the area of the base, and multiply it by the depth; then divide the product (if in cubic inches) by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Find the content, for an inch deep, by the pen; then set 1 on B, to this content, on A, and against the depth on B, will be the whole content on A.

EXAMPLE.

Let ABCDEFG be a vessel in the form of a prism with a regular hexagonal base, the side of the base being 40 inches, and the depth of the vessel, 32 inches; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

For Wine Gallons.

·011246 tabular number for hexagons
40 × 40 = 1600 square of the side

67476
11246

Wine gallons 17·9936 *content at an inch deep.*
Multiply by 32 depth of the vessel

359872
539808

Product 575·7952 *whole content* in wine gallons.

For Ale Gallons.

	·009212	tabular number for hexagons
40 × 40 =	1600	square of the side
	55272	
	9212	
Ale gallons	14·7392	*content at an inch deep.*
Multiply by	32	depth of the vessel
	294784	
	442176	
Product	471·6544	*whole content* in ale gallons.

For Malt Bushels.

	·001208	tabular number for hexagons
40 × 40 =	1600	square of the side
	7248	
	1208	
Malt bushels	1·9328	*content at an inch deep.*
Multiply by	32	depth of the vessel
	38656	
	57984	
Product	61·8496	*whole content* in malt bushels.

BY THE SLIDING RULE.

The content, for an inch deep, has been found by the pen to be,

17·99 wine gallons
14·74 ale gallons
1·93 malt bushels

Wherefore it will be,

$$\begin{array}{cccccccc} & \text{on A.} & & \text{on B.} & & \text{on B.} & & \text{on A.} \\ \text{Inch Area} & \left\{\begin{matrix}17\cdot99\\14\cdot74\\1\cdot93\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\1\\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\32\\ \end{matrix}\right\} & \text{are} & \left\{\begin{matrix}575\cdot8\text{ W.G.}\\471\cdot6\text{ A. G.}\\61\cdot8\text{ M. B.}\end{matrix}\right\}\end{array}$$

PROBLEM V.

To determine the Content of any Solid with plane parallel Ends, and Sides perpendicular to them.

RULE.

By the Pen.

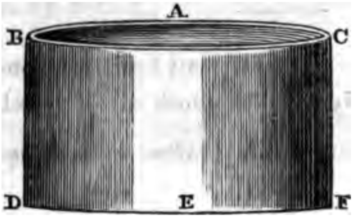
Find the content for an inch deep, and multiply it by the depth.

By the Sliding Rule.

Find the content for an inch deep, and set it, when found, on A, to 1 on B; then against the depth on B, will be the content on A.

EXAMPLE.

Let ABCDEF be a vessel in the form of a cylindroid, that is, having equal and parallel ovals for ends, and sides perpendicular to them. And let the depth be 50 inches, the transverse axis of the base being 154 inches, and the conjugate axis 54·96 inches; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

1. *For one inch deep.*

54·96 conjugate axis
Multiply by 154 transverse axis

21984
27480
5496

W.G. circ. divisor 294) 8463·84 (28·86 wine gallons.

A. G. circ. divisor 359) 8463·84 (23·57 ale gallons.

M. B. circ. divisor 2738) 8463·84 (3·09 malt bushels.

2. *For the whole depth.*

28·86 wine gallons at one inch
Multiply by 50 depth of the vessel

Product 1443·00 *content in wine gallons.*

23·57 ale gallons at one inch
Multiply by 50 depth of the vessel
Product 1178·50 *content in ale gallons.*

3·09 malt bushels at one inch
Multiply by 50 depth of the vessel
Product 154·50 *content in malt bushels.*

BY THE SLIDING RULE.

We set 154 on C, to 154 on D, and opposite to 54·96 on C, we find 92, *the mean proportional between the diameters of the base*, on D.

Then we set the circular gage-point, for the proposed integer, on D, to 1 on C, and opposite 92, the mean proportional on D, we find the content, at 1 inch deep, on C, as follows:

on D. on C. on D. on C.

$$\text{C. G. P.}\left\{\begin{array}{r}17{\cdot}15\\18{\cdot}95\\52{\cdot}32\end{array}\right\}\text{to}\left\{\begin{array}{c}\\1\\ \\ \end{array}\right\}\text{against}\left\{\begin{array}{c}\\92\\ \\ \end{array}\right\}\text{are}\left\{\begin{array}{r}28{\cdot}86\text{ W.G.}\\23{\cdot}57\text{ A. G.}\\3{\cdot}09\text{ M. B.}\end{array}\right\}$$

Lastly, we discover the whole content by the lines A and B, as below:

on A. on B. on B. on A.

$$\text{Inch areas}\left\{\begin{array}{r}28{\cdot}86\\23{\cdot}57\\3{\cdot}09\end{array}\right\}\text{to}\left\{\begin{array}{c}\\1\\ \\ \end{array}\right\}\text{against}\left\{\begin{array}{c}\\50\\ \\ \end{array}\right\}\text{are}\left\{\begin{array}{r}1443{\cdot}0\text{ W.G.}\\1178{\cdot}5\text{ A. G.}\\154{\cdot}5\text{ M. B.}\end{array}\right\}$$

In solving this example by the Sliding Rule we have adhered to the method necessary for figures with irregular bases: but in the instance before us the content may be found by one operation as below:

$$\begin{array}{lcccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{Cir.G.P.} & \left\{\begin{matrix}17{\cdot}15\\18{\cdot}95\\52{\cdot}32\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\50\\ \\\end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\92\\ \\\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}1443{\cdot}0\ \text{W.G.}\\1178{\cdot}5\ \text{A.G.}\\154{\cdot}5\ \text{M.B.}\end{matrix}\right\}\end{array}$$

PROBLEM VI.

To determine the Content of a Cone.

RULE.

By the Pen.

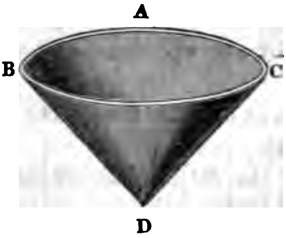
Find the area of the base, and multiply it by *one-third* part the altitude: then divide the product (if in cubic inches) by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the circular-gage point, for the proposed integer, on D, to *one-third* part the altitude on C, and opposite to the diameter of the base on D, will be the content on C.

EXAMPLE.

Let ABCD be a vessel in the form of a cone, the diameter BC being 60 inches, and the perpendicular depth, 36 inches; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

60 diameter
Multiply by 60 diameter
Product 3600 square of the diameter
Multiply by ·7854 constant decimal
47124
23562
Product 2827·44 *area of the base.*
Multiply by 12 one-third the altitude
W.G. divisor 231)33929·28 (146·88 wine gallons.
A. G. divisor 282)33929·28 (120·31 ale gallons.
M. B. divisor 2150)33929·28 (14·39 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{lccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C.G.P.} & \left\{\begin{array}{c}17{\cdot}15\\18{\cdot}95\\52{\cdot}32\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\12\\\\\end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\60\\\\\end{array}\right\} & \text{are} & \left\{\begin{array}{l}146{\cdot}88\ \text{W.G.}\\120{\cdot}31\ \text{A. G.}\\\ \ 14{\cdot}40\ \text{M.B.}\end{array}\right\}\end{array}$$

PROBLEM VII.

To determine the Content of a Pyramid.

RULE.

By the Pen.

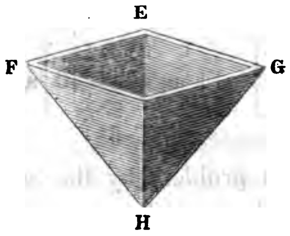
Find the area of the base, and multiply it by *one-third* part the altitude: then divide the product (if in cubic inches) by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Find the content for an inch deep on the base, and set it on A, to 1 on B, then against *one-third* part the depth on B, will be the content on A.

EXAMPLE.

Let EFGH be a vessel in the form of a pyramid with a square base, the altitude or depth being 30 inches; and EF, the side of the base, 22 inches: the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

22 side of the base
Multiply by 22 side

44
44

Product 484 *area of the base.*
Multiply by 10 one-third the depth

W.G. divisor 231) 4840 (20·95 wine gallons.

A. G. divisor 282) 4840 (17·16 ale gallons.

M. B. divisor 2150) 4840 (2·25 malt bushels.

BY THE SLIDING RULE.

1. *For the content an inch deep on the base.*

$$\begin{array}{ccccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} & \\ \text{Sq. G. P.} & \left\{\begin{matrix}15{\cdot}19\\16{\cdot}79\\46{\cdot}37\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\1\\\end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\22\\\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}2{\cdot}095\ \text{W.G.}\\1{\cdot}716\ \text{A. G.}\\0{\cdot}225\ \text{M. B.}\end{matrix}\right\} & \end{array}$$

2. *For the content of the whole pyramid.*

$$\begin{array}{ccccccccc} & \text{on A.} & & \text{on B.} & & \text{on B.} & & \text{on A.} & \\ \text{Inch areas} & \left\{\begin{matrix}2{\cdot}095\\1{\cdot}716\\0{\cdot}225\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\1\\\end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\10\\\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}20{\cdot}95\ \text{W.G.}\\17{\cdot}16\ \text{A. G.}\\2{\cdot}25\ \text{M. B.}\end{matrix}\right\} & \end{array}$$

In solving this problem by the Sliding Rule, we have adhered to the method in general to be pursued, although in the instance of a pyramid on a square base, the content may be found at one operation, as follows:

$$\begin{array}{lccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{Sq. G.P.} & \left\{\begin{matrix}15\cdot19\\16\cdot79\\46\cdot37\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\10\\ \\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\22\\ \\ \end{matrix}\right\} & \text{are} & \left\{\begin{matrix}20\cdot95\text{ W.G.}\\17\cdot16\text{ A.G.}\\2\cdot25\text{ M.B.}\end{matrix}\right\}\end{array}$$

PROBLEM VIII.

To determine the Content of a Sphere.

RULE.

By the Pen.

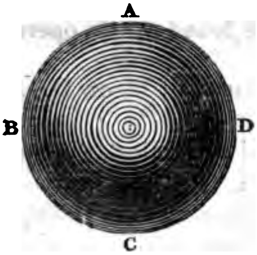
Multiply the cube of the diameter in inches, by the decimal ·5236, and the product will be the content in cubic inches: then divide by the number of cubic inches in the proposed integer.

By the Sliding Rule.

Set the circular gage-point, for the proposed integer, on D, to *two-thirds* of the diameter on C, and against the entire diameter on D, will be the content on C.

EXAMPLE.

Let ABCD be a globe or sphere 30 inches in diameter; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

	30	diameter
Multiply by	30	diameter
Product	900	square of the diameter
Multiply by	30	diameter
Product	27000	cube of the diameter
Multiply by	·5236	constant decimal
	36652	
	10472	
Product	14137·2	*content in cubic inches.*

Wherefore it will be,

W.G. divisor 231) 14137·2 (61·20 wine gallons.

A. G. divisor 282) 14137·2 (50·13 ale gallons.

M. B. divisor 2150) 14137·2 (6·57 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{lcccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G. P.} & \left\{\begin{array}{l} 17{\cdot}15 \\ 18{\cdot}95 \\ 52{\cdot}32 \end{array}\right\} & \text{to} & \left\{\begin{array}{c} \\ 20 \\ \\ \end{array}\right\} & \text{against} & \left\{\begin{array}{c} \\ 30 \\ \\ \end{array}\right\} & \text{are} & \left\{\begin{array}{l} 61{\cdot}20 \text{ W.G.} \\ 50{\cdot}13 \text{ A. G.} \\ 6{\cdot}57 \text{ M.B.} \end{array}\right\} \end{array}$$

The Rule by the pen is founded on this property, that globes are to one another as the cubes of their diameters, and that when the diameter is 1 the content is ·5236.

The reason of setting *two-thirds* of the diameter on the line C, to the circular gage-point on D, is, that every globe, is equal *to two-thirds* of its circumscribing cylinder; or to a cylinder two-thirds the altitude of the globe, on a base, having for diameter the diameter of the globe.

All this being well understood, it will be easy to construct a set of divisors and gage-points for spheres, as follows:

Divisor	Dividend	Quotient	Square roots
·5236)	231·0000	(441·18	(21·0 wine gallons.
·5236)	282·0000	(538·58	(23·2 ale gallons.
·5236)	2150·4200	(4107·0	(64·1 malt bushels.
Divisor	*Dividend*	*Sph. Divisors*	*Spherical Gage-points.*

In like manner may spherical divisors, and spherical gage-points, be obtained for any other integer whatever.

USE OF THE SPHERICAL DIVISORS AND SPHERICAL GAGE-POINTS.

To determine the Content of a Globe by the Pen.

RULE.

Divide the cube of the diameter by the spherical divisor for the proposed integer, and the quotient will be the content.

To determine the Content of a Globe by the Sliding Rule.

RULE.

Set the spherical gage-point, for the proposed integer on D, to the diameter of the globe on C, and against the diameter on D, will be the content on C.

EXAMPLE.

The content of a globe, 30 inches in diameter, is required in wine gallons, ale gallons, and malt bushels.

SOLUTION BY THE PEN.

30 diameter
Multiply by 30 diameter

Product 900 square of the diameter
Multiply by 30 diameter

Product 27000 cube of the diameter

Wherefore it will be,

W.G. sph. divisor 441·18) 27000 (61·19 wine gallons.

A. G. sph. divisor 538·58) 27000 (50·13 ale gallons.

M.B. sph. divisor 4107) 27000 (6·57 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{cccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{Sph. G. P.} & \left\{\begin{matrix}21\cdot0\\23\cdot2\\64\cdot1\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}30\end{matrix}\right\} & \text{against} & \left\{\begin{matrix}30\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}61\cdot2\text{ W.G.}\\50\cdot1\text{ A. G.}\\6\cdot6\text{ M.B.}\end{matrix}\right\}\end{array}$$

PROBLEM IX.

To determine the Content of a Spheroid.

RULE.

By the Pen.

Multiply the transverse diameter by the square of the conjugate, and divide the product by the spherical divisor for the proposed integer.

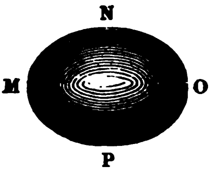
By the Sliding Rule.

Set the spherical gage-point, for the proposed integer, on D, to the transverse diameter on C, and against the conjugate on D, will be the content on C.

EXAMPLE.

Let MNOP be a spheroid, whereof the longer axis, MO, is 42 inches, and the shorter axis, NP, 31 inches;

the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

42 transverse axis MO.
$31 \times 31 = 961$ square of NP.

1922
3844

W.G. sph. divisor 441·2) 40362 (91·48 wine gallons.

A. G. sph. divisor 538·6) 40362 (74·93 ale gallons.

M. B. sph. divisor 4107) 40362 (9·83 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{ccccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} & \\ \text{Sph. G. P.} & \left\{\begin{array}{c}21{\cdot}0\\23{\cdot}2\\64{\cdot}1\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\42\\ \\ \end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\31\\ \\ \end{array}\right\} & \text{are} & \left\{\begin{array}{ll}91{\cdot}48 & \text{W.G.}\\74{\cdot}93 & \text{A. G.}\\9{\cdot}83 & \text{M.B.}\end{array}\right\} & \end{array}$$

PROBLEM X.

To determine the Content of a Parabolic Conoid or Paraboloid.

RULE.

By the Pen.

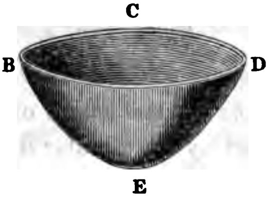
Multiply the square of the diameter of the base, by *half* the altitude, and divide the product by the circular divisor for the proposed integer.

By the Sliding Rule.

Set the circular gage-point, for the proposed integer, on D, to half the altitude on C, and against the diameter of the base, on D, will be the content on C.

EXAMPLE.

Let BCDE be a vessel in the form of a paraboloid, the diameter BD, of the base, being 40 inches, and the altitude or depth, 18 inches; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

40 diameter of the base

Multiply by 40 diameter of the base

Product 1600 square of the diameter

Multiply by 9 half the altitude

W.G. circ. divisor 294) 14400 (48·99 wine gallons.

A. G. circ. divisor 359) 14400 (40·11 ale gallons.

M. B. circ. divisor 2738) 14400 (5·25 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{cccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G. P.} & \left\{\begin{array}{c}17{\cdot}15\\18{\cdot}95\\52{\cdot}32\end{array}\right\} & \text{to} & \left\{\begin{array}{c}9\end{array}\right\} & \text{against} & \left\{\begin{array}{c}40\end{array}\right\} & \text{are} & \left\{\begin{array}{r}49{\cdot}0\text{ W.G.}\\40{\cdot}1\text{ A.G.}\\5{\cdot}2\text{ M.B.}\end{array}\right\}\end{array}$$

Every parabolic conoid is equal to half its circumscribing cylinder, and on this property is founded the Rule.

PROBLEM XI.

To determine the Content of a Parabolic Spindle.

RULE.

By the Pen.

Multiply the square of the middle or conjugate axis, by *eight-fifteenths* of the length of the spindle, and divide the product by the circular divisor for the proposed integer.

By the Sliding Rule.

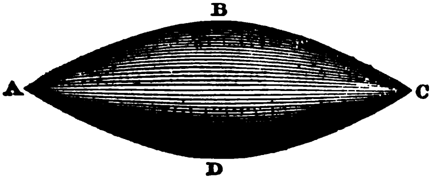
Set 15 on A, to the length of the spindle on B, then against 8 on A, will be *eight-fifteenths* of the length on B, which call the *reserved quotient.*

Then set the circular gage-point, for the proposed integer, on D, to the *reserved quotient* on C, and opposite to the conjugate or middle axis on D, will be the content on C.

EXAMPLE.

Let ABCD be a spheroid, in length 60 inches, and

20 inches in diameter at the middle; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

20 shorter axis BD.
Multiply by 20 shorter axis BD.

Product 400 square of BD.
Multiply by 32 eight-fifteenths of AC.

W.G. cir. divisor 294)12800 (43·53 wine gallons.

A. G. cir. divisor 359)12800 (35·65 ale gallons.

M.B. cir. divisor 2738)12800 (4·67 malt bushels.

BY THE SLIDING RULE.

$$\begin{array}{l} \text{on A.}\quad\text{on B.}\qquad\quad\text{on A.}\qquad\text{on B.} \\ \text{Set}\left\{15\right\}\text{to}\left\{60\right\}\text{against}\left\{8\right\}\text{are}\left\{32\right\}\textit{reserved quot.} \end{array}$$

Then,

$$\begin{array}{l} \qquad\qquad\text{on D.}\quad\text{on C.}\qquad\quad\text{on D.}\qquad\quad\text{on C.} \\ \text{C. G. P.}\left\{\begin{matrix}17.15\\18{\cdot}95\\52{\cdot}32\end{matrix}\right\}\text{to}\left\{32\right\}\text{against}\left\{20\right\}\text{are}\left\{\begin{matrix}43{\cdot}53 \text{ W.G.}\\35{\cdot}65 \text{ A. G.}\\4{\cdot}67 \text{ M. B.}\end{matrix}\right\} \end{array}$$

Every parabolic spindle is equal to *eight-fifteenths* of its circumscribing cylinder, and on this property is founded the Rule.

It is evident that divisors and gage-points may be constructed for this particular solid, and applied in the manner that the circular divisors and circular gage-points are applied to the cylinder, or the spherical divisors and spherical gage-points to the spheroid. Thus, the divisors would be respectively fifteen times the circular divisors divided by 8, and the gage-points would be the square roots of these parabolic divisors. Now 15, divided by 8, will quote 1·875; therefore the circular divisors multiplied by 1·875 will be the proper divisors for parabolic spindles.

Having thus pointed out the method of construction, we shall subjoin the divisors and gage-points, by which the content of a spindle may, with the greatest readiness, be found.

TABLE OF DIVISORS AND GAGE-POINTS FOR PARABOLIC SPINDLES.

DENOMINATION	DIVISORS	GAGE-POINTS
Wine gallon	551·47	23·48
Ale gallon	673·22	25·94
Malt bushel	5133·75	71·65

The circular divisor, for any other integer, multiplied by 1·875, will be the parabolic-spindle divisor for that integer, and the square root of the spindle divisor will be the spindle gage-point.

USE OF THE TABLE.

To determine the Content of a Parabolic Spindle by the Pen.

RULE.

Multiply the square of the middle diameter by the length of the spindle, and divide the product by the parabolic-spindle divisor.

To determine the Content of a Parabolic Spindle by the Sliding Rule.

RULE.

Set the parabolic-spindle gage-point, for the proposed integer, on D, to the transverse or longest diameter, on C, and against the middle diameter on D, will be the content on C.

EXAMPLE.

The content of a parabolic spindle, 60 inches in length, and 20 inches in diameter at the middle, is required in wine gallons, ale gallons, and malt bushels.

SOLUTION BY THE PEN.

Middle diameter	20
Middle diameter	20
Square of the conjugate	400
Length of the spindle	60
Product	24000

And,

W.G. par. sp. divisor 551·47) 24000 (43·52 W.G.

A. G. par. sp. divisor 673·22) 24000 (35·65 A. G.

M. B. par. sp. divisor 5133·75) 24000 (4·67 M.B.

BY THE SLIDING RULE.

$$\begin{array}{llllllll} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{P.sp.G.P.} & \left\{\begin{array}{c}23{\cdot}5\\25{\cdot}9\\71{\cdot}6\end{array}\right\} & \text{to} & \left\{\begin{array}{c}\\60\\ \end{array}\right\} & \text{against} & \left\{\begin{array}{c}\\20\\ \end{array}\right\} & \text{are} & \left\{\begin{array}{l}43{\cdot}52\ \text{W.G.}\\35{\cdot}65\ \text{A. G.}\\4{\cdot}67\ \text{M.B.}\end{array}\right\}\end{array}$$

PROBLEM XII.

To determine the Content of the Frustum of a Cone.

RULE.

By the Pen.

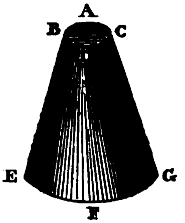
To the square of the difference of the diameters of the two ends, add three times the product of the greater diameter multiplied by the less. Then multiply the sum by *one-third* the altitude, and divide the result by the circular divisor of the proposed integer, for the content.

By the Sliding Rule.

Find a mean proportional between the diameters of the ends, and set the circular gage-point, for the proposed integer, on D, to *one-third* the depth on C, then against each of the diameters, and also the mean proportional between them, on D, take the content on C. The sum of the three contents will be the content of the frustum.

EXAMPLE.

Let ABDCEFG be the frustum of a cone; BC, the diameter at the top, being 24 inches; the altitude, 75 inches; and EG, the diameter of the base, 64 inches: the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

	64	greater diameter
Subtract	24	less diameter
Remainder	40	*difference of the diameters.*
	40	
	1600	square of the difference
64 × 24 × 3 =	4608	triple product
Sum	6208	
Multiply by	25	one-third the altitude
	31040	
	12416	
Product	155200	

Hence it will be,

W.G. circ. divisor 294)155200(527·89 wine gallons.

A. G. circ. divisor 359)155200(432·31 ale gallons.

M. B. circ. divisor 2738)155200(56·68 malt bushels.

BY THE SLIDING RULE.

1. *For a mean proportional between the diameters.*

Set 64 on C, to 64 on D, and against 24 on C, are 39·2 on D, for the *mean proportional.*

2. *For the solution by help of the mean proportional.*

$$\begin{array}{lccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G.P.} & \left\{\begin{matrix}\text{W.G.}\\ 17\cdot15\\ \end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\ 25\\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}24\\ 39\cdot2\\ 64\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}48.99\\ 130\cdot61\\ 348\cdot16\end{matrix}\right\} \end{array}$$

SUM 527·76 W.G.

$$\begin{array}{lccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G.P.} & \left\{\begin{matrix}\text{A. G.}\\ 18\cdot95\\ \end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\ 25\\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}24\\ 39\cdot2\\ 64\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}40\cdot10\\ 106\cdot99\\ 285\cdot19\end{matrix}\right\} \end{array}$$

SUM 432·28 A. G.

$$\begin{array}{lccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G.P.} & \left\{\begin{matrix}\text{M.B.}\\ 52\cdot32\\ \end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\ 25\\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}24\\ 39\cdot2\\ 64\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}5\cdot25\\ 14\cdot14\\ 37\cdot21\end{matrix}\right\} \end{array}$$

SUM 56·60 M. B.

PROBLEM XIII.

To determine Intermediate Diameters at any assigned Depth of the Frustum of a Cone, having the top and bottom Diameters given.

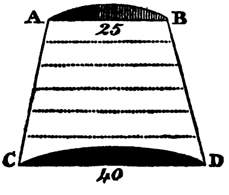
RULE.

From the greater diameter subtract the less, and divide the remainder by the depth of the frustum: the quotient will be the *differential factor* for any assigned depth.

To employ this factor, multiply the proposed depth by it, and the product will be the *defect* of the less diameter to equal the diameter of the frustum at that depth, when the depth is counted from the smaller end; but the *excess* of the greater diameter above the length of the diameter of the frustum at that depth, when the depth is counted from the larger end.

EXAMPLE.

Let ABCD be a perpendicular section of the frustum of a cone by a plane co-inciding with the axis: and let the less diameter AB, be 25 inches; the greater diameter, CD, 40 inches; and the depth 30 inches: intermediate diameters at every 5 inches are required.



SOLUTION.

40 greater diameter
Subtract 25 less diameter

Depth 30) 15 remainder

Quotient ·5 *differential factor.*

Then

5 × ·5 = 2·5 increase of diameter at 5 inches
10 × ·5 = 5·0 ....at 10 inches
15 × ·5 = 7·5 ....at 15 inches
20 × ·5 = 10·0 ....at 20 inches
25 × ·5 = 12·5 ....at 25 inches

But

25 + 2·5 = 27·5 diameter at 5 inches deep
25 + 5·0 = 30·0 ....at 10 inches deep
25 + 7·5 = 32·5 ....at 15 inches deep
25 + 10·0 = 35·0 ....at 20 inches deep
25 + 12·5 = 37·5 ....at 25 inches deep

It is evident that, had the wider end of the frustum been uppermost, the numbers found by the factor would have been decrease of diameter, and must have been severally subtracted from 40 the greatest diameter.

The Rule for the frustum of a square pyramid, in respect of intermediate breadths, is manifestly the same as for the cone.

PROBLEM XIV.

To determine the Content of the Frustum of a Pyramid.

RULE.

By the Pen.

Find the area of each end, and, to the sum of the two areas, add the geometrical mean between them; then multiply by *one-third* the altitude, and the product will be the content in cubic inches; which being divided by the number of cubic inches in any proposed integer, will give the content in the denomination of that integer.

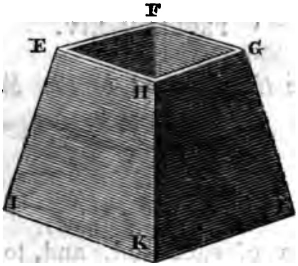
By the Sliding Rule.

Find the area of each of the two ends separately, and set either of them on C, to the same point on D; then against the other on C, will be the geometrical mean between them on D.

Next, set the gage-point, for the proposed integer, on A, to *one-third* the altitude on B, and against the sum of the three areas on A, will be the content on B.

EXAMPLE.

Let EFGHIKL be a vessel in the form of the frustum of a square-based pyramid, the depth being 48 inches, and the breadth of the ends, 21 inches, and 39 inches, respectively; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION.

21 side of the less square
Multiply by 21 side
21
42
Product 441 *area of the less end* EFGH.

39 side of the greater square
Multiply by 39 side
Product 1521 *area of the base.*
Multiply by 441 area of the less end
1521
6084
6084
Product 67|07|61 (819 *geometrical mean.*
64
161) 307
161
1629) 14661
14661
. . . . .

Lastly, for the sum of the three areas multiplied by one-third the depth, it will be,

441 area of the less end
819 mean area
1521 area of the base
2781 *sum.*
Multiply by 16 one-third the depth
16686
2781

W.G. divisor 231) 44496 (192·62 wine gallons.

A. G. divisor 282) 44496 (157·79 ale gallons.

M.B. divisor 2150) 44496 (20·69 malt bushels.

BY THE SLIDING RULE.

We set 1 on D, to 1 on C, and opposite to 21 on D, we find 441 on C, for the *area of the less end,* and opposite to 39 on D, we find 1521 on C, for the *area of the greater end.*

We now set 1521 on C, to 1521 on D, and looking against 441 on C, we discover 819 as the *geometrical mean proportional between the areas of the ends.*

Next it is,

441 less end
819 mean proportional
1521 base or greater end
2781 SUM.

Lastly,

$$\begin{array}{lcccccccc} & \text{on A.} & & \text{on B.} & & \text{on A.} & & \text{on B.} & \\ \text{G. P.} & \left\{\begin{matrix}231\\282\\2150\end{matrix}\right\} & \text{to} & \left\{\begin{matrix}\\16\\ \\ \end{matrix}\right\} & \text{against} & \left\{\begin{matrix}\\2781\\ \\ \end{matrix}\right\} & \text{are} & \left\{\begin{matrix}192{\cdot}62 & \text{W.G.}\\157{\cdot}79 & \text{A. G.}\\20{\cdot}69 & \text{M.B.}\end{matrix}\right\} & \end{array}$$

PROBLEM XV.

To determine the Content of a Solid with Parallel (but Unequal and Dissimilar) Ellipses for Ends, and Sides every way straight.

RULE.

To the transverse diameter of the greater end, add half the transverse diameter of the less end, and multiply the sum by the conjugate axis of the greater end; reserving the product.

Next, to the transverse diameter of the less end, add half the transverse diameter of the greater end, and multiply the sum by the conjugate axis of the less end.

Then to this product add the reserved product, and multiply the sum by *one-third* part the altitude. The result will be the square of the diameter of a circle, which, being an inch deep, shall be of equal content with the given figure. This last product, divided by the circular divisor of the proposed integer, will consequently be the content.

EXAMPLE.

A solid, 27 inches in depth, whose ends are ellipses (the transverse and conjugate diameters of the one being, respectively, 40 inches, and 30 inches; and of the other, 30 inches, and 20 inches) is given to be determined in wine gallons, ale gallons, and malt bushels.

SOLUTION.

	40	transverse of the greater end
Add	15	half the less transverse
	55	sum
Multiply by	30	greater conjugate
	1650	*reserved product.*
	30	transverse of the less end
Add	20	half the greater transverse
	50	sum
Multiply by	20	the less conjugate
	1000	product
Add	1650	reserved product
	2650	total
Multiply by	9	*one-third* the altitude
W.G. divisor 294)	23850	(81·12 wine gallons.
A. G. divisor 359)	23850	(66·43 ale gallons.
M.B. divisor 2738)	23850	(8·71 malt bushels.

COROLLARY.—Either end of the solid may be a circle without impairing the truth of the result.

PROBLEM XVI.

To determine the Content of a Solid with Parallel (but Unequal and Dissimilar) Rectangles for Ends, and Sides every way straight.

RULE.

To the length of the greater end, add half the length of the less, and multiply the sum by the breadth of the greater end; reserving the product.

Next, to the length of the less end, add half the length of the greater, and multiply the sum by the breadth of the less end.

Then to this product add the reserved product, and multiply the sum by *one-third* the altitude. The result will be the area of a square, which, being an inch deep, shall equal in content, the given solid. This area, divided by the number of cubic inches in the proposed integer, will consequently give the content required.

EXAMPLE.

A solid 15 inches deep, whereof one of the ends is a square, having 60 inches for its side; and the other end, a rectangle, 40 inches by 24; is given to be determined in wine gallons, ale gallons, and malt bushels.

SOLUTION.

	60	greater length
Add	20	half the less length
	80	sum
Multiply by	60	greater breadth
	4800	*reserved product.*

	40	the less length
Add	30	half the greater length
	70	sum
Multiply by	24	the less breadth
	1680	product
Add	4800	reserved product
	6480	total
Multiply by	5	*one-third* the depth
Product	32400	cubic inches.

Wherefore it will be,

W.G. divisor 231) 32400 (140·26 wine gallons.

A. G. divisor 282) 32400 (114·89 ale gallons.

M.B. divisor 2150) 32400 (15·07 malt bushels.

PROBLEM XVII.

To determine the Content of the UNGULA *of the Frustum of a Cone.*

RULE.

FOR THE GREATER HOOF.

Find a mean proportional between the greatest and least diameters of the frustum, and multiply it by the least diameter: then subtract the product from the square of the greatest diameter, and multiply the remainder by the greatest diameter.

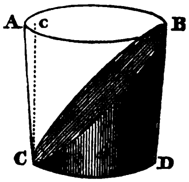
The last product multiplied by *one-third* the altitude, and divided, first by the difference of the diameters, and afterwards by the circular divisor corresponding to any proposed integer, will be the content in the denomination of that integer.

FOR THE LESS HOOF.

Find a mean proportional between the least and greatest diameters, as for the greater hoof, and multiply this proportional by the greatest diameter; then from the product subtract the square of the least diameter, and multiply the remainder by the least diameter. The last product multiplied by *one-third* part the frustum's height, and divided, first by the difference of the diameters, and afterwards by the circular divisor for any proposed integer, will give the content in the denomination of that integer.

EXAMPLE.

Let ABC be the greater hoof of the frustum of a cone, and BCD, the less hoof, the altitude C*c* being 39 inches; the diameter AB, where the frustum is the widest, 35 inches; and the least diameter CD, 31 inches: the content of each hoof is required in wine gallons, ale gallons, and malt bushels.



SOLUTION.

For the Greater Hoof.

35 greatest diameter

Multiply by 31 least diameter

35

105

Product 10|85 (33 *nearly*, mean proportional.

9

63) 185

189

4 deficient.

Again,

33 mean proportional
Multiply by 31 least diameter

33
99

Product 1023 *to be subtracted.*

And,

35 greatest diameter
Multiply by 35 greatest diameter

175
105

Product 1225 square of the greatest diameter
1023 product to be subtracted

202 remainder
Multiply by 35 greatest diameter

1010
606

7070 *product.*
Multiply by 13 one-third the altitude

21210
7070

35 — 31 = 4) 91910 product

Quotient 22977·5

Wherefore it will be,

W.G. circ. divisor 294) 22977·5 (78·15 wine gallons.

A. G. circ. divisor 359) 22977·5 (64·00 ale gallons.

M. B. circ. divisor 2738) 22977·5 (8·39 malt bushels.

For the Less Hoof.

The mean proportional between the greatest and least diameters, as already found, is,

33 mean proportional
Multiply by 35 greatest diameter
165
99
Product 1155
31 × 31 = 961 sq. of the least diameter
194 difference
Multiply by 31 least diameter
194
582
6014 *product.*
Multiply by 13 one-third the altitude
18042
6014
35 — 31 = 4) 78182 product
Quotient 19545·5

And,

W.G. circ. divisor 294) 19545·5 (66·48 wine gallons.

A. G. circ. divisor 359) 19545·5 (54·44 ale gallons.

M.B. circ. divisor 2738) 19545·5 (7·13 malt bushels

PROBLEM XVIII.

To determine the Content of the Ungula of a Square Pyramid.

RULE.

FOR THE GREATER HOOF.

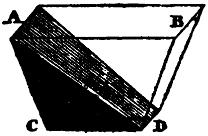
Multiply the greatest breadth by the least, and to the product add twice the square of the greatest: then multiply the sum by *one-sixth part* the altitude or depth, and divide the product by the number of cubic inches in the proposed integer.

FOR THE LESS HOOF.

Multiply the greatest breadth by the least, and to the product add twice the square of the least: then multiply the sum by *one-sixth* part the altitude or depth, and divide the product by the number of cubic inches in the proposed integer.

EXAMPLE.

Let ABD be the greater hoof, and ACD, the less hoof of the frustum of a square pyramid; the greatest width, AB, being 43 inches; the least, CD, 25 inches; and the perpendicular depth, 24 inches: the content of each hoof is required in wine gallons, ale gallons, and malt bushels.



SOLUTION.

For the Greater Hoof.

43 greatest breadth
Multiply by 25 least breadth
215
86
Product 1075
$43 \times 43 \times 2 = 3698$ twice the square of AB.
Sum 4773
Multiply by 4 one-sixth the depth
W.G. divisor 231) 19092 (82·65 wine gallons.

A. G. divisor 282) 19092 (67·70 ale gallons.

M.B. divisor 2150) 19092 (8·88 malt bushels.

For the Less Hoof.

43 greatest breadth
Multiply by 25 least breadth
215
86
Product 1075
$25 \times 25 \times 2 =$ 1250 twice the square of CD.
Sum 2325
Multiply by 4 one-sixth the altitude
W.G. divisor 231) 9300 (40·26 wine gallons.

A. G. divisor 282) 9300 (32·97 ale gallons.

M. B. divisor 2150) 9300 (4·32 malt bushels.

PROBLEM XIX.

To determine the Content of the Segment of a Sphere.

RULE.

By the Pen.

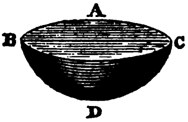
To the square of the altitude, add three times the square of half the diameter of the base, and multiply the sum by the altitude; then divide the product by the spherical divisor for the proposed integer, and the quotient will be the content.

By the Sliding Rule.

Set the circular gage-point, for the proposed integer, on D, to half the altitude on C, and opposite to the diameter of the segment's base on D, note the number on C; also without shifting the slide, note the number on C opposite to the altitude on D. Add these two numbers, and one-third part the latter, together, and the sum will be the content.

EXAMPLE.

Let ABCD be a segment of a globe, whereof the altitude is 20 inches, and BC the diameter of the base 80 inches; the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

20 altitude
Multiply by 20 altitude

400 square of the altitude
40×40×3 = 4800 thrice the sq. of half BC.

5200 sum
Multiply by 20 altitude

104000 *product*

Consequently it will be,

W.G. sph. divisor 441·18) 104000 (235·73 W.G.

A. G. sph. divisor 538·58) 104000 (193·10 A. G.

M. B. sph. divisor 4106·2) 104000 (25·32 M. B.

BY THE SLIDING RULE.

For Wine Gallons.

on D. on C. on D. on C.

$$\text{C.G.P.}\left\{17\cdot15\right\}\text{to}\left\{10\right\}\text{against}\left\{\begin{matrix}80\\20\end{matrix}\right\}\text{are}\left\{\begin{matrix}217\cdot6\\13\cdot6\end{matrix}\right\}$$

Add one-third the last 4·5

Sum 235·7 W. G.

For Ale Gallons.

on D. on C. on D. on C.

$$\text{C. G. P.}\left\{18\cdot95\right\}\text{to}\left\{10\right\}\text{against}\left\{\begin{matrix}80\\20\end{matrix}\right\}\text{are}\left\{\begin{matrix}178\cdot2\\11\cdot1\end{matrix}\right\}$$

Add one-third the last 3·7

Sum 193·0 A. G

For Malt Bushels.

on D. on C. on D. on C.

$$\text{C. G. P.}\left\{52\cdot32\right\}\text{to}\left\{10\right\}\text{against}\left\{\begin{matrix}80\\20\end{matrix}\right\}\text{are}\left\{\begin{matrix}23\cdot4\\1\cdot4\end{matrix}\right\}$$

Add one-third the last 0·5

Sum 25·3 B

PROBLEM XX.

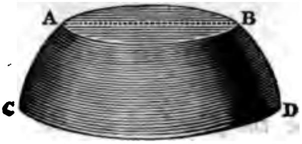
To determine the Content of the Frustum of a Globe.

RULE.

To two-thirds the square of the altitude, add half the square of the diameter of the frustum's base, and half the square of the top diameter; then multiply the sum by the altitude, and divide the product by the circular divisor for the proposed integer.

EXAMPLE.

Let ABCD be the frustum of a sphere, the altitude being thirty inches; the diameter of the base, CD, 70 inches; and the top-diameter, AB, 45 inches: the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION.

30 altitude
30 altitude
900 square of the altitude

Square of the altitude 900 brought over
Multiply by 2
Divide by 3) 1800 product
Quotient 600 two-thirds the sq. of the alt.
70 × 70 ÷ 2 = 2450 half the square of CD.
45 × 45 ÷ 2 = 1012·5 half the square of AB.
4062·5 *sum.*

Again,

4062·5 sum of the 3 numbers
Multiply by 30 altitude
W.G. cir. divisor 294)121875 (414·54 wine gallons.
A. G. cir. divisor 359)121875 (339·48 ale gallons.
M. B. cir. divisor 2738)121875 (44·51 malt bushels.

PROBLEM XXI.

To determine the Content of the Frustum of a Paraboloid.

RULE.

By the Pen.

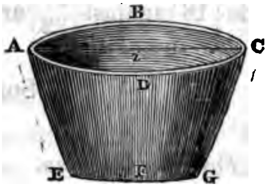
To the square of the diameter of the base, add the square of the diameter of the top, and multiply the sum by half the altitude: then divide the product by the circular divisor for the proposed integer, and the quotient will be the content.

By the Sliding Rule.

Set the circular gage point, for the proposed integer, on D, to half the altitude on C, and opposite to the two diameters respectively on D, take the content on C. The sum of the two contents will be the content required

EXAMPLE.

Let ABCDEFG be the frustum of a paraboloid, whereof the greater diameter, AC, is 56 inches; the less diameter, EG, 32 inches; and the altitude iF 36 inches: the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION BY THE PEN.

56 × 56 = 3136 square of AC.
32 × 32 = 1024 square of EG.
4160 sum

Sum of the squares 4160 brought over
Multiply by 18 half the altitude

$$\begin{array}{r}\hline 33280\\ 4160\\ \hline\end{array}$$

W.G. cir. divisor 294) 74880 (254·69 wine gallons.

A. G. cir. divisor 359) 74880 (208·58 ale gallons.

M. B. cir. divisor 2738) 74880 (27·34 malt bushels.

BY THE SLIDING RULE.

For Wine Gallons.

$$\begin{array}{lcccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G. P.} & \left\{17{\cdot}15\right\} & \text{to} & \left\{18\right\} & \text{against} & \left\{\begin{matrix}56\\32\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}191{\cdot}92\\62{\cdot}69\end{matrix}\right\} \\ & & & & & & \text{Sum} & \underline{\overline{254{\cdot}61}}\ \text{W.G.}\end{array}$$

For Ale Gallons.

$$\begin{array}{lcccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G. P.} & \left\{18{\cdot}95\right\} & \text{to} & \left\{18\right\} & \text{against} & \left\{\begin{matrix}56\\32\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}157{\cdot}16\\51{\cdot}51\end{matrix}\right\} \\ & & & & & & \text{Sum} & \underline{\overline{208{\cdot}67}}\ \text{A.}\end{array}$$

For Malt Bushels.

$$\begin{array}{cccccccc} & \text{on D.} & & \text{on C.} & & \text{on D.} & & \text{on C.} \\ \text{C. G. P.} & \left\{52{\cdot}32\right\} & \text{to} & \left\{18\right\} & \text{against} & \left\{\begin{matrix}56\\32\end{matrix}\right\} & \text{are} & \left\{\begin{matrix}20{\cdot}61\\6{\cdot}73\end{matrix}\right\} \end{array}$$

Sum 27·34 M.B.

PROBLEM XXII.

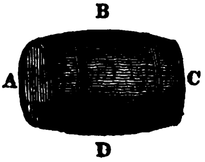
To determine the Content of the middle Frustum of a Spheroid, the Diameters at the Ends being Equal.

RULE.

To the square of the diameter at the end, add twice the square of the diameter in the middle; and multiply the sum by half the length of the frustum. Then divide by the spherical divisor for the proposed integer, and the quotient will be the content.

EXAMPLE.

Let ABCD be a cask in the form of the middle frustum of a spheroid; the middle diameter, BD, being 26 inches; the length, AC, 34 inches; and the diameter of either end, 22 inches: the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION.

	22	diameter of the end
Multiply by	22	diameter of the end
	44	
	44	
Product	484	sq. of the end diam.
Add 26 × 26 × 2 =	1352	twice the sq. of BD.
	1836	sum
Multiply by	17	half the length
	12852	
	1836	
W.G. sph. divisor 441·18)	31212	(70·74 wine gallons.
A. G. sph. divisor 538·58)	31212	(57·95 ale gallons.
M. B. sph. divisor 4106·2)	31212	(7·60 malt bushels.

PROBLEM XXIII.

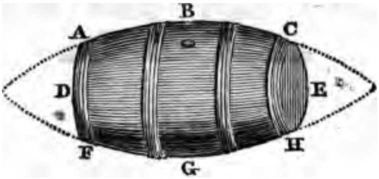
To determine the Content of the middle Frustum of a Parabolic Spindle.

RULE.

To 8 times the square of the diameter in the middle, add 3 times the square of the diameter at the end, and 4 times the product of the greatest and least diameters: then multiply the sum of these three numbers by one tenth part the length, and divide by the spherical divisor of the proposed integer.

EXAMPLE.

Let ABCDEFGH be the middle frustum of a parabolic spindle; the length, DE, being 60 inches; the middle diameter, BG, 35 inches; and the end diameters, AF and HC, each 21 inches: the content is required in wine gallons, ale gallons, and malt bushels.



SOLUTION

35 middle diameter

Multiply by 35 middle diameter

175

105

Product 1225 square of BG.

Multiply by 8

9800 product

$21 \times 21 \times 3 = 1323$ thrice the sq. of AF.

$35 \times 21 \times 4 = 2940$ quadruple product

14063 sum of the 3 numbers

Sum of the 3 numbers is 14063 brought over
Multiply by 6 one tenth the length

W.G. sph. divisor 441·18) 84378 (191·25 wine gallons.

A. G. sph. divisor 538·58) 84378 (156·66 ale gallons.

M.B. sph. divisor 4106·2) 84378 (20·54 malt bushels.

Thus have we finished the introductory and theoretical part of Gaging, and come now to *Practical Gaging*, properly so called.

OF GAGING AND TABLING

COMMON BREWERS' UTENSILS.

WHEN an Officer of Excise visits a Brewhouse for the purpose of gaging and fixing the utensils, the most regular method of proceeding is, to begin at the upper part of the house with the *Coppers*, and finish below with the *Backs* and *Squares*.

For this reason we shall begin with a COPPER.

PROBLEM I.

To Gage and Inch a Copper.

Most of the Coppers used by common brewers have the bottom in the form of a spherical segment, sometimes of greater, and sometimes of less altitude; the convex part generally projecting upwards into the Copper, when it has the name of a *Rising Crown*; but at other times the convexity projecting downwards (and of course leaving a concave space within,) when it is termed a *Falling Crown*.

Coppers with flat bottoms are very seldom made; and of large Coppers, those with rising crowns are more frequently met with than any others. We shall, therefore, treat first of a Copper with a rising crown.

CASE I.

Of a Copper with a Rising Crown.

METHOD OF GAGING.

Lay a straight pole or rod diametrically across the mouth of the Copper, or, where that cannot conveniently be done, extend a small cord like a diameter across the mouth, causing it to be held tight in that situation. Then along this cord or rod move a line, furnished with a plummet, till the lead hangs directly over the centre of the crown, and with a dimension-cane take the perpendicular distance between the centre of the crown and the cord at the level of the mouth; this distance will be the depth of the Copper in the middle. Again, slide the plumb-line along the rod or cord above, till the plummet hangs freely over the lowest part of the bottom of the Copper, just at the beginning of the convexity of the crown, where the crown rises from the sides. And with a rule, or dimension-cane, take the length of this perpendicular also; then without shifting the line, measure the distance it has been moved from the middle of the cord or rod, since the first perpendicular was taken. Twice this distance will be the diameter of the base of the spherical segment forming the crown, and the difference of the two perpendiculars will evidently be the altitude of the segment, or *rise* of the *crown.*

The next step is to take two cross diameters of the Copper within, at the level of the highest part of the crown, first striking a mark at that level all round the Copper with a pair of compasses. Those cross diameters being added together, and the sum divided by 2, will quote the mean diameter of the Copper, at the level of the top of the crown. This mean diameter multiplied by the decimal ·707 (*see page* 135,) will give the side of the inscribed square for the circle described round the Copper at the level of the crown-head. Having, therefore, either by the pen or Sliding Rule, found the side of this square, extend a sliding-cane or a tape to the exact length, and mark the Copper with chalk at the four angles of the square, precisely level with the top of the crown; that is, on the circle described with the compasses.

If the Copper be large, a lighted candle is now to be set on the centre of the crown, and the Copper darkened from without. Then a plumb-line is to be held between the candle and each of the four marks at the angles of the square, so that the shadow of the line may be chalked down the sides of the Copper all the way from the level of the mouth, to the circle described with the compasses.

In these chalk-lines will be the extremities of the cross diameters, which are to be taken at stated altitudes above the centre of the crown, or, more properly, at stated depths below the level of the mouth.

To determine the place of these diameters, lay off *ten-inch* distances along one of the chalked lines, beginning at the level of the crown head, if the altitude consist of integral inches, (but if not, beginning at such distance above the crown, as shall leave the altitude the greatest whole number in inches,) and with these ten-inch distances proceed up to the mouth. Then half way between each of those distances will be the place of the cross diameters. Wherefore make a mark at the half of each distance, transferring the several altitudes of the last marks to the other three lines, and at these middle distances take cross diameters, considering the half of each pair as the *mean* diameter of the Copper at that depth. These mean diameters are to be transferred into the Dimension Book, and severally placed opposite the depths at which they were taken.

Although we have here recommended the distances to be taken from the bottom upwards, we would be understood, that in reality, the divisions begin at the mouth: but since we leave all odd tenths at the bottom, and are not confined to any particular number o inches in either the first or last division, this method beginning below is not attended with any inconve ence. We shall, however, for the sake of variety, and, perhaps, as the best practical mode for a Copper, begin, in all our examples except the first, at the level of the mouth.

It may here be observed, that when we begin at the bottom, and the ten-inch distances laid off, end not at the mouth, but within a few inches of the mouth, it is customary, but not on that account necessary, to consider the upper half of the last ten inches, and the portion of the Copper above it, as one division, making a mark at the half of its altitude; and in this case we carry the other marks severally to all the tens below, instead of making new marks at the middle of each ten: we may now take the mean of the cross diameters at the top of the crown, as the diameter of the lowermost division including the crown, provided we calculate, by the pen, the quantity of liquor required to cover the crown: but if we ascertain this quantity by measure, then the last pair of cross diameters is not taken close to the crown, but about two inches and a half above it, that is, at the middle of the lowermost division of the Copper, whatever it may be, excluding the crown. This division evidently can never be less than five inches, nor so much as six inches.

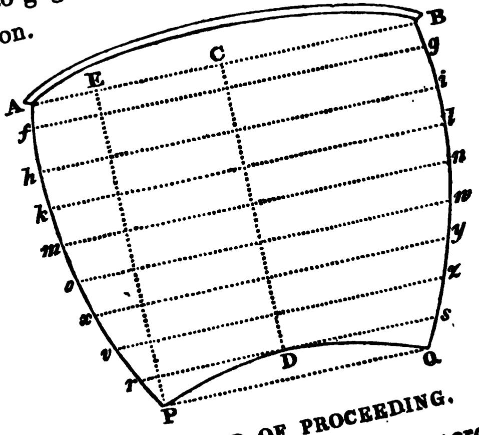
In all cases of gaging Coppers, odd tenths, on the depth, as we have already stated, are to be left at the bottom, as they would otherwise occasion much trouble in the calculation.

Having ascertained the several cross diameters, and found the *arithmetical mean* between each pair, we are to consider each mean as the true diameter of the Copper throughout the space, regarded as cylindrical, to the middle of which the particular mean refers.

This being premised we shall proceed with an example in illustration, having no doubt whatever that the reader will readily comprehend our aim and method at every step.

EXAMPLE.

Let ABPQ be a section of a Copper with a rising crown, cut by a plane passing perpendicularly through the middle, and let PDQ represent the crown: it is required to gage and inch the Copper, of which ABPQ is a section.



METHOD OF PROCEEDING.

Extend a cord, as AB, diametrically across the mouth of the Copper, and let CD be a plumb-line, touching AB, and hanging freely over D the centre of the also let the length of CD be 78 inches.

Next move the plumb-line along AB, towards A, till it arrive at E, perpendicularly over P; and let the length of EP be 84 inches, the distance of E from C, being 39 inches. Then it will be,

84 — 78 = 6 inches *altitude of the crown.*
39 + 39 = 78 inches *diameter of the segment's base.*

Here there are no odd tenths on the depth, and, consequently, we may, without inconvenience, begin our marks from the level of the crown-head, since every division of the Copper will consist of integral inches.

Descending now into the Copper, and with a pair of compasses opened to six inches full, striking a line all round, at the height of 6 inches from the angle formed by the crown and sides, we take two cross diameters of that circle, whereof the one, we shall suppose, is 81·5 inches, and the other 82·3 inches. If these two diameters be added together, and the sum divided by 2, we shall have 81·9 inches for the mean diameter of the Copper at the top of the crown.

But,

81·9 × ·707 = 57·9 *side of the inscribed square.*

With a sliding-cane or tape, therefore, extended to 57·9 inches, we inscribe the square, and, either with a chalk-line strike perpendiculars on the sides of the Copper at the four angles, or by the shadow of a plumb-line from a candle in the centre, draw the lines with the hand.

We next begin at the level of the top of the crown, and proceed to set off ten-inch distances towards the mouth on one of the chalk-lines; when, having made *seven* marks, we find the remaining portion above, to be eight inches over even tens, that is, 13 inches over the middle of the space included between the two marks made for the *last two* tens. Wherefore, regarding these 13 inches as one division, we make a mark at 6.5 inches from the mouth of the Copper, for the middle of this division; and now the lowermost six ten-inch marks will each be in the middle of a division; hence the other three perpendicular lines must be similarly marked.

This part of the operation being finished, we carefully measure the cross diameters, and enter them in a memorandum, with the mean diameter between each pair, and every other particular, such as the division to which they refer of the Copper, and the distance from the mouth at which they were taken. The form of the memorandum will, with reference to the figure, be as follows:

Memorandum of A. B.'*s Copper, No.* 1, *Gaged,* 10 *May,* 1820.

Divisions in Inches	Distance from the mouth	Cross Diameters	Mean Diameter
13	6·5 inches	105·3...105·9	105·6 inches...*fg*
10	18·0 inches	105·1...105.5	105·3 inches...*hi*
10	28·0 inches	104·5...104·7	104·6 inches...*kl*
10	38·0 inches	103·9...103·5	103·7 inches...*mn*
10	48 0 inches	98·0... 97·4	97·7 inches...*ow*
10	58·0 inches	94·1... 93·9	94·0 inches...*xy*
10	68·0 inches	88·6... 88·6	88·6 inches...*vz*
11	78·0 inches	82·0... 81·8	81·9 inches...*rs*

CALCULATION.

The content of a circle, an inch deep, in ale gallons, for each of the mean diameters respectively, is now to be multiplied by the depth of the space to which the particular diameter applies, including the lowest portion of the Copper, as if the crown were flat: and the sum of these contents, after deducting the content of the spherical segment answering to the crown, will be the true content of the Copper.

Thus,

Depth from the Mouth.	Mean Diameter in Inches	Content for one inch deep in ale gallons	Depth of the several parts	Content of each part in ale gallons
First 13 inches	105·6	31·05	× 13 =	403·65
Next 10 inches	105·3	30·88	× 10 =	308·80
Next 10 inches	104·6	30·47	× 10 =	304·70
Next 10 inches	103·7	29·95	× 10 =	299·50
Next 10 inches	97·7	26·58	× 10 =	265·80
Next 10 inches	94·0	24·61	× 10 =	246·10
Next 10 inches	88·6	21·86	× 10 =	218·60
Last 11 inches	81·9	18·68	× 11 =	205·48
Whole content including the crown			A. G.	2252·63
Deduct for the crown			A. G.	51·23
True Content of the Copper			A. G.	2201·40

The exact allowance to be made for the convexity of the crown, we deduce from calculation founded on PROB. XIX. SEC. III. of MENSURATION; hence the process will be as follows:

CALCULATION OF THE SPHERICAL SEGMENT FORMING THE CROWN.

[*See page* 212.]

36 square of the altitude
39 × 39 × 3 = 4563 thrice the sq. of half the base
4599 sum
Multiply by 6 the altitude
A.G.sp.div.538·58)27594 (51·23 ale gallons.

But the content of the cylinder circumscribing the crown, for one inch deep, and mean diameter 81·9 as found by measurement at *rs*, is 60·85 ale gallons, obtained as follows:

18·68 ale gallons at an inch deep
Multiply by 6 the height of the crown
Product 112·08 *content of the cylinder.*
Subtract 51·23 for the crown
Remainder 60·85 ale gallons TO COVER THE CROWN.

It will now be proper to exhibit a Specimen of a Dimension Book, with the manner of entering the above Copper; where it will be seen that the content of each division of the Copper in gallons, is reduced in the last column, into Barrels, Firkins, and Gallons. This is done by dividing the gallons by 36 to integer places *only*, for barrels; and then dividing the remainder by 9 in the same way: for the first quotient being Barrels, the second will be Firkins, and the last remainder Gallons.

DIMENSION BOOK.

A. B.'s *Copper*, *No.* 1, *Gaged*, 10 *May*, 1820.

Depths in Inches	Mean Diameters	Areas at an inch deep	Content in Gallons	Content in		
				B.	F.	G.
13·0	105·6	31·05	403·65	11	0	7·65
10·0	105·3	30·88	308·80	8	2	2·80
10·0	104·6	30·47	304·70	8	1	7·70
10·0	103·7	29·95	299·50	8	1	2·50
10·0	97·7	26·58	265·80	7	1	4·80
10·0	94·0	24·61	246·10	6	3	3·10
10·0	88·6	21·86	218·60	6	0	2·60
5·0	81·9	18·68	93·40	2	2	3·40
6·0	To cover the crown		60·85	1	2	6·85
84·0	*Content of the Copper*		2201·40	61	0	5·40

Having taken the dimensions of the Copper, and ascertained the content, we are next to tabulate it; that is, to enter in a Table the content of the Copper at every inch deep throughout. The use of this Table will be great, inasmuch as it will enable us, by inspection, to tell the quantity of liquor in the Copper, having either the wet or the dry inches given.

By wet inches is understood the depth of liquor; and by dry inches, the distance of its surface from the level of the mouth of the Copper.

It must be apparent, that to construct the Table, it will only be necessary to subtract the first inch-area from the whole content; the second from the remainder; and so on to the bottom of the Copper. This is done in Barrels, Firkins, and Gallons, as follows:

A. B.'s Copper, No. 1, Tabled.

Dry Inches	B. F. G.	Dry Inches	B. F. G.
Full	61..0..5·40 0..3..4·05	13 *	49..3..6·75 0..3..3·88
1	60..1..1·35 0..3..4·05	14	49..0..2·87 0..3..3·88
2	59..1..6·30 0..3..4·05	15	48..0..7·99 0..3..3·88
3	58..2..2·25 0..3..4·05	16	47..1..4·11 0..3..3·88
4	57..2 .7·20 0..3..4·05	17	46..2..0·23 0..3..3·88
5	56..3..3·15 0..3..4·05	18	45..2..5·35 0..3..3·88
6	55..3..8·10 0..3..4·05	19	44..3..1·47 0..3..3·88
7	55..0..4·05 0..3..4·05	20	43..3..6·59 0..3..3·88
8	54..1..0·00 0..3..4·05	21	43..0..2·71 0..3..3·88
9	53..1..4·95 0..3..4·05	22	42..0..7·83 0..3..3·88
10	52..2..0·90 0..3..4·05	23 *	41..1..3·95 0..3..3·47
11	51..2..5·85 0..3..4·05	24	40..2..0·48 0..3..3·47
12	50..3..1·80 0..3..4·05	25	39..2..6·01 0..3..3·47
13	49..3..6·75	26	38..3..2·54

A. B.'s COPPER, No. 1, TABLED.

Dry Inches	B. F. G.	Dry Inches	B. F. G.
26	38..3..2·54 0..3..3·47	39	27..3..5·55 0..3..2·95
27	37..3..8·07 0..3..3·47	40	27..0..2·60 0..3..2·95
28	37..0..4·60 0..3..3·47	41	26..0..8·65 0..3..2·95
29	36..1..1·13 0..3..3·47	42	25..1..5·70 0..3..2·95
30	35..1..6·66 0..3..3·47	43 *	24..2..2·75 0..2..8·58
31	34..2..3·19 0..3..3·47	44	23..3..3·17 0..2..8·58
32	33..2..8·72 0..3..3·47	45	23..0..3·59 0..2..8·58
33 *	32..3..5·25 0..3..2·95	46	22..1..4·01 0..2..8·58
34	32..0..2·30 0..3..2·95	47	21..2..4·43 0..2..8·58
35	31..0..8·35 0..3..2·95	48	20..3..4·85 0..2..8·58
36	30..1..5·40 0..3..2·95	49	20..0..5·27 0..2..8·58
37	29..2..2·45 0..3..2·95	50	19..1..5·69 0..2..8·58
38	28..2..8·50 0..3..2·95	51	18..2..6·11 0..2..8·58
39	27..3..5·55	52	17..3..6·53

A. B.'s Copper, No. 1, Tabled.

Dry Inches	B. F. G.	Dry Inches	B. F. G.
52	17..3..6·53 0..2..8·58	65	9..0..5·13 0..2..3·86
53 *	17..0..6·95 0..2..6·61	66	8..2..1·27 0..2..3·86
54	16..2..0·34 0..2..6 61	67	7..3..6·41 0..2..3·86
55	15..3..2·73 0..2..6·61	68	7..1..2·55 0..2..3·86
56	15..0..5·12 0..2..6·61	69	6..2..7·69 0..2..3·86
57	14..1..7·51 0..2..6·61	70	6..0..3·83 0..2..3·86
58	13..3..0·90 0..2..6·61	71	5..1..8·97 0..2..3·86
59	13..0..3·29 0..2..6·61	72	4..3..5·11 0..2..3·86
60	12..1..5·68 0..2..6·61	73 *	4..1..1·25 0..2..0·68
61	11..2..8·07 0..2..6·61	74	3..3..0·57 0..2..0·68
62	11..0..1·46 0..2..6·61	75	3..0..8·89 0..2..0 68
63 *	10..1..3·85 0..2..3·86	76	2..2..8·21 0..2..0·68
64	9..2..8·99 0..2..3·86	77	2..0..7·53 0..2..0·68
65	9..0..5·13	78	1..2..6·85

In tabling every description of utensil, it will save much time, in case of error, to check the work, by frequently examining its correctness in some counter-way. In the example before us, we check as follows.

	B.	F.	G.
Whole content of the copper	61	0	5·40
Subtract the content for 13 inches	11	0	7·65
Content when 13 *inches are dry*	49	3	6·75
Subtract for the next 10 inches	8	2	2·80
Content when 23 *inches are dry*	41	1	3·95
Subtract for the next 10 inches	8	1	7·70
Content when 33 *inches are dry*	32	3	5·25
Subtract for the next 10 inches	8	1	2·50
Content when 43 *inches are dry*	24	2	2·75
Subtract for the next 10 inches	7	1	4·80
Content when 53 *inches are dry*	17	0	6·95
Subtract for the next 10 inches	6	3	3·10
Content when 63 *inches are dry*	10	1	3·85
Subtract for the next ten inches	6	0	2·60
Content when 73 *inches are dry*	4	1	1·25
Subtract for the next 5 inches	2	2	3·40
CONTENT WHEN THE CROWN IS JUST COVERED	1	2	6·85

By which it appears that the operation has been rightly performed.

The practical and most expeditious method of determining the quantity of liquor necessary to cover the crown of a Copper, is, after the other dimensions are taken, to admit water into the Copper till the crown is barely covered, and then to draw off the water into some vessel of which the content is known.

Some experienced officers of the Excise prefer measuring the water as it is put into the Copper, and for this purpose they cause it to be poured in, by a barrel at a time, till the crown is nearly covered: then a kilderkin is emptied into the Copper, if necessary; and lastly, any number of gallons requisite, from a half-gallon measure.

Both these methods are in effect the same, and consequently it matters not which of them we employ; although, perhaps the first has the advantage, if there be plenty of water on the premises, since, by opening the plug in the Copper-Back, we can with the greatest ease admit the exact quantity necessary.

Again, to prevent the great trouble of calculating, in the first instance, the content, for an inch deep, of each circle, whereof cross diameters have been taken, a Table of the Content of Cylinders, in ale, for one inch deep, to every diameter, in inches and tenths, from one inch to 210 inches inclusively, will be found highly serviceable, and which Table we shall now subjoin.

TABLE

OF

ALE GALLONS

(FOR ONE INCH DEEP)

In Cylinders from 1 inch to 210 inches in diameter,
with all the intermediate tenths;

VERY USEFUL IN GAGING,

AND COMMONLY

CALLED THE TABLE OF CIRCULAR AREAS

FOR

ALE.

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
1	0·0028	0·0034	0·0040	0·0047	0·0054
2	0·0111	0·0123	0·0135	0·0147	0·0160
3	0·0251	0·0268	0·0285	0·0303	0·0322
4	0·0445	0·0468	0·0491	0·0515	0·0540
5	0·0696	0·0724	0·0753	0·0782	0·0812
6	0·1002	0·1036	0·1070	0·1106	0·1141
7	0·1365	0·1404	0·1444	0·1484	0·1525
8	0·1782	0·1827	0·1873	0·1918	0·1965
9	0·2256	0·2306	0·2357	0·2409	0·2461
10	0·2785	0·2841	0·2898	0·2955	0·3012
11	0·3370	0·3431	0·3494	0·3556	0·3619
12	0·4011	0·4078	0·4146	0·4214	0·4282
13	0·4707	0·4780	0·4853	0·4927	0·5001
14	0·5459	0·5537	0·5616	0·5695	0·5775
15	0·6266	0·6350	0·6435	0·6520	0·6604
16	0·7130	0·7219	0·7309	0·7400	0·7491
17	0·8049	0·8144	0·8239	0·8335	0·8432
18	0·9024	0·9124	0·9225	0·9327	0·9429
19	1·0054	1·0160	1·0267	1·0374	1·0482
20	1·1140	1·1252	1·1364	1·1477	1·1590
21	1·2282	1·2400	1·2517	1·2635	1·2754
22	1·3480	1·3602	1·3726	1·3850	1·3974
23	1·4733	1·4861	1·4990	1·5120	1·5250
24	1·6042	1·6176	1·6310	1·6445	1·6581
25	1·7406	1·7546	1·7687	1·7827	1·7968
26	1.8827	1·8972	1·9118	1·9264	1·9411
27	2·0303	2·0454	2·0605	2·0757	2·0909
28	2·1835	2·1991	2·2148	2·2305	2·2463
29	2·3422	2·3584	2·3746	2·3910	2·4073
30	2·5066	2·5233	2·5401	2·5570	2·5739
31	2·6764	2·6938	2·7111	2·7285	2·7460
32	2·8519	2·8698	2·8877	2·9057	2·9237
33	3·0330	3·0514	3·0698	3·0884	3·1069
34	3·2196	3·2385	3·2576	3·2766	3·2958
35	3·4117	3·4312	3·4508	3·4705	3·4902

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
1	0·0063	0·0071	0·0080	0·0090	0·0100
2	0·0174	0·0188	0·0203	0·0218	0·0234
3	0·0341	0·0361	0·0381	0·0402	0·0424
4	0·0564	0·0589	0·0615	0·0642	0·0669
5	0·0842	0·0873	0·0905	0·0937	0·0969
6	0·1176	0·1213	0·1250	0·1287	0·1326
7	0·1566	0·1609	0·1651	0·1694	0·1738
8	0·2012	0·2060	0·2108	0·2157	0·2206
9	0·2513	0·2567	0·2620	0·2675	0·2730
10	0·3070	0·3129	0·3189	0·3248	0·3309
11	0·3683	0·3748	0·3812	0·3878	0·3944
12	0·4352	0·4422	0·4492	0·4563	0·4635
13	0·5076	0·5151	0·5227	0·5304	0·5381
14	0·5856	0·5937	0·6018	0·6100	0·6183
15	0·6691	0·6778	0·6865	0·6953	0·7041
16	0·7582	0·7674	0·7767	0·7861	0·7954
17	0·8529	0·8627	0·8725	0·8824	0·8924
18	0·9532	0·9635	0·9739	0·9843	0·9948
19	1·0590	1·0699	1·0808	1·0918	1·1029
20	1·1704	1·1819	1·1934	1·2049	1·2165
21	1·2874	1·2994	1·3114	1·3236	1·3357
22	1·4099	1·4225	1·4351	1·4478	1·4605
23	1·5380	1·5511	1·5643	1·5775	1·5908
24	1·6717	1·6854	1·6991	1·7129	1·7267
25	1·8110	1·8252	1·8395	1·8538	1·8682
26	1·9558	1·9706	1·9854	2·0003	2·0153
27	2·1062	2·1215	2·1369	2·1524	2·1679
28	2·2621	2·2780	2·2940	2·3100	2·3261
29	2·4237	2·4401	2·4567	2·4732	2·4899
30	2·5908	2·6078	2·6249	2·6420	2·6592
31	2·7635	2·7811	2·7987	2·8163	2·8341
32	2·9418	2·9599	2·9781	2·9963	3·0146
33	3·1256	3·1443	3·1630	3·1818	3·2007
34	3·3150	3·3342	3·3535	3·3728	3·3923
35	3·5099	3·5297	3·5496	3·5695	3·5894

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
36	3·6095	3·6295	3·6497	3·6699	3·6901
37	3·8128	3.8334	3·8541	3·8749	3·8957
38	4·0217	4·0428	4·0641	4·0854	4·1068
39	4·2361	4·2578	4·2796	4·3015	4·3234
40	4·4562	4·4785	4·5008	4·5233	4·5457
41	4·6818	4·7046	4·7275	4·7505	4·7735
42	4·9129	4·9363	4·9598	4·9834	5·0069
43	5·1496	5·1736	5·1977	5·2218	5·2459
44	5·3920	5·4165	5·4411	5·4657	5·4904
45	5·6398	5·6649	5·6901	5·7153	5·7405
46	5·8933	5·9189	5·9446	5·9704	5·9962
47	6·1523	6·1785	6·2048	6·2311	6·2575
48	6·4169	6·4436	6·4705	6·4973	6·5243
49	6·6870	6·7143	6·7417	6·7692	6·7966
50	6·9628	6·9906	7·0186	7·0466	7·0746
51	7·2440	7·2725	7·3010	7·3295	7·3581
52	7·5309	7·5599	7·5890	7·6181	7·6472
53	7·8233	7·8529	7·8825	7·9122	7·9419
54	8·1214	8·1515	8·1816	8·2118	8·2421
55	8·4249	8·4556	8·4863	8·5171	8·5479
56	8·7341	8·7653	8·7966	8·8279	8·8593
57	9·0488	9·0806	9·1124	9·1443	9·1762
58	9·3691	9·4014	9·4338	9·4662	9·4988
59	9·6949	9·7278	9·7608	9·7938	9·8268
60	10·0264	10·0598	10·0933	10·1269	10·1605
61	10·3634	10·3974	10·4314	10·4655	10·4997
62	10·7059	10·7405	10·7751	10·8098	10·8445
63	11·0541	11·0892	11·1244	11·1596	11·1949
64	11·4078	11·4434	11·4792	11·5150	11·5508
65	11·7670	11·8033	11·8396	11·8759	11·9123
66	12·1319	12·1687	12·2055	12·2424	12·2794
67	12·5023	12·5397	12·5771	12·6145	12·6520
68	12·8723	12·9162	12·9542	12·9922	13·0303
69	13·2599	13.2982	13·3368	13·3754	13·4140
70	13·6470	13·6860	13·7251	13·7642	13·8034

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
36	3·7104	3·7308	3·7512	3·7717	3·7922
37	3·9165	3·9374	3·9584	3·9794	4·0005
38	4·1282	4·1496	4·1712	4·1927	4·2144
39	4·3454	4·3674	4·3895	4·4117	4·4338
40	4·5683	4·5908	4·6135	4·6362	4·6589
41	4·7966	4·8198	4·8430	4·8662	4·8895
42	5·0306	5·0543	5·0780	5·1019	5·1257
43	5·2701	5·2944	5·3187	5·3430	5·3675
44	5·5152	5·5400	5·5649	5·5898	5·6148
45	5·7659	5·7912	5·8167	5·8421	5·8677
46	6·0221	6·0480	6·0740	6·1000	6·1261
47	6·2839	6·3104	6·3369	6·3635	6·3902
48	6·5513	6·5783	6·6054	6·6325	6·6597
49	6·8242	6·8518	6·8794	6·9072	6·9349
50	7·1027	7·1309	7·1591	7·1873	7·2157
51	7·3868	7·4155	7·4443	7·4731	7·5020
52	7·6764	7·7057	7·7350	7·7644	7·7939
53	7·9717	8·0015	8·0314	8·0613	8·0913
54	8·2724	8·3028	8·3333	8·3638	8·3943
55	8·5788	8·6097	8·6407	8·6718	8·7029
56	8·8907	8·9222	8·9538	8·9854	9·0171
57	9·2082	9·2403	9·2724	9·3046	9·3368
58	9·5313	9·5639	9·5966	9·6293	9·6621
59	9·8600	9·8931	9·9263	9·9596	9·9930
60	10·1942	10·2279	10·2617	10·2955	10·3294
61	10·5339	10·5682	10·6026	10·6370	10·6714
62	10·8793	10·9141	10·9490	10·9840	11·0190
63	11·2302	11·2656	11·3011	11·3366	11·3721
64	11·5867	11·6227	11·6587	11·6947	11·7309
65	11·9488	11·9853	12·0219	12·0585	12·0952
66	12·3164	12·3535	12·3906	12·4278	12·4650
67	12·6896	12·7272	12·7649	12·8027	12·8405
68	13·0684	13·1066	13·1448	13·1831	13·2215
69	13·4527	13·4915	13·5302	13·5691	13·6080
70	13·8426	13·8819	13·9213	13·9607	14·0002

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
71	14·0397	14·0793	14·1189	14·1586	14·1983
72	14·4380	14·4781	14·5183	14·5585	14·5988
73	14·8418	14·8825	14·9232	14·9640	15·0049
74	15·2512	15·2925	15·3338	15·3751	15·4165
75	15·6662	15·7080	15·7499	15·7918	15·8337
76	16·0867	16·1291	16·1715	16·2140	16·2565
77	16·5129	16·5558	16·5988	16·6418	16·6849
78	16·9445	16·9880	17·0316	17·0751	17·1188
79	17·3818	17·4258	17·4699	17·5141	17·5583
80	17·8246	17·8692	17·9139	17·9586	18·0033
81	18·2730	18·3182	18.3634	18·4086	18·4540
82	18·7270	18·7727	18·8185	18·8643	18·9102
83	19·1866	19·2328	19·2791	19·3255	19·3719
84	19·6517	19·6985	19·7454	19·7923	19·8393
85	20·1223	20·1697	20·2172	20·2646	20·3122
86	20·5986	20·6465	20·6945	20·7426	20·7907
87	21·0804	21·1289	21·1775	21·2261	21·2747
88	21·5678	21·6169	21·6660	21·7151	21·7643
89	22·0608	22·1104	22·1600	22·2098	22·2595
90	22·5593	22·6095	22·6597	22·7100	22·7603
91	23·0634	23·1141	23·1649	23·2157	23·2666
92	23·5731	23·6244	23·6757	23·7271	23·7785
93	24·0883	24·1402	24·1920	24·2440	24·2960
94	24·6091	24·6615	24·7140	24·7665	24·8190
95	25·1355	25·1885	25·2415	25·2945	25·3476
96	25·6675	25·7210	25·7745	25·8282	25·8818
97	26·2050	26·2591	26·3132	26·3673	26·4216
98	26·7481	26·8027	26·8574	26·9121	26·9669
99	27·2968	27·3519	27·4072	27·4625	27·5178
100	27·8510	27·9067	27·9625	28·0184	28·0743
101	28·4108	28·4671	28·5234	28·5798	28·6363
102	28·9762	29·0330	29·0899	29·1469	29·2039
103	29·5471	29·6045	29·6620	29·7195	29·7771
104	30·1236	30·1816	30·2396	30·2977	30·3558
105	30·7057	30·7642	30·8228	30·8814	30·9401

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
71	14·2381	14·2780	14·3179	14·3579	14·3978
72	14·6392	14·6796	14·7201	14·7606	14·8012
73	15·0458	15·0868	15·1278	15·1689	15·2100
74	15·4580	15·4995	15·5411	15·5827	15·6244
75	15·8758	15·9178	15·9600	16·0022	16·0444
76	16·2991	16·3417	16·3844	16 4272	16·4700
77	16·7280	16·7712	16·8145	16·8578	16·9011
78	17·1625	17·2062	17·2500	17·2939	17·3378
79	17·6025	17·6468	17·6912	17·7356	17·7801
80	18·0481	18·0930	18·1379	18·1829	18·2280
81	18·4993	18·5448	18·5902	18·6358	18·6814
82	18·9561	19·0021	19·0481	19·0942	19·1403
83	19 4184	19·4650	19·5115	19·5582	19·6049
84	19·8863	19·9334	19·9806	20·0278	20·0750
85	20·3598	20·4074	20·4551	20·5029	20·5507
86	20·8388	20·8870	20·9353	20·9836	21·0320
87	21·3234	21·3722	21·4210	21·4699	21·5188
88	21·8136	21 8629	21·9123	21·9617	22·0112
89	22·3093	22·3592	22·4092	22·4592	22·5092
90	22·8107	22·8611	22·9116	22·9621	23·0128
91	23·3176	23·3685	23·4196	23·4707	23·5220
92	23·8300	23·8816	23·9332	23·9848	24·0367
93	24·3480	24·4001	24·4523	24·5045	24·5568
94	24·8716	24·9243	24·9770	25·0298	25·0826
95	25·4008	25·4540	25·5073	25·5606	25·6140
96	25·9355	25·9893	26·0432	25·0971	26·1510
97	26·4759	26·5302	26·5846	26·6390	26·6935
98	27·0217	27·0766	27·1316	26·1866	27·2416
99	27·5732	27·6286	27·6841	27·7397	27·7953
100	28·1302	28·1862	28·2423	28·2984	28·3546
101	28·6928	28·7494	28·8060	28·8627	28·9194
102	29·2610	29·3181	29·3753	29 4325	29·4898
103	29·8347	29·8924	29·9501	30·0079	30·0657
104	30·4140	30·4722	30·5305	30·5889	30·6473
105	30·9989	31·0577	31·1165	31·1754	31·2344

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
106	31·2934	31·3525	31·4116	31·4708	31·5300
107	31·8866	31·9462	32·0059	32·0657	32·1255
108	32·4854	32·5456	32·6058	32·6661	32·7265
109	33·0898	33·1505	33·2113	33·2722	33·3331
110	33·6997	33·7610	33·8224	33·8838	33·9452
111	34·3152	34·3771	34·4390	34·5010	34·5630
112	34·9363	34·9987	35·0612	35·1237	35·1863
113	35·5629	35·6259	35·6889	35·7520	35·8152
114	36·1952	36·2587	36·3223	36·3859	36·4496
115	36·8329	36·8970	36·9612	37·0254	37·0896
116	37·4763	37·5409	37·6056	37·6704	37·7352
117	38·1252	38·1904	38·2557	38·3210	38·3864
118	38·7797	38·8455	38·9113	38·9772	39·0431
119	39·4398	39·5061	39·5725	39·6389	39·7054
120	40·1054	40·1723	40·2392	40·3062	40·3733
121	40·7766	40·8441	40·9116	40·9791	41·0467
122	41·4534	41·5214	41·5895	41·6575	41·7257
123	42·1358	42·2043	42·2729	42·3416	42·4103
124	42·8237	42·8928	42·9619	43·0312	43·1004
125	43·5172	43·5868	43·6566	43·7263	43·7961
126	44·2162	44·2865	44·3567	44·4271	44·4974
127	44·9209	44·9916	45·0625	45·1334	45·2043
128	45·6311	45·7024	45·7738	45·8452	45·9167
129	46·3468	46·4187	46·4907	46·5627	46·6347
130	47·0682	47·1406	47·2131	47·2857	47·3583
131	47·7951	47·8681	47·9412	48·0143	48·0874
132	48·5276	48·6011	48·6747	48·7484	48·8221
133	49·2656	49·3397	49·4139	49·4880	49·5624
134	50·0093	50·0839	50·1586	50·2334	50·3083
135	50·7584	50·8336	50·9089	50·9842	51·0596
136	51·5132	51·5890	51·6650	51·7407	51·8166
137	52·2735	52·3500	52·4262	52·5030	52·5792
138	53·0394	53·1164	53·1933	53·2703	53·3474
139	53·8109	53·8884	53·9659	54·0434	54·1211
140	54·5880	54·6660	54·7440	54·8222	54·9003

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
106	31·5893	31·6487	31·7080	31·7675	31·8270
107	32·1853	32·2452	32·3052	32·3652	32·4252
108	32·7869	32·8474	32·9079	32·9685	32·0291
109	33·3940	33·4551	33·5161	33·5773	33·6385
110	34·0068	34·0683	34·1300	34·1917	34·2534
111	34·6251	34·6872	34·7494	34·8116	34·8739
112	35·2489	35·3116	35·3744	35·4372	35·5000
113	35·8784	35·9416	36·0049	36·0683	36·1317
114	36·5134	36·5772	36·6410	36·7049	36·7689
115	37·1539	37·2183	37·2827	37·3472	37·4117
116	37·8001	37·8650	37·9300	37·9950	38·0601
117	38·4518	38·5173	38·5828	38·6484	38·7140
118	39·1091	39·1751	39·2412	39·3073	39·3735
119	39·7719	39·8385	39·9052	39·9719	40·0386
120	40·4403	40·5075	40·5747	40·6420	40·7093
121	41·1143	41·1820	41·2498	41·3176	41·3855
122	41·7939	41·8622	44·9305	41·9989	42·0673
123	42·4790	42·5479	42·6167	42·6856	42·7547
124	43·1697	43·2391	43·3086	43·3780	43·4476
125	43·8660	43·9360	44·0059	44·0760	44·1461
126	44·5679	44·6384	44·7089	44·7795	44·8502
127	45·2753	45·3463	45·4174	45·4886	45·5598
128	45·9883	46·0599	46·1315	46·2032	46·2750
129	46·7068	46·7790	46·8512	46·9235	46·9958
130	47·4309	47·5037	47·5764	47·6492	47·7222
131	48·1606	48·2339	48·3073	48·3806	48·4541
132	48·8959	48·9697	49·0436	49·1176	49·1916
133	49·6367	49·7111	49·7856	49·8601	49·9346
134	50·3832	50·4581	50·5331	50·6082	50·6833
135	51·1351	51·2106	51·2861	51·3620	51·4374
136	51·8927	51·9688	52·0450	52·1210	52·1972
137	52·6558	52·7324	52·8090	52·8860	52·9628
138	53·4245	53·5017	53·5789	53·6562	53·7335
139	54·1987	54·2765	54·3543	54·4321	54·5100
140	54·9786	55·0569	55·1352	55·2136	55·2921

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
141	55·3706	55·4491	55·5278	55·6064	55·6852
142	56·1587	56·2379	56·3170	56·3963	56·4756
143	56·9525	57·0322	57·1119	57·1917	57·2716
144	57·7518	57·8321	57·9124	57·9927	58·0731
145	58·5567	58·6375	58·7184	58·7993	58·8802
146	59·3672	59·4485	59·5299	59·6114	59·6929
147	60·1832	60·2651	60·3471	60·4291	60·5112
148	61·0048	61·0873	61·1698	61·2524	61·3350
149	61·8320	61·9150	61·9981	62·0812	62·1644
150	62·6648	62·7483	62·8320	62·9157	62·9994
151	63·5030	63·5872	63·6714	63·7556	63·8400
152	64·3470	64·4316	64·5164	64·6012	64·6861
153	65·1964	65·2817	65·3670	65·4523	65·5377
154	66·0514	66·1372	66·2231	66·3090	66·3950
155	66·9120	66·9983	67·0848	67·1712	67·2578
156	67·7781	67·8651	67·9520	68·0391	68·1262
157	68·6499	68·7374	68·8249	68·9125	69·0001
158	69·5272	69·6152	69·7033	69·7915	69·8797
159	70·4101	70·4987	70·5873	70·6760	70·7648
160	71·2985	71·3877	71·4769	71·5661	71·6554
161	72·1925	72·2822	72·3720	72·4618	72·5517
162	73·0921	73·1824	73·2727	73·3631	73·4535
163	73·9973	74·0881	74·1790	74·2699	74·3609
164	74·9080	74·9994	75·0908	75·1823	75·2739
165	75·8243	75·9162	76·0082	76·1003	76·1924
166	76·7462	76·8387	76·9312	77·0238	77·1165
167	77·6736	77·7667	77·8598	77·9529	78·0461
168	78·6066	78·7002	78·7939	78·8876	78·9814
169	79·5452	79·6394	79·7336	79·8279	79·9222
170	80·4893	80·5841	80·6788	80·7737	80·8686
171	81·4391	81·5343	81·6297	81·7251	81·8205
172	82·3943	82·4902	82·5861	82·6820	82·7780
173	83·3552	83·4516	83·5480	83·6446	83·7411
174	84·3216	84·4186	84·5156	84·6127	84·7098
175	85·2936	85·3911	85·4887	85·5863	85·6840

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
141	55·7640	55·8428	55·9217	56·0007	56·0797
142	56·5549	56·6343	56·7138	56·7933	56·8729
143	57·3515	57·4314	57·5114	57·5915	57·6716
144	58·1536	58·2341	58·3147	58·3952	58·4760
145	58·9613	59·0423	59·1235	59·2047	59·2859
146	59·7745	59·8561	59·9378	60·0196	60·1014
147	60·5933	60·6755	60·7578	60·8401	60·9224
148	61·4177	61·5005	61·5833	61·6661	61·7490
149	62·2477	62·3310	62·4143	62·4978	62·5812
150	63·0832	63·1671	63·2510	63·3350	63·4190
151	63·9243	64·0087	64·0932	64·1777	64·2623
152	64·7709	64·8560	64·9410	65·0261	65·1112
153	65·6232	65·7088	65·7943	65·8800	65·9657
154	66·4810	66·5671	66·6533	66·7395	66·8257
155	67·3444	67·4311	67·5178	67·6045	67·6913
156	68·2133	68·3005	68·3878	68·4751	68·5625
157	69·0878	69·1756	69·2634	69·3513	69·4392
158	69·9679	70·0562	70·1446	70·2330	70·3215
159	70·8536	70·9425	71·0314	71·1204	71·2094
160	71·7448	71·8343	71·9237	72·0133	72·1029
161	72·6416	72·7316	72·8217	72·9118	73·0019
162	73·5440	73·6345	73·7251	73·8158	73·9065
163	74·4519	74·5430	74·6342	74·7254	74·8167
164	75·3655	75·4571	75·5488	75·6406	75·7324
165	76·2845	76·3767	76·4690	76·5613	76·6537
166	77·2092	77·3020	77·3948	77·4877	77·5806
167	78·1394	78·2327	78·3261	78·4196	78·5131
168	79·0752	79·1691	79·2630	79·3570	79·4511
169	80·0166	80·1110	80·2055	80·3001	80·3947
170	80·9635	81·0585	81·1536	81·2487	81·3438
171	81·9160	82·0116	82·1072	82·2028	82·2986
172	82·8741	82·9702	83·0664	83·1626	83·2589
173	83·8377	83·9344	84·0311	84·1279	84·2247
174	84·8069	84·9042	85·0015	85·0988	85·1962
175	85·7817	85·8795	85·9774	86·0752	86·1732

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
176	86·2712	86·3693	86·4674	86·5656	86·6638
177	87·2543	87·3530	87·4516	87·5504	87·6492
178	88·2431	88·3422	88·4415	88·5408	88·6401
179	89·2373	89·3371	89·4369	89·5367	89·6366
180	90·2372	90·3375	90·4378	90·5382	90·6387
181	91·2426	91·3435	91·4444	91·5453	91·6463
182	92·2536	92·3550	92·4565	92·5580	92·6596
183	93·2702	93·3721	93·4741	93·5762	93·6783
184	94·2923	94·3948	94·4974	94·6000	94·7027
185	95·3200	95·4231	95·5262	95·6294	95·7326
186	96·3533	96·4569	96·5606	96·6643	96·7681
187	97·3921	97·4963	97·6005	97·7049	97·8092
188	98·4365	98·5413	98·6461	98·7509	98·8558
189	99·4865	99·5918	99·6972	99·8026	99·9081
190	100·5421	100·6479	100·7538	100·8598	100·9658
191	101·6032	101·7096	101·8161	101·9226	102·0292
192	102·6699	102·7769	102·8839	102·9910	103·0981
193	103·7421	103·8497	103·9573	104·0649	104·1726
194	104·8200	104·9281	105·0362	105·1444	105·2527
195	105·9034	106·0120	106·1207	106·2295	106·3383
196	106·9924	107·1016	107·2108	107·3201	107·4295
197	108·0869	108·1967	108·3065	108·4163	108·5263
198	109·1870	109·2973	109·4077	109·5181	109·6286
199	110·2927	110·4036	110·5145	110·6255	110·7365
200	111·4040	111·5154	111·6269	111·7384	111·8500
201	112·5208	112·6328	112·7448	112·8569	112·9691
202	113·6432	113·7557	113·8683	113·9810	114·0937
203	114·7711	114·8842	114·9974	115·1106	115·2239
204	115·9047	116·0183	116·1320	116·2458	116·3596
205	117·0438	117·1580	117·2723	117·3866	117·5010
206	118·1885	118·3032	118·4181	118·5329	118·6479
207	119·3387	119·4540	119·5694	119·6849	119·8004
208	120·4945	120·6104	120·7263	120·8423	120·9584
209	121·6559	121·7723	121·8888	122·0054	122·1220
210	122·8229	122·9399	123·0569	123·1740	123·2912

AREAS OF CIRCLES IN ALE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
176	86·7621	86·8604	86·9588	87·0573	87·1558
177	87·7480	87·8469	87·9459	88·0449	88·1439
178	88·7395	88·8390	88·9385	89·0380	89·1377
179	89·7366	89·8366	89·9366	90·0368	90·1370
180	90·7392	90·8398	90·9404	91·0411	91·1418
181	91·7474	91·8485	91·9497	92·0510	92·1523
182	92·7612	92·8629	92·9646	93·0664	93·1683
183	93·7805	93·8828	93·9851	94·0874	94·1898
184	94·8055	94·9082	95·0111	95·1140	95·2170
185	95·8359	95·9393	96·0427	96·1462	96·2497
186	96·8720	96·9759	97·0799	97·1839	97·2880
187	97·9136	98·0181	98·1226	98·2272	98·3318
188	98·9608	99·0658	99·1709	99·2761	99·3813
189	100·0136	100·1192	100·2248	100·3305	100·4363
190	101·0719	101·1781	101·2843	101·3905	101·4968
191	102·1358	102·2425	102·3493	102·4561	102·5630
192	103·2053	103·3126	103·4199	103·5272	103·6347
193	104·2804	104·3882	104·4960	104·6040	104·7119
194	105·3610	105·4693	105·5778	105·6863	105·7948
195	106·4472	106·5561	106·6651	106·7741	106·8832
196	107·5389	107·6484	107·7579	107·8675	107·9772
197	108·6363	108·7463	108·8564	108·9665	109·0767
198	109·7392	109·8498	109·9604	110·0711	110·1819
199	110·8476	110·9588	111·0700	111·1813	111·2926
200	111·9617	112·0734	112·1851	112·2970	112·4088
201	113·0813	113·1935	113·3059	113·4182	113·5307
202	114·2065	114·3193	114·4322	114·5451	114·6581
203	115·3372	115·4506	115·5640	115·6775	115·7911
204	116·4735	116·5875	116·7015	116·8155	116·9296
205	117·6154	117·7299	117·8445	117·9591	118·0737
206	118·7629	118·8779	118·9930	119·1082	119·2234
207	119·9159	120·0315	120·1472	120·2629	120·3787
208	121·0745	121·1907	121·3069	121·4232	121·5395
209	122·2387	122·3554	122·4722	122·5890	122·7059
210	123·4084	123·5257	123·6430	123·7604	123·8779

USE OF THE TABLE.

When the diameter of a circle is given in inches and tenths, the content in ale gallons, for an inch deep, may be found by inspection as follows: look for the inches in the left hand column of the Table under the word *Diameter*, then in the same horizontal line under the tenths will be the content required.

EXAMPLE.

What is the content, for an inch deep, in ale gallons, of a circle whereof the diameter is 137·7 inches?

SOLUTION.

Looking in the column of diameters for 137 inches, we find this integral number to be the fourth from the bottom in the fourth pair of pages of the Table: then proceeding horizontally along till we arrive under ·7, we discover the content in that angular space to be 52·8090 *ale gallons* required.

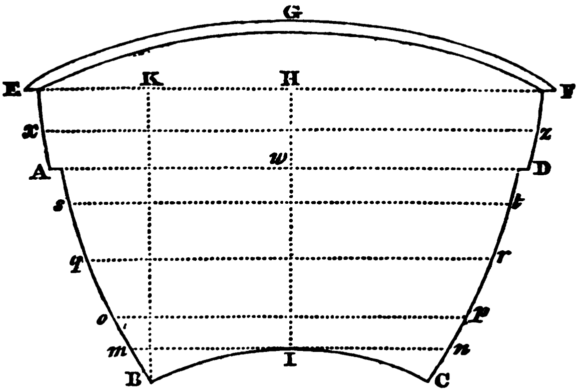
To the mouth of many Coppers is fastened a copper hoop, called a curb, of considerably greater diameter than the mouth of the Copper, and of no fixed depth. When a Copper with a curb is to be gaged, the method of proceeding is the same as that laid down for the Copper with the rising crown, only cross diameters are to be taken in the middle of the curb considered as one division of the Copper, provided its depth be an integral number of inches. But if the depth of the curb consist of inches and tenths, we are to take cross diameters at stated distances from the level of the top of the curb all the way down to the bottom of the Copper. The first distance will be at half the number of integral inches constituting the depth of the curb; the second will be at as many tenths within the Copper, as the odd tenths of the curb want of an inch; and half the mean diameter here, added to half the mean diameter first taken, must be considered the diameter for the dry inch immediately under the whole inches in the depth of the curb. This is on the supposition that as many tenths are within the Copper as in the curb, but if otherwise an allowance is to be made accordingly.

The remaining distances will be respectively at the middle of every ten inches below the place of the second pair of cross diameters, thus leaving (as we always must, where dry inches are taken) the odd tenths, if any, at the bottom.

On the contrary, in all cases where wet inches are taken, the odd tenths on the depth, must be left at the top.

EXAMPLE OF A COPPER WITH A CURB.

Let ABCD, be a perpendicular medial section of a Copper with a rising crown, having at the mouth the curb EADF; the content of the whole in ale gallons is required.



METHOD OF GAGING.

Lay a straight rod diametrically across the mouth of the curb from E to F, and take the perpendicular HI by means of a plumb-line hanging freely over the centre of the crown, and let this perpendicular be 45·8 inches. Then extend a small cord from A to D, diametrically across the mouth of the Copper at the shelf formed by the bottom of the curb, and take the distance H*w*, which let be 15 inches *the depth of the curb.*

Also take the perpendicular KB, which let be 50·9 inches: now the difference of HI and KB, that is, of 45·8 inches and 50·9 inches, which is 5·1 inches, will be *the height of the crown.*

Descend now into the Copper, and with a pair of compasses, opened to 5·1 inches, strike a line at that height from C, all round the Copper, and inscribe in the circle formed by this line a square as already directed. Then draw, on the sides of the Copper, perpendicular chalk-lines from the angles of the square up to the mouth. Next consider the depth, and determine on the divisions to be made; at the middle of each of which cross diameters are to be taken. These, in the present case, will do extremely well as follows:

First division.........The curb 15 inches.
Second division.......The first 10 inches from the curb.
Third division........The next 10 inches.
Last division.........The next 10·8 inches.

These four divisions constitute all above the level of the crown, and at the middle of these we are to make marks for the ends of the cross diameters, on the four perpendicular chalk-lines.

The distance of the marks from the circle described with the compasses will be,

First......(*op*)... 5·4 inches from the crown.
Second...(*qr*)...10·4 inches higher.
Third....(*st*)...10·0 inches higher.
Fourth...(*xz*). . 7·5 inches from the mouth.

Let the following be a copy of the memorandum now to be taken of the Copper, as we proceed to gage it.

The dimensions entered in the memorandum are, we shall suppose, the dimensions found by actual admeasurement.

Memorandum of A. B.'s *Copper*, No. 2, *Gaged*, 10 *May*, 1820.

Divisions in inches	Inches from the mouth	Cross Diameters	Mean Diameter
15	7.5	87·8.........87·2	87·5
10	20·0	78·1.........77·7	77·9
10	30·0	72·4.........72·8	72·6
10·8	40·4	63·7.........64.3	64·0
Crown by measure 30 gallons			....

Having correctly obtained the necessary dimensions, and carefully measured the quantity of liquor requisite to cover the crown, we proceed, after thus noting them in a memorandum, to make an entry of them in the Dimension Book; and the account will stand as below:

DIMENSION BOOK.

A. B.'s *Copper, No.* 2, *Gaged*, 10 *May*, 1820.

Depth	Diameter	Area.	Content in Gallons.	Content in B. F. G.
15	87·5	21·32	319·80	8..3..4·80
10	77·9	16·90	169·00	4..2..7·00
10	72·6	14·68	146·80	4..0..2·80
10·8	64·0	11·40	123·12	3..1..6·12
45·8	To cover the crown		30·00	0..3..3·00
Whole content of the Copper			788·72	21..3..5·72

The next step is to table the Copper, by subtracting from the whole content in barrels, firkins and gallons, the first area fifteen times; then the next, ten times;

also the third, ten times; and in like manner the fourth, ten times: thus leaving only *eight-tenths* of an inch on the crown.

As the process of Tabling has been sufficiently elucidated in gaging and inching the first Copper, any thing further on the subject would be here superfluous.

CASE II.

Of a Copper with a Falling Crown.

METHOD OF GAGING.

Lay a straight rod diametrically across the mouth, and with a plumb-line, take the greatest depth of the Copper in the middle; take likewise the least depth, by means of the same line, at the joining of the crown and sides of the Copper: then the difference of these perpendiculars will be the depth of the crown.

Next by cross diameters at the level of the mouth find the mean diameter, and inscribe a square in the Copper at that level; the side of which square, as has already been said, will be the mean diameter multiplied by ·707.

Now from the angles of the square, strike, with a chalked plumb-line, perpendiculars all the way down the sides of the Copper.

Having done this, set off 5 inches from the mouth upon each of the perpendiculars, then 10 inches, and again 10, as long as 10 can be taken so as to leave 5 inches clear below the last mark. But when 10 inches, so as to leave 5 inches clear on the side, cannot any longer be set off, make a faint mark at five inches below the last 10, and then at half the remainder make a mark for the middle of the bottom division of the Copper, not including the crown.

Proceed now to take cross diameters at each of the marks, which are respectively in the middle of every ten inches downwards, except the last, which is in the middle of the remainder over ten-inch divisions.

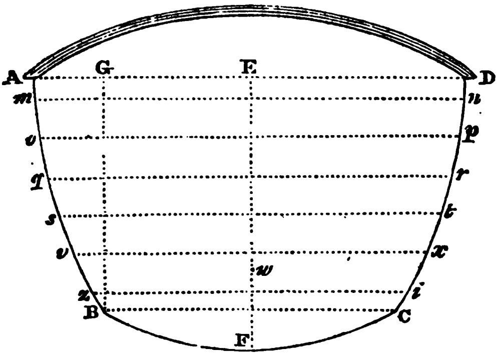
This part of the operation having been performed, let liquor into the Copper, till the crown is completely filled, and draw it carefully off into some vessel of known capacity: or, should you prefer the other method, water may be poured either from a filled firkin or gallon measure, into the Copper, till the surface of the water is level with the joining of the crown and sides; in which case you will keep an exact account of the quantity of water emptied into the Copper.

Should it, however, be deemed better to calculate the content of the crown by the pen, take a pair of cross diameters at its joining with the sides, and you have every dimension requisite, since the depth is already known.

Lastly, make a memorandum of your proceedings, from which you are to enter the Copper in the Dimension Book, and afterwards inch it.

EXAMPLE.

Let ABCD be a perpendicular medial section of a Copper with a falling crown, which it is required to gage and inch.



METHOD OF PROCEEDING.

Lay a straight rod AED diametrically across the Copper at the level of the mouth, and along the rod slide a line furnished with a plummet till the lead hangs freely from E over F, the centre of the crown. Then measure EF, which let be 68·7 inches, and slide the plumb-line back to G, perpendicularly over B, where the crown and sides meet, accurately measuring GB, which let be 58·7 inches. Here it is evident that the depth of the crown is 68·7 — 58·7 = 10 inches.

Next take two cross diameters at the mouth, which we shall suppose are 108·5 inches, and 109·3 inches: the mean is 108·9 inches, and this, multiplied by ·707, becomes 77 very nearly, for the side of the square to be inscribed in the Copper at the level of the mouth.

Having, therefore, either with a tape or cane, extended to 77 inches, inscribed the square, let fall from the four angles, perpendiculars down the sides of the

Copper, with a chalked line, for the cross diameters to end in, at the following depths:

Namely, the first pair at 5 inches from the mouth, the next at 15, the next at 25, and so on.

Wherefore we make four marks at 5 inches from the mouth, on each of the chalk-lines, as at *m* and *n*; again, four marks 10 inches lower, as at *o* and *p*: and at 10 inches still lower, as at *q* and *r*: a fourth set 10 inches lower, as at *s* and *t*: and a fifth set 10 inches lower, as at *v* and *x*.

If we now attempt another set of marks, we reach within less than 5 inches of the beginning of the bottom, and consequently, as directed, we desist, making a faint mark at *w*, five inches under the last set of marks.

But below *w*, there are still 8·7 inches left on the side, the half of which being 4·3 inches *nearly*, we make the last mark at that distance either from *w*, or from the bottom.

Lastly, if it be intended to calculate the content of the crown by the pen, a pair of cross diameters must be taken at that part of the Copper where the crown begins. These will, of course, be unnecessary, if we are to fill the crown with water.

The process of marking having been finished, we lose no time in taking the necessary cross diameters, and entering them faithfully in the Memorandum Book, of which let the following be a copy.

Memorandum of A. B.'s *Copper,* No. 3, *Gaged,* 10 *May,* 1820.

Divisions in Inches	Inches from the mouth	Cross Diameters	Mean Diameter
10	5	112·3......111·9	112·1
10	15	110·1......109·7	109·9
10	25	103·5......103.1	103·3
10	35	100·0...... 99·6	99·8
10	45	94·2...... 93·7	93·9
8·7	54·3	81·4...... 81·2	81·3
Crown	58·7	76·9...... 76·7	76·8
Crown by measure 8·4 gallons.			

It is evident, as we have just now stated, that the last pair of cross diameters, with the mean arising from them, would be superfluous, were we not to calculate, by the pen, the content of the crown considered as a spherical segment.

But first let us make an entry of this Copper, No. 3, in the Dimension Book, stating the number of gallons, found by measure, contained in the crown.

DIMENSION BOOK.

A. B.'s *Copper, No.* 3, *Gaged,* 10 *May,* 1820.

Depth	Diameter	Area	Content in Gallons	Content in B. F. G.
10	112·1	34·99	349·90	9..2..7·90
10	109·9	33·64	336·40	9..1..3·40
10	103·3	29·72	297·20	8..1..0·20
10	99·8	27·74	277·40	7..2..7·40
10	93·9	24·55	245·50	6..3..2·50
8·7	81·3	18·41	160·16	4..1..7·16
10	Crown by measure		8·40	0..0..8·40
68·7	*Whole content*		1674·96	46..2..0·96

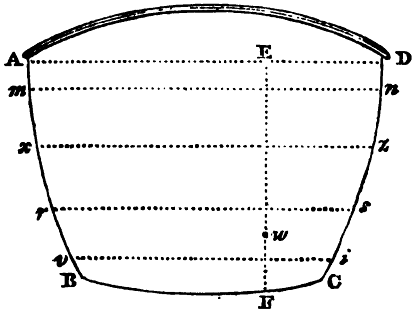
The next step would be, from the Dimension Book, to inch or table the Copper, but this we shall omit, as a reference to Copper, No. 1, will render the operation of inching perfectly intelligible and easy. We shall, therefore, conclude the present process with deducing the content of the crown from the dimensions taken; and it will be, [*See page* 212.]

2)	76·8	diameter of the base
	38·4	radius of the base
Multiply by	38·4	radius of the base
	1536	
	3072	
	1152	
Product	1474·56	square of half BC.
Multiply by	3	according to rule
Product	4423·68	triple square
10 × 10 =	100·00	square of the depth
	4523·68	sum
Multiply by	10	altitude
A.G. sph. div. 538·58)	45236·8	(8·39 ale gallons.

When the concavity formed by the falling crown of a Copper is very inconsiderable, or in other words, when the bottom appears to be a spherical segment of very diminutive altitude compared with the diameter of the sphere, a mean depth, as in the following diagram, may be taken on the base, half way between the centre and side, as E*w*F: then, disregarding the crown, cross diameters at stated depths are to be taken as in the preceding Coppers.

EXAMPLE.

Let ABCD be a Copper to be gaged and inched, of which the fall of the bottom is very inconsiderable.



METHOD OF GAGING.

Lay a rod diametrically across the mouth from A to D, and along the rod slide a plumb-line, till the lead hangs over F, half-way between the centre of the bottom and the side.

The distance between E and F, which let be 40·6 inches, must be considered as the *mean* depth of the Copper.

Cross diameters are now to be taken five inches from the mouth, as at *mn*; then at 10 inches lower, as at *xz*; and again 10 inches lower, as at *rs*. Now setting off ten inches downwards from *rs*, there remain 5 inches and 6 tenths. But the last pair of cross diameters would not, in this case, be exactly in the middle of the last division, (for we cannot well take cross diameters in the middle of the remaining 6 tenths, so as to consider the ten inches above, as a division,) wherefore we make, at five inches below *rs*, a faint mark as at *w*, and take cross diameters half way betwixt *w* and the bottom, as at *vi.*

These being all the necessary dimensions, we enter them, one by one, in the Memorandum Book, as we take them, and shall suppose them to stand as follows:

Memorandum of A. B.'*s Copper*, No. 4, *Gaged*,
10 *May*, 1820.

Divisions in Inches	Inches from the mouth	Cross Diameters	Mean Diameters
10	5	61·2......62·0	61·6
10	15	58·6......58·8	58·7
10	25	52·0......53·0	52·5
10·6	35·3	46·8......46·4	46·6

From this memorandum we make the following entry in the Dimension Book, and with which we shall conclude this first problem, having sufficiently explained and illustrated the method of inching or tabling a Copper, in PROBLEM I., to which the reader may, whenever either a difficulty or a doubt occurs, advantageously refer.

DIMENSION BOOK.

A. B.'s *Copper, No.* 4, *Gaged,* 10 *May,* 1820.

Depth	Diameter	Area	Content in Gallons	Content in B. F. G.
10	61·6	10·57	105·70	2..3..6·70
10	58·7	9·60	96·00	2..2..6·00
10	52·5	7·67	76·70	2..0..4·70
10·6	46·6	6·05	64·13	1..3..1·13
40·6	*Content of the Copper*		342·53	9..2..0·53

PROBLEM II.

To Gage and Inch a Copper-Back.

Copper-Backs are vessels with flat bottoms, generally rectilineal, and constructed sometimes of copper but more frequently of wood, placed horizontally over the mouth of a Copper, so that the steam from the Copper may impart a portion of its heat to the liquor in the Back, and thus prepare it for the purposes to which it is subsequently to be applied.

The most common form of a Copper-Back is a quadrilateral with two adjutments in front, between which is the entrance to the mouth of the Copper, with a firm door (when on a large scale) to prevent the steam from issuing out in this direction, and to secure its action more completely on the Back over the Copper.

METHOD OF GAGING.

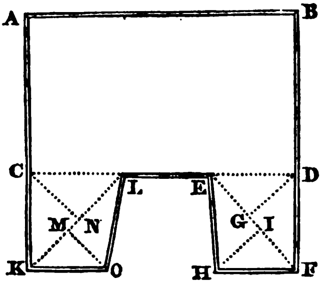
Divide the base into such geometrical figures as can be most easily and conveniently measured with the fewest dimensions, on the principles fully explained in the First and Second Sections of MENSURATION. Then take the lengths of the necessary lines, carefully noting them in a memorandum.

Next chuse a constant dipping place by the side of the Back, to which you can at all times approach without labour or interruption. The dipping place having been chosen, cause liquor to be admitted into the back, till the bottom is just covered, and take the depth of the liquor at the dipping place: after which, draw the whole off into a vessel of known capacity.

Lastly, observe whether the sides of the Back are all perpendicular or nearly so, and conclude with calculating the area, at an inch deep, in ale gallons, adding it first to the drip (if any) and then continually to the sum, in barrels, firkins, and gallons, for wet inches; leaving all odd tenths on the depth, at the top.

EXAMPLE.

Let ABFHELOK be a Copper-Back with a flat bottom and perpendicular sides, to be gaged and inched.



PROCESS OF TAKING THE DIMENSIONS.

Prolong the line EL both ways to C and D, then draw the diagonals EF, and KL; after which, demit on them the perpendiculars DG, HI, CN, OM.

Having performed this operation with a chalked line on the base of the Back, take the length and breadth of the parallelogram ABCD, with the lengths of the two diagonals, and the four perpendiculars; and let these eight dimensions be as follows:

AB = 140·0 inches
AC = 80·5 inches
EF = 64·0 inches
DG = 30·0 inches
HI = 28·4 inches
KL = 68·0 inches
CN = 34·6 inches
OM = 27·0 inches

The lengths of the lines having been taken and entered in a memorandum as above, chuse a constant dipping place, which we shall suppose to be half way betwixt F and H; and desire liquor to be admitted into the Back to cover the bottom.

As soon as this is effected, put the dipping rod perpendicularly down to the bottom at the dipping place; and then drawing it up, observe the number of tenths that are wet. These (which in this instance we shall suppose to be 7) are to be conspicuously marked on the Back near the dipping place, with the sign *minus* prefixed to them, to shew that at all dips, 7 tenths are to be deducted from the apparent depth of the Back, for the true depth.

Thus,

[C. B. —00·7]

The two letters mean Copper-Back, and the figures with the negative sign before them, is the correction on all dips, as above explained.

The next step is to draw the water carefully off into some vessel, such as a firkin, or half gallon measure, and then to enter the whole quantity as the *Drip* of the BACK.

We shall suppose the drip of the Back before us to be 19 gallons; and then proceed with the calculation.

CALCULATION.

	140	side AB.
Multiply by	80·5	breadth AC.
	700	
	11200	
Product	11270·0	*area of* ABCD.
	30·0	perpendicular DG.
Add	28·4	perpendicular HI.
	58·4	sum
Multiply by	32	half of EF.
	1168	
	1752	
Product	1868·8	*area of* DFHE.
	34·6	perpendicular CN.
Add	27·0	perpendicular OM.
	61·6	sum
Multiply by	34	half of KL.
	2464	
	1848	
Product	2094·4	*area of* CKOL.

Now it is evident that the sum of these three areas, will be the area of the whole base: wherefore,

11270·0 area of ABCD
1868·8 area of DFHE.
2094·4 area of CKOL.

A.G. divisor 282) 15233·2 (54·02 ale gallons.

Now 54·02 ale gallons are equal to *one* barrel, *two* firkins, and 0·02 gallons, the content of the Back for one inch deep, when the base is even and level. Consequently, as the sides are supposed to be perpendicular, this content added to the Drip, and continually to the sum, as many times in all as there are inches in the depth, will give the content of the Back at every wet inch throughout.

Hence the process of tabling the Back, will be the following:

A. B.'s Copper-Back Tabled.

Wet Inches	Content in B. F. G.	Wet Inches	Content in B. F. G.
Drip	0..2..1·00 1..2..0·02	5	8..0..1·10 1..2..0·02
1	2..0..1·02 1..2..0·02	6	9..2..1·12 1..2..0·02
2	3..2..1·04 1..2..0·02	7	11..0..1·14 1..2..0·02
3	5..0..1·06 1..2..0·02	8	12..2..1·16 1..2..0·02
4	6..2..1·08 1..2..0·02	9	14..0..1·18 1..2..0·02
5	8..0..1·10	10	15..2..1·20

A. B.'s Copper-Back Tabled.

Wet Inches	Content in B. F. G.	Wet Inches	Content in B. F. G.
10	15..2..1·20 1..2..0·02	20	30..2..1·40 1..2..0·02
11	17..0..1·22 1..2..0·02	21	32..0..1·42 1..2..0·02
12	18..2..1·24 1..2..0·02	22	33..2..1·44 1..2..0·02
13	20..0..1 26 1..2..0·02	23	35..0..1·46 1..2..0·02
14	21..2..1·28 1..2..0·02	24	36..2..1·48 1..2..0·02
15	23..0..1·30 1..2..0·02	25	38..0..1·50 1..2..0·02
16	24..2..1·32 1..2..0·02	26	39..2..1·52 1..2..0·02
17	26..0..1·34 1..2..0·02	27	41..0..1·54 1..2..0·02
18	27..2..1·36 1..2..0·02	28	42..2..1·56 1..2..0 02
19	29..0..1·38 1..2..0·02	29	44..0..1·58 1..2..0 02
20	30..2..1·40	30	45..2..1·60

Here the tabling has been carried to 30 inches, and may be continued at pleasure to the whole depth of the Back, whatever it may be, by the continual addition of 1 barrel, 2 firkins, 0·02 gallons, the equivalent of 54·02 ale gallons, as already stated ; which we found by calculation to be the area of the base, for an inch deep.

PROBLEM III.

To Gage and Inch a Copper Pan in the Form of the Segment of a Hollow Sphere.

Take two cross diameters (which ought to be equal) at the level of the lowest part of the mouth, and consider half their sum as the mean diameter of the segment's base: take also the depth of the Pan. These will be all the dimensions necessary, if the Pan be really part of the segment of a sphere.

METHOD OF DETERMINING THE DIAMETERS AT EVERY INCH.

Divide the square of half the base by the altitude, and to the quotient add the altitude: the sum will be the *diameter of the sphere* whereof the Pan is a part.

Then for the diameter of the segment at *one* inch deep, subtract *one* inch from the diameter of the sphere, and multiply the remainder by 1. Twice the square root of the product will be the diameter required.

Next, for the diameter of the segment at *two* inches deep, subtract *two* inches from the diameter of the sphere, and multiply the remainder by 2. Twice the square root of the product will be the diameter required.

Again, for the diameter of the segment at *three* inches deep, subtract *three* inches from the diameter of the sphere, and multiply the remainder by 3. Twice the square root of the product will be the diameter required.

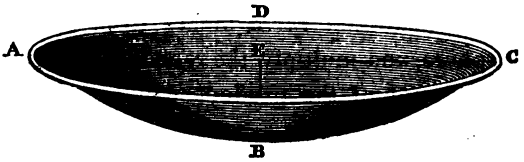
So, for the diameter of the segment at *four* inches deep, subtract *four* inches from the diameter of the sphere, and multiply the remainder by 4. Twice the square root of the product will be the diameter required.

Likewise, for the diameter of the segment at *five* inches deep, subtract *five* inches from the diameter of the sphere, and multiply the remainder by 5. Twice the square root of the product will be the diameter required.

In this manner proceed through the whole depth of the Pan, and from the Table of the Areas of Circles in Ale Gallons, take out the areas respectively answering to half the sum of every two contiguous diameters at the various depths. Then divide three times the square of half the lowest diameter, by the spherical divisor for ale gallons, and the quotient will be the content of the segment at the bottom, in ale gallons. Add, now, the second inch-area to the content of this segment; the third to the sum; the fourth to that sum; and so on, all in barrels, firkins, and gallons, up to the mouth of the Pan.

EXAMPLE.

Let ABCD be a copper Pan in the form of a spherical segment, to be gaged and inched.



METHOD OF PROCEEDING.

Lay a rod diametrically across the mouth of the Pan from A to C, and take the depth EB, which let be 15 inches. Then from two cross diameters at the mouth, suppose 89·9 inches, and 90·1 inches, find the mean diameter of the mouth, which in this event will be 90 inches.

These dimensions having been taken, and noted in a memorandum, we come to determine the several diameters: and first for the diameter of the sphere, it will be,

2) 90 diameter of the mouth
Quotient 45 radius of the mouth
Multiply by 45 radius of the mouth
225
180
Depth 15) 2025 (135 quotient
Add 15 altitude
Sum 150 *diameter of the* SPHERE.

Next, for the diameters of the Pan at every inch upwards from the bottom, it will be,

Inches	Diam. in Inches	Wet Inches
— 1 = 149		
ltiply by 1		
149 whereof twice the sq. root is	24·4	1
) — 2 = 148		
ltiply by 2		
296 whereof twice the sq. root is	34·4	2
) — 3 = 147		
ltiply by 3		
441 whereof twice the sq. root is	42·0	3
) — 4 = 146		
ltiply by 4		
584 whereof twice the sq. root is	48·3	4
) — 5 = 145		
ltiply by 5		
725 whereof twice the sq. root is	53·9	5
0 — 6 = 144		
ultiply by 6		
864 whereof twice the sq. root is	58·7	6
0 — 7 = 143		
ultiply by 7		
1001 whereof twice the sq. root is	63·2	7

Inches	Diam. in Inches	Wet Inches
150 — 8 = 142		
Multiply by 8		
1136 whereof twice the sq. root is	67·4	8
150 — 9 = 141		
Multiply by 9		
1269 whereof twice the sq. root is	71·2	9
150 — 10 = 140		
Multiply by 10		
1400 whereof twice the sq. root is	74·8	10
150 — 11 = 139		
Multiply by 11		
1529 whereof twice the sq. root is	78·2	11
150 — 12 = 138		
Multiply by 12		
1656 whereof twice the sq. root is	81·3	12
150 — 13 = 137		
Multiply by 13		
1781 whereof twice the sq. root is	84·4	13
150 — 14 = 136		
Multiply by 14		
1904 whereof twice the sq. root is	87·2	14

The diameter of the mouth of the Pan was found by admeasurement to be 90 inches, and therefore it is not necessary to calculate it by the pen: yet should any one be curious enough to try in this manner the truth of the result, he will find it correspond to the greatest nicety.

As the diameters at every inch of the Pan are now ascertained, the entry of half the sum of each contiguous pair, in the Dimension Book, with the area in gallons, and content in *barrels*, *firkins*, and *gallons*, opposite to each mean, will be easily made.

Last of all must be entered the segment at the bottom, the content of which, as has been already said, will be found in ale gallons, by dividing three times the square of half the diameter at one inch from the vertex, by the spherical divisor for ale gallons.

It would here be idle to exhibit at length the process of obtaining each *mean* from the sum of every two contiguous diameters: and therefore we shall only instance the first, which will be as follows:

90·0 diameter at 15 inches
87·2 diameter at 14 inches
Divide by 2) 177·2 sum
Quotient 88·6 *mean diameter at* 14·5 *inches.*

We shall now submit the entry of this Pan in the Dimension Book, where it will be seen that each inch, in depth, constitutes a division.

DIMENSION BOOK.

A. B.'s *Copper Pan Gaged,* 10 *May,* 1820.

Depth	Diameter	Area in Gallons	Content in B. F. G.
1	88·6	21·86	0..2..3·86
1	85·8	20·50	0..2..2·50
1	82·8	19·09	0..2..1·09
1	79·7	17·69	0..1..8·69
1	76·5	16·30	0..1..7·30
1	73·0	14·84	0..1..5·84
1	69·3	13·37	0..1..4·37
1	65·3	11·87	0..1..2·87
1	60·9	10·33	0..1..1·33
1	56·3	8·82	0..0..8·82
1	51·1	7·27	0..0..7·27
1	45·1	5·66	0..0..5·66
1	38·2	4·06	0..0..4·06
1	29·4	2·40	0..0..2·40
1	* 24·4	SEG. 1·01	0..0..1·01
15	*Whole content*	175·07	4..3..4·07

The concluding process, that of tabling the Pan, will be as under:

A. B.'s COPPER PAN TABLED.

Wet Inches	Content in B. F. G.
1	0..0..1·01 0..0..2·40
2	0 0..3·41 0..0..4·06
3	0..0..7·47

A. B.'s COPPER PAN TABLED.

Wet Inches	Content in B. F. G.
3	0..0..7.47 0..0..5·66
4	0..1..4·13 0..0..7·27
5	0..2..2·40 0..0..8·82
6	0..3..2·22 0..1..1·33
7	1..0..3·55 0..1..2·87
8	1..1..6·42 0..1..4·37
9	1..3..1·79 0..1..5·84
10	2..0..7·63 0..1..7·30
11	2..2..5·93 0..1..8·69
12	3..0..5·62 0..2..1·09
13	3..2..6·71 0..2..2·50
14	4..1..0·21 0..2..3·86
15	4..3..4·07

Thus have we given the true method of gaging and inching a Pan in the form of the segment of a hollow sphere; but as Pans so accurately co-inciding with this shape are seldom to be met with, the diameters we have calculated by the pen, will, for the most part, have to be taken by admeasurement.

PROBLEM IV.

To Gage and Inch a Round Mash-Tun.

The most common form of a Mash Tun is that of the frustum of a cone standing on the greater base, and seldom differing much from a true frustum; but Mash-Tuns of other forms are likewise occasionally met with.

METHOD OF GAGING A ROUND TUN.

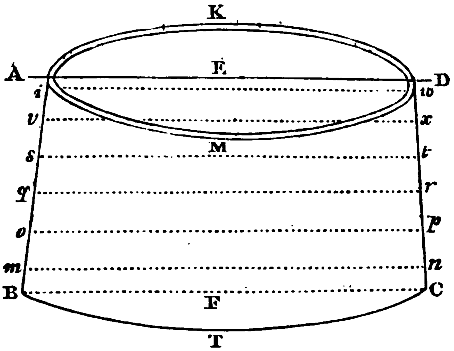
From two cross diameters at the bottom, find the mean diameter, and multiply it by ·707, for the side of the square to be inscribed at the bottom: then with a rule, or tape, extended to the number of inches in the product, inscribe the square; and from the four angles strike perpendiculars on the sides with a chalked line up to the mouth of the Tun. Next take the perpendicular depth; and at the middle of every 10 inches from the bottom, cross diameters. In case the depth of the Tun is not a multiple of ten inches, the last pair of cross diameters is to be taken at the middle of the excess above tens.

Of the several dimensions found by measure make a memorandum, and from the memorandum fill up the Dimension Book, calculating the areas at an inch deep in Mash-Tun Gallons, and afterwards converting them to *Quarters, Bushels, and Gallons.*

The last step of all will be to inch the Tun by continually adding to each area, in quarters, bushels, and gallons, the content under it in the Tun.

EXAMPLE.

Let AKDMBTC be a Mash Tun resembling the frustum of a cone standing on its greater end, to be gaged and inched.



METHOD OF TAKING THE DIMENSIONS.

Descend into the Tun, and from two cross diameters at the bottom, (which we shall suppose are 107·2 inches, and 106·8 inches,) find a mean diameter, which will in this case be 106 inches. Then multiply this mean by ·707, and it will be,

107 mean diameter
Multiply by ·707 constant decimal

749
7490

Product 75·649 *side of the inscribed square.*

Extending, therefore, a dimension-cane, or a tape, to 75·6 inches, inscribe a square in the circle forming the bottom of the Tun. Then with a chalked line strike perpendiculars on the sides, from the four angles of the square, up to the mouth.

Next, at five inches from the bottom make a mark on each of the chalk-lines, for the middle of the first ten inches, as at *mn*.

A rod being now laid diametrically across the mouth of the Tun from A to D, the perpendicular depth EF, is to be taken, which we can imagine to be 55 inches.

The marks, for the ends of the several pairs of cross diameters, will consequently be as follows:

Second pair 15 inches from the bottom, as at *op*.
Third ...... 25 inches from the bottom, as at *qr*.
Fourth ...... 35 inches from the bottom, as at *st*.
Fifth ...... 45 inches from the bottom, as at *vx*.
Sixth ...... 2·5 inches from the mouth, as at *iw*.

The necessary dimensions having been all carefully taken, let the copy of the memorandum be as under:

Memorandum of A. B.'*s Round Mash-Tun*,
Gaged, 10 *May*, 1820.

Divisions in Inches	Depth from the mouth	Cross Diameters	Mean Diameter
5	2.5	95·2......... 94·8	95·0
10	10·0	98·0......... 98·2	98·1
10	20·0	100·4.........100·0	100·2
10	30·0	102·6.........102·4	102·5
10	40·0	104·0.........104·0	104·0
10	50·0	106·2.........106·6	106·4

From this memorandum we make the following entry in the Dimension Book, dividing the square of each diameter by 289, (the circular divisor for Mash-Tun Gallons,) and thus obtaining the *Gallons*. These, divided by 8, become *Bushels;* and bushels, divided by 8, give *Quarters.*

DIMENSION BOOK.

A. B.'s *Round Mash-Tun Gaged*, 10 *May*, 1820.

Depth	Diameter	Area in Gallons	Area in Q. B. G.	Content in Q. B. G.
5	95·0	31·2	0..3..7·2	2..3..4·0
10	98·1	33·3	0..4..1·3	5..1..5·0
10	100·2	34·7	0..4..2·7	5..3..3·0
10	102·5	36·3	0..4..4·3	5..5..3·0
10	104·0	37·4	0..4..5·4	5..6..6·0
10	106·4	39·1	0..4..7·1	6..0..7·0
55	*Whole Content*			30..5..4·0

The area, for an inch deep, in *Quarters*, *Bushels*, and *Gallons*, having been now ascertained for each of the diameters taken, we are to construct a Table of the Tun, in which the content for any assigned depth may be seen by inspection.

To do this, we have only to add the area for the first inch at the bottom, continually, to itself, till the sum contains the first area 10 times; then to the content thus found at 10 inches deep, we are to add, in like manner, the area for an inch of the second division, 10 times also: and so of all the rest except the last, which, since the uppermost division of the Tun consists of no more than 5 inches, must be added only 5 times.

In the course of the operation it will be of advantage to check the result at the end of every division of the Tun, as this will, in case of error, prevent a great deal of trouble.

The practical mode of checking the process of tabling has been fully explained, and elucidated by an example in PROBLEM I., as will be seen on referring to page 237. The only difference is, that in Coppers the areas are subtracted from the top for dry inches, but in Tuns added from the bottom for wet inches.

A. B.'s ROUND MASH-TUN TABLED.

Wet Inches	Q. B. G.	Wet Inches	Q. B. G.
1	0..4..7·1 0..4..7·1	8	4..7..0·8 0..4..7·1
2	1..1..6·2 0..4..7·1	9	5..3..7·9 0..4..7·1
3	1..6..5·3 0..4..7·1	10 *	6..0..7·0 0..4..5·4
4	2..3..4·4 0..4..7·1	11	6..5..4·4 0..4..5·4
5	3..0..3·5 0..4..7·1	12	7..2..1·8 0..4..5·4
6	3..5..2·6 0..4..7·1	13	7..6..7·2 0..4..5·4
7	4..2..1·7 0..4..7·1	14	8..3..4·6 0..4..5·4
8	4..7..0·8	15	9..0..2·0

A. B.'s ROUND MASH-TUN TABLED.

Wet Inches	Q. B. G.	Wet Inches	Q B. G.
15	9..0..2·0 0..4..5·4	28	16..3..7·4 0..4..4·3
16	9..4..7·4 0..4..5·4	29	17..0..3·7 0..4..4·3
17	10..1..4·8 0..4..5·4	30 *	17..5..0·0 0..4..2·7
18	10..6..2·2 0..4..5·4	31	18..1..2·7 0..4..2·7
19	11..2..7·6 0..4..5·4	32	18..5..5·4 0..4..2·7
20 *	11..7..5·0 0..4..4·3	33	19..2..0·1 0..4..2·7
21	12..4..1·3 0..4..4·3	34	19..6..2·8 0..4..2·7
22	13..0..5·6 0..4..4·3	35	20..2..5·5 0..4..2·7
23	13..5..1·9 0..4..4·3	36	20..7..0·2 0..4..2·7
24	14..1..6·2 0..4..4·3	37	21..3..2·9 0..4..2·7
25	14..6..2·5 0..4..4·3	38	21..7..5·6 0..4..2·7
26	15..2..6·8 0..4..4·3	39	22..4..0·3 0..4..2·7
27	15..7..3·1 0..4..4·3	40 *	23..0..3.0 0..4..1·3
28	16..3..7·4	41	23..4..4·3

A. B.'s ROUND MASH-TUN TABLED.

Wet Inches	Q. B. G.	Wet Inches	Q. B. G.
41	23..4..4·3 0..4..1·3	48	27..1..5·4 0..4..1·3
42	24..0..5.6 0..4..1·3	49	27..5..6·7 0..4..1·3
43	24..4..6·9 0..4..1·3	50 *	28..2..0·0 0..3..7·2
44	25..1..0·2 0..4..1·3	51	28..5..7·2 0..3..7·2
45	25..5..1·5 0..4..1·3	52	29..1..6·4 0..3..7·2
46	26..1..2·8 0..4..1·3	53	29..5..5·6 0..3..7·2
47	26..5..4·1 0..4..1·3	54	30..1..4·8 0..3..7·2
48	27..1..5·4	55	30..5..4·0

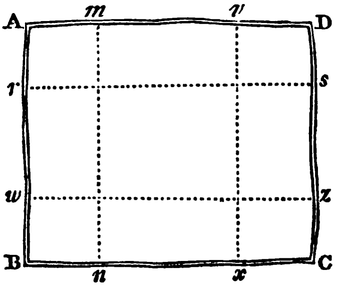
As the Goods in a Mash Tun are not of a nature to assume a surface perfectly level, but on the contrary have, for the most part, the surface very uneven, no account of drip needs be taken, since a mean depth, from several dips, is invariably to be considered the depth of the Goods.

PROBLEM V.

To Gage and Inch a Square Mash-Tun.

At the middle of every ten inches from the bottom, take the breadth of the Tun half way between the middle of the side and each of the ends: and at the same time take the length on the same level, half way between the middle of the end and each of the sides.

Thus, if ABCD were a horizontal section of a square Mash-Tun, five inches from the bottom, the two breadths would be *mn*, *vx*; and the two lengths would be *rs*, *wz*.



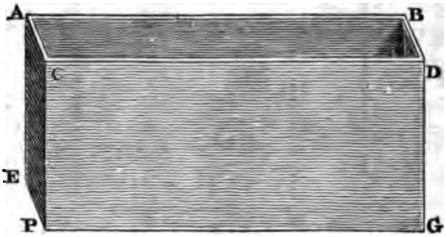
Then half the sum of *mn* and *vx* would be the mean breadth of the Tun five inches from the bottom; and half the sum of *rs* and *wz* would be the mean length on the same level.

Precisely in this way the mean breadth and mean length at 15 inches, 25 inches, 35 inches, &c. from the bottom will be obtained from two breadths and two lengths respectively at those altitudes. If any inches and tenths remain at the mouth, above the last division of ten inches, two breadths, and two lengths are to be taken at half the altitude of this smaller division from the mouth.

Of your proceedings make a memorandum, from which fill up the Dimension Book as explained for the Round Tun, and conclude with constructing a Table from the Dimension Book, in every respect similar to the Table last exhibited.

EXAMPLE.

Let ABCDEPG be a square Mash Tun, to be gaged and inched.



METHOD OF GAGING.

With a tape extended from A to B, ascertain the length of the Tun, which let be 96 inches: then at 24 inches from A, towards B, (being *one-fourth* part of 96 inches,) strike with a chalked line a perpendicular on the side within, and at the same distance from B towards A, strike another perpendicular. Repeat this operation on the opposite side of the Tun; and then measure the breadth AC which let be 80 inches.

Next at 20 inches (one fourth part of 80) from A towards C, strike a perpendicular line down the end of the Tun within, and another at 20 inches from C towards A.

This operation having been repeated on the opposite end, descend into the Tun, and make marks at five inches high on each of the perpendiculars; with another set at 15 inches high; and a third set at 25 inches; carrying the ten-inch divisions as far as possible up the side, but always leaving 5 inches clear above the last mark.

If any number of inches, more than five, but less than ten, remain at the mouth, make marks at the half of the difference between 5 inches, and the excess above 5 inches, reckoning from the mouth.

As soon as the marking is finished, proceed to take the lengths and breadths of the Tun, noting them in a memorandum, with the mean length and breadth arising from them.

In the instance before us, we shall suppose the following to be a copy of the memorandum furnished by actual admeasurement.

Memorandum of A. B.'s *Square Mash-Tun.*

Divisions	Lengths	Mean Length	Breadths	Mean Breadth
10	95·8...96·0	95·9	79·6...79·8	79·7
10	96·0...96·2	96·1	80·2...80·0	80·1
10	95·9...95·7	95·8	80·4...80·2	80·3
10	96·8...97·2	97·0	81·0...81·0	81·0

In this memorandum each mean length is deduced from two lengths, and each mean breadth from two breadths; but it sometimes happens that three or four lengths and as many breadths are taken on the same level. In all cases, however, the mean length is the quotient of the sum of all the lengths divided by the number of lengths taken, and the mean breadth is the quotient of the sum of the breadths divided by the number of breadths.

Being now in possession of every requisite for the Dimension Book and Table, we make the following entry, dividing the product of each length and breadth by 227, the number of cubic inches in a Mash-Tun Gallon, and thus obtaining *Gallons.* These, divided by 8, become *Bushels;* and bushels, divided by 8, give *Quarters.*

DIMENSION BOOK.

A. B.'*s Square Mash-Tun Gaged*, 10 *May*, 1820.

Depth	Length	Breadth	Area in Gallons	Area in Q. B. G.	Content in Q. B. G.
10	95·9	79·7	33·6	0..4..1·6	5..2..0·0
10	96·1	80·1	33·9	0..4..1·9	5..2..3·0
10	95·8	80·3	33·8	0..4..1·8	5..2..2·0
10	97·0	81·0	34·6	0..4..2·6	5..3..2·0
40	*Whole content of the Tun*				21..1..7·0

The Dimension Book being thus complete, we shall proceed to tabulate as far as 30 inches, which will be amply sufficient for a specimen.

A. B.'s SQUARE MASH-TUN TABLED.

Wet Inches	Q. B. G.	Wet Inches	Q. B. G.
....	.... 0..4..2·6	13	6..7..7·4 0..4..1·8
1	0..4..2·6 0..4..2·6	14	7..4..1·2 0..4..1·8
2	1..0..5·2 0..4..2·6	15	8..0..3·0 0..4..1·8
3	1..4..7·8 0..4..2·6	16	8..4..4·8 0..4..1·8
4	2..1 .2·4 0..4..2·6	17	9..0..6·6 0..4..1·8
5	2..5..5·0 0..4..2·6	18	9..5..0·4 0..4..1·8
6	3..1..7·6 0..4..2·6	19	10..1..2·2 0..4..1·8
7	3..6..2·2 0..4..2·6	20 *	10..5..4·0 0..4..1·9
8	4..2..4·8 0..4..2·6	21	11..1..5·9 0..4..1·9
9	4..6..7·4 0..4..2·6	22	11..5..7·8 0..4..1·9
10 *	5..3..2·0 0..4..1·8	23	12..2..1·7 0..4..1·9
11	5..7..3·8 0..4..1·8	24	12..6..3·6 0..4..1·9
12	6..3..5·6 0..4..1·8	25	13..2..5·5 0..4..1·9
13	6..7..7·4	26	13..6..7·4

A. B.'s SQUARE MASH-TUN TABLED.

Wet Inches	Q. B. G.	Wet Inches	Q. B. G.
26	13..6..7·4 0..4..1·9	28	14..7..3·2 0..4..1·9
27	14..3..1·3 0..4..1·9	29	15..3..5·1 0..4..1·9
28	14..7..3·2	30	15..7..7·0

PROBLEM VI.

To Gage and Inch an Oval Mash-Tun.

Extend a small cord or rule, longitudinally across the mouth, shifting the ends of the cord or rule backwards and forwards, till the greatest length of the oval is found: then make two marks with chalk on the inside of the mouth to represent the extremities of the transverse axis of the upper oval. Next extend the cord or rule diametrically across the middle of the mouth-shifting the ends of the cord or rule till the greatest width is found, and make two marks with chalk as in the first instance. Having thus found the extremities of the transverse and conjugate diameters, strike perpendiculars down the inside of the Tun at each of the four marks, with a chalked line, and take the depth.

After this, let marks be made on each of the chalk, lines, at the middle of every ten inches, counting from the bottom; and take the transverse and conjugate diameters of the Tun at the several marks. If any inches over tens remain at the top, diameters are to be taken at half the number from the top.

A faithful memorandum of every one of the dimensions having been made on the spot, the Dimension Book will be easily filled up, if the preceding problems have been carefully noticed. The area, for one inch deep, at any part of the Tun will evidently be the product of the transverse and conjugate axis at that depth, divided by 289, the circular divisor for Mash-Tun Gallons.

The process of tabulation, which is the last step of all, has already been sufficiently explained and illustrated.

PROBLEM VII.

To Gage and Inch a Hop-Back or Jack-Back.

Hop-Backs, or Jack-Backs, are usually in the form of a cube or parallelopipedon, though some are cylindrical, and others cylindroidal.

METHOD OF GAGING A SQUARE JACK-BACK.

Take the depth at the fixed dipping place, and from several lengths of the Back at various places ascertain the mean length, by dividing the sum of all the lengths, by the number of lengths taken. In like manner, from several breadths, deduce a mean breadth.

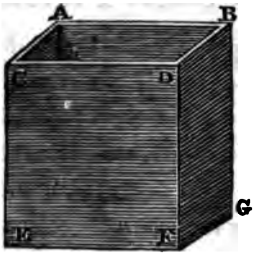
Then multiply the *mean length* by the *mean breadth*, for the area of the Back in cubic inches, which divided by 282, the number of cubic inches in an ale gallon, will give the area in ale gallons.

This area added to itself will be the content of the Back at 2 inches; and thus, by continual addition, may the content be found for any number of inches. It should here be observed that in gaging worts in a Jack-Back, an allowance in the depth must be made for hops; which allowance is left to the discretion of the Gager. This deduction is always more for strong than for table beer, because more hops are used for the former than for the latter: the abatement must also be in proportion to the quantity of wort or beer actually in the Back.

The best method of ascertaining exactly the allowance to be made for hops, false bottom, &c. in a Jack-Back, is, to compare the gages of wort or beer in the Back, and in the Cooler; for the difference divided by the area of the Back must evidently give the number of inches to be allowed.

EXAMPLE.

Let ABCDEFG be a square Hop-Back, with perpendicular sides, to be gaged and inched.



OF TAKING THE DIMENSIONS.

With a tape or dimension-cane measure the length of the Back in several places, and at various depths; and suppose the result to be as follows:

1st length	102·0	inches
2nd ———	101·9	inches
3rd ———	102·1	inches
4th ———	102·0	inches
5th ———	101·8	inches
6th ———	102·2	inches
Sum	612·0	inches

Here six different lengths have been taken, and their sum is 612·0 inches. Now the mean length will be this sum divided by the number of lengths; that is, 612 divided by 6. But 612 divided by 6 will quote 102, as follows:

Divide by 6) 612 sum of six lengths
Quotient 102 inches, *mean length.*

In like manner take several breadths in various parts of the Back, and at different depths; which breadths let be the following:

1st breadth 86·6 inches
2nd ——— 87·0 inches
3rd ——— 87·1 inches
4th ——— 86·9 inches
5th ——— 86·7 inches
6th ——— 86·5 inches
Sum 520·8 inches

Here the sum of six different breadths is 520·8 inches, which divided by 6 (the number of breadths) will quote 86·8 inches for the mean breadth, as below:

6) 520·8 sum of six breadths
Quotient 86·8 inches, *mean breadth.*

Consequently, for the area of the Back, it will be,

86·8 mean breadth
Multiply by 102 mean length
1736
8680
A. G. divisor 282) 8853·6 (31·39 ale gallons.

The content of the Back, for one inch deep, being 31·39 ale gallons, it will, for *one-tenth* of an inch, be 3·139 gallons. Wherefore by the continual addition of 3·139 gallons to the drip (if any,) we shall obtain a Table of the content of the Back for every inch and tenth of its depth.

It will be sufficient for our present purpose to table the above Back to a few tenths only, since, by the simple process of addition, the Table may be extended to any depth at pleasure; and here it may be observed that, in general, Backs (unless very large) are inched in the tabulation, and not tenthed.

A. B.'s JACK-BACK, No. 1, TABLED.

Wet Inches	B. F. G.	Wet Inches	B. F. G.
0·1	0..0..3·14 0..0..3·14	1.1	0..3..7·54 0..0..3·14
0·2	0..0..6·28 0..0..3·14	1·2	1..0..1·68 0..0..3·14
0·3	0..1..0·42 0..0..3·14	1·3	1..0..4·82 0..0..3·14
0·4	0..1..3·56 0..0..3·14	1·4	1..0..7·96 0..0..3·14
0·5	0..1..6·70 0..0..3·14	1·5	1..1..2·10 0..0..3·14
0·6	0..2..0·84 0..0..3·14	1·6	1..1..5·24 0..0..3·14
0·7	0..2..3·98 0..0..3·14	1·7	1..1..8·38 0..0..3·14
0·8	0..2..7·12 0..0..3·14	1·8	1..2..2·52 0..0..3·14
0·9	0..3..1·26 0..0..3·14	1·9	1..2..5·66 0..0..3·14
1·0	0..3..4·40 0..0..3·14	2·0	1..2..8·80 0..0..3·14
1·1	0..3..7·54	2·1	1..3..2·94

In transcribing the content from any tabulation, it is customary to omit the decimal parts of a gallon; but to compensate for this omission, every decimal greater than 50 is considered a gallon. Hence the above Table would be transcribed as follows:

TABLE BOOK.

Wet Inches	B. F. G.	Wet Inches	B. F. G.
0·1	0..0..3	1·2	1..0..2
0·2	0..0..6	1·3	1..0..5
0·3	0..1..0	1·4	1..0..8
0·4	0..1..4	1·5	1..1..2
0·5	0..1..7	1·6	1..1..5
0·6	0..2..1	1·7	1..1..8
0·7	0..2..4	1·8	1..2..3
0·8	0..2..7	1·9	1..2..6
0·9	0..3..1	2·0	1..3..0
1·0	0..3..4	2·1	1..3..3
1·1	0..3..8	2·2	....

PROBLEM VIII.

To Gage and Inch a Round Hop-Tun.

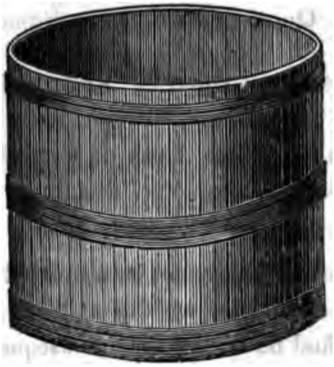
If the sides be perpendicular to the bottom, several cross diameters at various depths are to be taken, and the sum of all the diameters, divided by the number taken, will be the *mean diameter* of the Tun. This mean diameter squared, and divided by 359, the circular divisor for ale gallons, will give the area for an inch deep on the bottom, and this added to the drip (if any) will be the content of the Tun at one inch wet.

The area, for an inch deep, added to the content at one inch wet, will be the content at two inches wet; and thus by continual addition, the Tun may be tabulated for every inch of its depth.

If the sides be not perpendicular to the bottom, cross diameters at the middle of every ten inches from the bottom must be taken, and half the sum of each pair must be considered the diameter of the Tun throughout the division at the middle of which the two diameters have been taken. Then the content is to be determined precisely in the manner directed for Coppers and Round Mash Tuns, in PROBLEMS I. and V. in the Brew-House; and lastly, a Table is to be constructed for every wet inch of the Tun from the bottom to the top.

EXAMPLE 1.

Let the following diagram represent a round Hop-Back with sides perpendicular to the bottom, given to be gaged and inched.



METHOD OF PROCEEDING.

Measure the depth of the Tun, which we may suppose is 67·3 inches: then with a dimension-cane take several diameters in various parts of the Tun, which let be the following:

1st diameter	82·6 inches
2nd ————	82·4 inches
2rd ————	83·0 inches
4th ————	82·8 inches
5th ————	82·2 inches
6th ————	82·0 inches

Here six different diameters have been taken, and if their sum be divided by their number, we shall obtain for quotient the mean diameter of the Tun: thus,

Divide by 6) 495 sum of six diameters

Quotient 82·5 *mean diameter.*

Now, instead of squaring 82·5, and dividing the product by the circular divisor for ale gallons, we enter the Table of the Areas of Circles for Ale Measure, with 82·5 as diameter, and find the content for one inch deep to be 18·95 ale gallons.

Next, causing liquor to be admitted into the Back to cover the bottom, we dip at the constant dipping place, and find no tenths wet: consequently there will be no abatement to be made from the apparent depth on any occasion whatever.

On drawing off and carefully measuring the liquor admitted to cover the bottom, we enter it as drip: suppose it to be a firkin, and the Table will begin as follows:

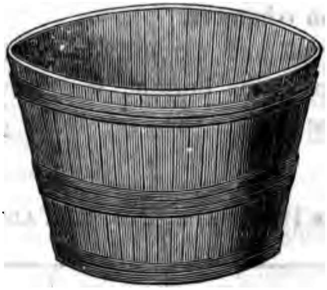
A. B.'s HOP-BACK, No. 2, TABLED.

Wet Inches	B. F. G.	Wet Inches	B. F. G.
Drip	0..1..0·00 0..2..0·95	5	2..3..4·75 0..2..0·95
1	0..3..0·95 0..2..0·95	6	3..1..5·70 0..2..0·95
2	1..1..1·90 0..2..0·95	7	3..3..6·65 0..2..0·95
3	1..3..2·85 0..2..0·95	8	4..1..7·60 0..2..0·95
4	2..1..3·80 0..2..0·95	9	4..3..8·55 0..2..0·95
5	2..3..4·75	10	5..2..0·50

In this manner may the Table be extended at pleasure to the whole depth of the Tun.

EXAMPLE 2.

Let the following diagram represent a Hop-Back, in the form of the frustum of a cone standing on its less end, given to be gaged and inched.



METHOD OF PROCEEDING.

Lay a rod diametrically across the mouth, and with a rule or dimension-cane take the perpendicular depth, which let be 50 inches.

Then five inches from the bottom take two cross diameters, and reserve half their sum for the mean diameter throughout the first ten inches from the bottom.

Again, take two cross diameters ten inches above the place of the first pair, and reserve half their sum likewise, as the mean diameter throughout the second ten-inch division from the bottom.

Also, ten inches still higher, take a pair of cross diameters, reserving half their sum as the mean diameter of the third division.

In this manner proceed to the top, taking cross diameters at distances of ten inches on the perpendicular.

Suppose now, the memorandum to stand as in the following page:

Memorandum of A. B.'s *Hop-Back, No.* 3, *Gaged,* 10 *May,* 1820.

Divisions in Inches	Depths from the top	Mean Diameters
10	5	76·2
10	15	72·4
10	25	68·1
10	35	64·3
10	45	60·0

This memorandum will furnish the means of inching the Back, for to the drip (if any) we have only to add the area, in ale, of a circle 60 inches in diameter, ten times: then the area of a circle 64·3 inches in diameter, likewise ten times; and next the area of a circle 68·1 inches in diameter, ten times; again, the area of a circle 72·4 inches in diameter, is to be added ten times; and afterwards, the area of a circle 76·2 inches, ten times; which will finish the Table.

Suppose the drip in this case to be 11 gallons by measure, wetting one inch of the dipping rod, at the constant dipping place; then one inch must be subtracted from each apparent depth by the rod, for the true depth, the following notice being conspicuously affixed to the Back near the dipping place.

[J. B. No. 3, — 01·0]

The letters here employed, signify Jack-Back: and the figures denote that in this Back, which is

Number *three*, an inch is to be deducted from each depth given by the rod, for the true depth; and thus the whole depth, instead of 50 inches, will be reduced to 49 inches. We shall not here exhibit the Dimension Book for this Back, since the manner of entry must by this time be perfectly familiar to the reader: we shall, however, subjoin a portion of the Table, deeming ten or twelve inches sufficient for illustration, as the process may be continued at pleasure, by mere addition, for the whole depth of the Back.

A. B.'s Hop-Back, No. 3, Tabled.

Wet Inches	B. F. G.	Wet Inches	B. F. G.
Drip	0..1..2·00 0..1..1·02	6	1..3..8·12 0..1..1·02
1	0..2..3·02 0..1..1·02	7	2..1..0·14 0..1..1·02
2	0..3..4·04 0..1..1·02	8	2..2..1·16 0..1..1·02
3	1..0..5·06 0..1..1·02	9	2..3..2·18 0..1..1·02
4	1..1..6·08 0..1..1·02	10 *	3..0..3·20 0..1..2·51
5	1..2..7·10 0..1..1·02	11	3..1..5·71 0..1..2·51
6	1..3..8·12	12	3..2..8·22

PROBLEM IX.

To Gage and Inch an Oval Hop-Tun.

The method of gaging an oval Tun is in all respects the same as that employed for a round Tun, except that, instead of cross diameters, we take the transverse and conjugate axes of the oval, and find a *mean proportional* between each pair, thus reducing the oval to a circle. For in the Table of the Areas of Circles in Ale Gallons, against each *mean proportional*, will be found the area of a circle precisely equal to the area of the oval at the depth where the transverse and conjugate axes, giving the proportional, were taken.

To enlarge further on this problem would be a waste of time, since the subject has been amply discussed in the MENSURATION.

We shall, therefore, only subjoin the form of a memorandum for a cylindroidal Back; and for a Back with similar, but unequal, ovals for the bottom and top.

Memorandum of A. B.'s *Oval Hop-Back Gaged,*
10 *May*, 1820.

Whole Depth	Mean Length	Mean Breadth	Mean Proportional
68·5 inches	108 inches	75 inches	90 inches

Hence the mean area of this Back, is 22·56 ale gallons, *nearly*, as may be seen on entering the Table of Circular Areas for Ale, with 90 inches as diameter.

Memorandum of A. B.'*s Oval Tapering Back, Gaged,* 10 *May*, 1820.

Divisions in Inches	Depths from the mouth	Transverse	Conjugate	Mean Proportional
8	4	72·0	60·0	65·7
10	13	76·4	63 6	69·7
10	23	81·5	67·9	74·3
10	33	86·5	72·1	79·1
10	43	91·4	76·2	83·4
10	53	96·5	80·4	88·0

PROBLEM X.

To Gage and Tenth a Cooler.

Coolers are generally oblong flat vessels, with perpendicular sides, seldom exceeding 8 or 10 inches in depth, because their use being to receive the wort hot from the Coppers, for the purpose of cooling, it is material to expose as great a surface as possible to the atmosphere.

METHOD OF GAGING A RECTANGULAR COOLER.

Take the length and breadth crossing each other in the middle. Then cause water or other liquid to be admitted to cover the bottom of the Cooler, an inch or more in the ebbest part; and chuse a constant dipping place.

Next, put on a pair of pattens, and walk over the Cooler, taking dips in several parts of it, and noting them in a memorandum. These dips are to be regulated in such manner as that with the least labour the whole Cooler may have been visited, and the last dip taken near the constant dipping place. Then divide the sum of all the depths, by the number of dips, and the quotient will be the mean depth.

Lastly, dip at the constant dipping place, and compare the depth here found, with the mean depth. If this dip be less than the mean dip, the *defect* is always to be *added* to the apparent depth at the dipping place, but if greater, the *excess* is always to be *subtracted.*

This DEFECT or EXCESS must be legibly written on a post or beam, as near as possible to the constant dipping place.

Suppose first, we have 7 tenths *deficient*, that is, the dip taken at the dipping place is 7 tenths less than the mean depth; then the mark on the post or beam will be,

[C. No. 1, + 00·7]

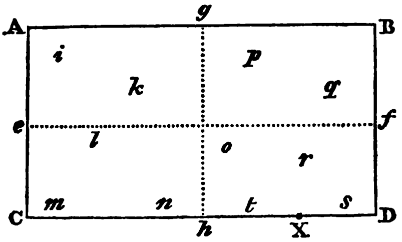
This expression signifies that Cooler, No. 1, gives 7 tenths too little for depth at the dipping place.

Suppose next, we have three tenths more on the dip at the constant dipping place, than the mean depth, and it will be,

[C. No. 1, — 00·3]

EXAMPLE.

Let ABCD be a rectangular Cooler, to be gaged and tenthed.



METHOD OF PROCEEDING.

Extend a tape from C to A, and at *e*, half the distance, make a mark with chalk. In like manner make marks at *f*, *g*, and *h*, half way respectively between D and B, B and A, C and D. Then take the distance of *e* from *f*, which let be 360 inches, the length of the Cooler. Again take the distance of *g* from *h*, which let be 200 inches, the breadth of the Cooler.

Fix, now, your constant dipping place, to which there can at all times be free and easy access, as at X, and cause liquor to be admitted into the Cooler, till the shallowest part of the bottom is covered at least an inch.

Lastly, take several dips, as at *i*, *k*, *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, and *t*, of which let the following be a memorandum:

Depth at *i* .... 2·5 inches
———— *k* .... 2·2 inches
———— *l* .... 1·8 inches
———— *m* .... 2·4 inches
———— *n* .... 3·0 inches
———— *o* .... 2·8 inches
———— *p* .... 2·4 inches
———— *q* .... 1·9 inches
———— *r* .... 2·3 inches
———— *s* .... 2·6 inches
———— *t* .... 2·8 inches

Number of dips 11) 26·7 *sum*

Quotient 2·4 MEAN DEPTH.

If the dip at X be now found to be 2·4 inches, there will be nothing additive or subtractive; but if the dip at X be 3 inches, it follows that the dipping place gives 6 tenths of an inch too much: or, if the dip at X be only 2 inches, then it is 4 tenths of an inch too little. Suppose it to be 2·9 inches, and the mark on the post or beam will be,

[C. No. 1, — 00·5]

The area of the base in ale gallons, for an inch deep, will be the length of the Cooler multiplied by the breadth, and divided by 282, the number of cubic inches in an ale gallon: hence, for one tenth of an inch deep, it will either be 360 multiplied by 200, and divided by 2820; or it will be the product of the length by *one-tenth* the breadth, divided by 282, as follows:

360 length of the Cooler
Multiply by 20 one-tenth the breadth
A. G. divisor 282) 7200 (25·5 ale gallons.

Wherefore to tabulate the Cooler we have only to add this content, namely, 25·5 ale gallons, continually to itself as below:

A. B.'s COOLER, No. 1, TABLED.

Wet Inches	B. F. G.	Wet Inches	B. F. G.
0·1	0..2..7·5 0..2..7·5	1·0	7..0..3·0 0..2..7·5
0·2	1..1..6·0 0..2..7·5	1·1	7..3..1·5 0..2..7·5
0·3	2..0..4·5 0..2..7·5	1·2	8..2..0·0 0..2..7·5
0·4	2..3..3·0 0..2..7·5	1·3	9..0..7·5 0..2..7·5
0·5	3..2..1·5 0..2..7·5	1·4	9..3..6·0 0..2..7·5
0·6	4..1..0·0 0..2..7·5	1·5	10..2..4·5 0..2..7·5
0·7	4..3..7·5 0..2..7·5	1·6	11..1..3·0 0..2..7·5
0·8	5..2..6·0 0..2..7·5	1·7	12..0..1·5 0..2..7·5
0·9	6..1..4·5 0..2..7·5	1·8	12..3..0·0 0..2..7·5
1·0	7..0..3·0	1·9	13..1..7·5

A. B.'s Cooler, No. 1, Tabled.

Wet Inches	B. F. G.	Wet Inches	B. F. G.
1·9	13..1..7·5 0..2..7·5	3·0	21..1..0·0 0..2..7·5
2·0	14..0..6·0 0..2..7·5	3·1	21..3..7·5 0..2..7·5
2·1	14..3..4·5 0..2..7·5	3·2	22..2..6·0 0..2..7·5
2·2	15..2..3·0 0..2..7·5	3·3	23..1..4·5 0..2..7·5
2·3	16..1..1·5 0..2..7·5	3·4	24..0..3·0 0..2..7·5
2·4	17..0..0·0 0..2..7·5	3·5	24..3..1·5 0..2..7·5
2·5	17..2..7·5 0..2..7·5	3·6	25..2..0·0 0..2..7·5
2·6	18..1..6·0 0..2..7·5	3·7	26..0..7·5 0..2..7·5
2·7	19..0..4·5 0..2..7·5	3·8	26..3..6·0 0..2..7·5
2·8	19..3..3·0 0..2..7·5	3·9	27..2..4·5 0..2..7·5
2·9	20..2..1·5 0..2..7·5	4·0	28..1..3·0 0..2..7·5
3·0	21..1..0·0	4·1	29..0..1·5

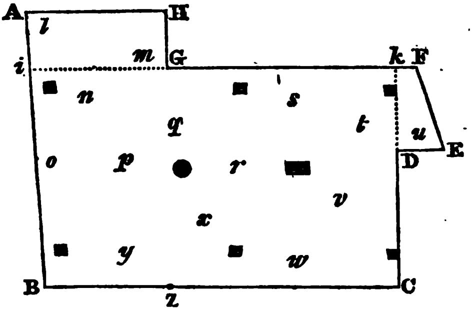
PROBLEM XI.

To Gage and Tenth a Multilateral Cooler.

Divide the base into such figures as, on the principles laid down in the Mensuration, can be most easily measured, and find the area for one tenth of an inch deep, as in the last Problem. Then make the necessary deductions for off-sets, that is, for pillars or other bodies occupying part of the area. Lastly, chuse a dipping place, and proceed in all other respects as in PROBLEM X.

EXAMPLE.

Let ABCDEFGH be a Cooler with off-sets, to be gaged and tenthed.



METHOD OF PROCEEDING.

Prolong the line CD to *k*; and the line FG, to *i*: then take the necessary dimensions with a box and tape. The dimensions, in the instance before us, we shall suppose to be as follows:

BC = 366 inches
C*k* = 224 inches
D*k* = 88 inches
DE = 48 inches

F*k* = 22 inches
F*i* = 400 inches
G*i* = 140 inches
GH = 60 inches

These data will be sufficient for our purpose, if we consider A*i*GH to be a rectangle; and that F*i* is parallel to BC, whilst C*k* is at right angles to both: and likewise that ED is parallel to F*k*.

Next take the seven rectangular offsets, which may be regarded as square pillars; and also the circular off-set in the middle, which may be the support of a fanner-wheel employed to accelerate the cooling of the worts in the Cooler.

Let the area of all the eight offsets be 740 square inches on the base, and the calculation will be as follows:

366 side BC.
400 line F*i*.
Divide by 2) 766 sum
Quotient 383 half sum
Multiply by 224 perpendicular C*k*.
Product 85792 *area of i*BC*k*.

140 line G*i*.
Multiply by 60 perpendicular GH.
Product 8400 *area of* A*i*GH.

48 side DE.
22 line F*k*.
Divide by 2) 70 sum
Quotient 35 half sum
Multiply by 88 perpendicular D*k*.
280
280
Product 3080 *area of* DEF*k*.

Now, if from the sum of the three areas, the sum of the off-sets be deducted, the remainder will be the area of the base of the Cooler, in square inches.

Wherefore it will be,

85792 area of *i*BC*k*.
8400 area of A*i*GH.
3080 area of DEF*k*.
97272 sum
Deduct 740 for off-sets
A. G. divisor 282) 96532 (342·3 ale gallons.

The area of the whole base for an inch deep, after deducting the off-sets, being 342·3 ale gallons, if the decimal point in that number be moved one step to the left, we shall have 34·23 ale gallons for the content at one-tenth of an inch deep.

Let liquor be now admitted into the Cooler till the bottom is covered an inch or more throughout; and then take dips at *l*, *m*, *n*, *o*, *p*, *q*, *r*, *s*, *t*, *v*, *w*, *x*, and *y*.

And suppose that the mean depth arising from the sum of all the depths, divided by the number of dips, is 3·1 inches, and that having fixed your constant dipping place at Z, the depth there is only 2·9 inches: the expression at the dipping place will be

[C. No. 2, + 00·2]

We are, finally, to table the Cooler; in doing which the same method is to be pursued that was laid down and practised in the last Problem; and of this a few tenths will be ample illustration.

A. B.'s Cooler, No. 2, Tabled.

Wet Inches	B. F. G.	Wet Inches	B. F. G.
0·1	0..3..7·23 0..3..7·23	0·4	3..3..1·92 0..3..7·23
0·2	1..3..5·46 0..3..7·23	0·5	4..3..0·15 0..3..7·23
0·3	2..3..3·69 0..3..7·23	0·6	5..2..7·38 0..3..7·23
0·4	3..3..1·92	0·7	6..2..5·61

PROBLEM XII.

To Gage and Inch an Under-Back.

Under-Backs are vessels, sometimes rectangular, but more frequently circular or oval, placed to receive the new wort from the Mash-Tun, and retain it till it can conveniently be pumped into the Copper-Back or into the Copper.

The method of gaging and inching an Under-Back differs nothing from that laid down for a Hop-Back, in PROBLEMS VII, VIII, and IX: consequently further directions would be superfluous.

PROBLEM XIII.

To Gage and Inch a Square Guile-Tun or Working Vat with Perpendicular Sides.

With your dimension-cane take the depth of the Tun at the constant dipping place, marked either by a notch or a plate of iron: then from several lengths of the Tun in various parts, and at different depths deduce a mean length; and in the same manner find a mean breadth, from a variety of breadths, as already directed for a square Hop-Back. The mean length, multiplied by the mean breadth, and divided by 282, the number of cubic inches in an ale gallon, will give the area of the Tun, or the content for an inch deep.

Next find the drip by pouring into the Tun liquor by measure, just sufficient to cover the bottom; and at the dipping place take the depth of the liquor by observing the wet inches and tenths on the cane. The depth of the liquor thus found being subtracted from the whole depth of the Tun will leave the nett depth at the dipping place. Wherefore, if the area be multiplied by the nett depth, and if to the product the drip be added, the sum will evidently be the whole content of the Tun.

Now to form a Table for *dry inches*, all gages in Working Vats being taken in dry inches with a rod and cork, we have only to proceed as in Tabling a Copper: that is, we must first reduce the whole content to *Barrels*, *Firkins*, and *Gallons*, as directed in page 232, and then from this content continually subtract the area reduced to the same denominations.

The Table may be presumed correct when nothing except the drip remains after the last dry inch. All odd tenths on the depth are of course to be left at the bottom.

PROBLEM XIV.

To Gage and Inch a Round or an Oval Guile-Tun with Perpendicular Sides.

If the Tun be round, take cross diameters at various depths, and from their sum deduce a mean diameter, as directed for circular Hop-Backs. Then with the mean diameter enter the Table of the Areas of Circles in Ale-Gallons, and observe the content. This content is to be multiplied by the depth of the Tun, and to the product the drip is to be added, for the whole content. Lastly, a Table for *dry inches* is to be constructed exactly as in the last problem.

But if the Tun be oval, from the sum of several transverse diameters at different depths, find a *mean transverse*, and from the sum of several conjugates at various depths also, find a *mean conjugate.* Then, between the mean transverse and mean conjugate, find a *geometrical mean proportional.*

With the geometrical mean, as diameter, enter the Table of Circular Areas for Ale, and take out the content answering to it. This content will be the area, which multiplied by the depth, and then added to the drip, will give the whole content. A Table for *dry inches* is now to be constructed, by continually subtracting from the whole content the inch area in *Barrels, Firkins,* and *Gallons,* as already clearly explained.

PROBLEM XV.

To Gage and Inch a Guile-Tun with Tapering Straight Sides.

1. If the base be rectangular, from two lengths find a mean length, and from two breadths a mean breadth of the Tun at the middle of every ten inches on the depth, beginning at the mouth: then multiply each length by the corresponding breadth, and divide the product by 282, the number of cubic inches in an ale gallon: the several quotients will be the content for an inch deep, respectively, throughout the ten-inch division of the Tun, to which the particular mean length and mean breadth refer. Next the drip is to be found as already directed, and afterwards the whole content, precisely in the manner of a Copper. *See page* 231. Lastly, a Table for *dry inches* is to be constructed as illustrated in page 234.

2. If the Tun or Vat be circular, take cross diameters at the middle of every ten inches from the mouth, and find the *arithmetical mean* between each pair of diameters, by adding them together, and dividing by 2: then enter the Table of Circular Areas in Ale, with this mean, and take out the content answering to it, which note in a memorandum. Next find the drip (the method of doing which has already been explained) and fix the dipping place. In all other

respects proceed as directed for the Tun with a rectangular base in the beginning of this problem.

EXAMPLE OF A ROUND GUILE-TUN WITH TAPERING SIDES.

Let the following diagram represent a round Guile-Tun, or Working-Vat, with tapering sides, to be gaged and inched.

METHOD OF GAGING.

Lay a rod diametrically across the mouth of the Tun, and take the perpendicular depth; then strike four vertical lines with chalk, centrally opposite to each other, from the mouth of the Tun to the bottom. Next, begin at the mouth, and set off 5 inches on each of the chalk-lines, for the extremities of the uppermost pair of cross diameters. Which repeat at every ten inches below the first marks, so long as five inches clear remain at the bottom: but as soon as one of the ten-inch distances would leave less than 5 inches clear below it, forbear to mark this ten-inch distance, and instead of it make a mark at 5 inches below the last ten marked. Then half way between this five-inch mark and the bottom, make a mark on each of the lines for cross diameters at the middle of this lowermost division of the Tun.

As this will be more intelligible by assuming an appropriate depth for the Tun before us, we shall suppose its depth to be, by admeasurement, 134·6 inches.

Here the cross diameters will be respectively at the following distances from the mouth:

First pair	5 inches
Second	15 inches
Third	25 inches
Fourth	35 inches
Fifth	45 inches
Sixth	55 inches
Seventh	65 inches
Eighth	75 inches
Ninth	85 inches
Tenth	95 inches
Eleventh	105 inches
Twelfth	115 inches
Thirteenth	125 inches

And now no ten-inch division remains; wherefore, a mark being made on each of the four lines, at 5 inches below the last marks respectively, that is, at 130 inches from the mouth, there will be 4·6 inches remaining over, at the bottom: but the half of 4·6 inches is 2·3 inches, the distance from the bottom at which the lowermost two cross diameters are to be taken.

Hence the fourteenth pair of cross diameters will be at 132·3 inches from the mouth, and thus we bring the odd tenths in the depth to the bottom, as we ought to do for dry inches.

As the method of finding the drip, and of tabling a circular Tun with tapering sides, has been amply illustrated in some of the foregoing problems, further instructions would be a waste of time. We shall, therefore, only once more remind the young officer, that in all Working Vats or Guile-Tuns the Tabulation must be for *Dry Inches.*

PROBLEM XVI.

To Gage and Inch a Round Guile Tun with Bending Sides.

Lay a straight rod across the mouth of the Tun, and with a rule or dimension-cane, take the perpendicular depth: then in the middle of every six inches, counting either from the bottom or top, where there are no odd tenths on the depth, take cross diameters; and record your proceedings and measurements in a memorandum, with the mean between each pair of diameters. Lastly, make an entry in the Dimension Book, precisely in the same way as in the preceding problems, having ascertained, as already directed, the drip of the Tun.

EXAMPLE.

Let the following diagram represent a Guile Tun with bending sides, given to be gaged and inched.

METHOD OF PROCEEDING.

Lay a straight rod across the mouth, and take the depth of the Tun, which let be 40 inches. Then at 3 inches from the bottom take a pair of cross diameters, and 6 inches higher, another pair: also, 6 inches higher a third pair; and so on at six-inch distances, till six pair of cross diameters have been taken; when it will be found that only seven inches are left between the last pair and the mouth: but of these seven inches three belong to the division of the Tun, to which the last pair of cross diameters refers. Consequently the uppermost division of the Tun will be only four inches in depth, and therefore a pair of diameters taken two inches from the mouth will be at the middle of this division.

Suppose, now, the following to be a copy of the memorandum made on the spot, with the mean diameters between each pair.

Memorandum of A. B.'*s Guile Tun, No.* 7, *Gaged,* 10 *May,* 1820.

Divisions in Inches	Distance from the mouth	Cross Diameters	Mean Diameter
4	2 inches	54·4......54·0	54·2
6	7 inches	56·0......55·6	55·8
6	13 inches	60·2......59·8	60·0
6	19 inches	56·6......56·4	56·5
6	25 inches	54·1......54·3	54·2
6	31 inches	52·2......52·2	52·2
6	37 inches	50·1......49·9	50·0
Drip of the Tun 1 kilderkin by measure. Dip,—00·8			

Here, then, is every requisite for the entry to be made in the Dimension Book, the manner of filling up which has been sufficiently explained and amply illustrated.

PROBLEM XVII.

To Gage a By-Tub.

By-Tubs are vessels generally of an irregular form, used by Victuallers in the country, and others that brew in small quantities at a time, for the purpose of cooling their worts.

When a By-Tub is to be gaged, it is customary to ascertain at once the quantity of wort or beer in it, without finding different areas, as practised for all *Fixed Utensils.*

METHOD OF GAGING.

Find, on the principles laid down in MENSURATION, the area of the Tub for one inch deep, in ale gallons, at half the depth of the wort or beer in it; and multiply this area by the depth of liquor, which depth, if the bottom of the Tub be level, you may ascertain at once with your dimension-cane: but if the bottom be uneven, a mean depth must be deduced from several dips in various parts of the Tub. The gallons are afterwards to be reduced to Barrels, Firkins, and Gallons.

PROBLEM XVIII.

To Deduct from Warm Worts the proper Allowance for Heat.

It is well known that heat expands almost every sort of body, for if any liquid whatever be poured boiling hot into a vessel so as to fill it, the liquid will shrink in bulk more and more as it cools down to the ordinary temperature of the atmosphere; and if any fixed fluid at pleasure be put into a vessel cold, and exposed to heat, its bulk will increase more and more as the fluid becomes hotter and hotter.

By experience it is found that every quantity of hot wort shrinks in cooling to little more than nine-tenths of its original bulk; that is, hot wort in cooling loses nearly *one-tenth* part of its volume or bulk. This being clearly understood, it will be easy to make the proper allowance for heat in worts, for in casting the gages of all hot worts, we must deduct *one-tenth* part the content. This is readily performed in *gallons;* for if the number of hot gallons be written twice, but in such manner as that the figures in the lower line be each one place more to the right, than those above them, (in which case the decimal point in the lower line will be one place more to the left,) we have only simply to subtract the lower line from the upper, and the remainder will be the content in cold gallons.

EXAMPLE.

If a gage of hot wort be 198 gallons, what quantity must be charged?

SOLUTION.

198·0 hot gallons
Deduct 19·8 one-tenth part

Remainder 178·2 gallons when cold, *Ans.*

But an officer must be likewise expert in deducting a tenth, in *Barrels, Firkins* and *Gallons.* This presents very little difficulty, for, under the Barrels in the Content hot, write one-tenth part the Barrels, units ranging with units; then, if there be a remainder over tens, multiply it by 4, and to the product add the Firkins in the Content hot: the sum divided by ten, will give the Firkins to be set under Firkins. The remainder is now to be multiplied by 9, and the Gallons, in the Content hot, added to the product; when one-tenth of the sum is to be set under the Gallons. Lastly, the lower line subtracted from the upper, will give the Content to be charged.

EXAMPLE.

If a gage of hot wort be B43..F3..G5·7, what will the charge be?

SOLUTION.

	B. F. G.	
	43..3..5·70	content hot
Deduct	4..1..5·07	one-tenth part
Remainder	39..2..0·63	*Answer.*

OF VICTUALLERS' UTENSILS.

The Utensils of common Brewers, and of Victuallers differ nothing except in the size, those of Victuallers being generally on a scale much smaller than that employed by a Wholesale Brewer. The method of gaging and fixing the Utensils, however, is the same in both cases, and hence further instructions are quite unnecessary.

We shall, therefore, conclude this part of an Exciseman's duty with one remark; *viz.*, that where great accuracy is deemed of consequence, we must not confine ourselves to ten-inch divisions in any sort of utensil, (although this particular number, on account of its being so convenient a multiplier and divisor, is exceedingly appropriate,) but on the contrary, make such divisions of the Utensil, as that the cross diameters, or mean lengths and breadths of any two contiguous divisions may never differ *more than an inch.* Hence, in vessels of great or sudden curvature, the altitudes of the divisions will be comparatively small; whereas in curves approaching more to straight and parallel lines, the divisions will be of greater depth. In vessels with straight sides, if the whole depth be divided by the difference of the diameters at the top and bottom when round, or by the difference of the lengths or breadths, above and below, when square, the quotient will be the perpendicular depth of each division whereof the difference of the corresponding dimensions at the surface and base shall be exactly an inch.

It might also be mentioned, that in tabling any sort of utensil, whether by subtraction for dry inches, or by addition for wet inches, the area to be subtracted or added need not be written at all in the Table, but on a moveable slip of paper; for, by shifting the paper, the quantity to be added or subtracted, may, with the greatest neatness and ease, be brought under the quantity to be operated upon, in regular succession. Several slips, it is true, will be wanted where the areas change, as in Coppers, and tapering utensils, but still the trouble will be less than if we write each area in the Table.

OF THE DISTILLERY.

In gaging and fixing the utensils of a Distillery more than ordinary care and skill are requisite, because the revenue arising from this branch of trade is very considerable, and of a description greatly affected by a small error, since the duty on a single gallon of spirits is high in comparison of the duty on a gallon of beer.

Hence, if the dimensions are not taken with the utmost precision, and the calculations made with much accuracy, an injury must arise either to the revenue or to the trader, which to prevent ought to be a first consideration in the mind of the Gager on whom it falls to make the charge.

This being premised, we shall endeavour to lay down as clear directions as possible for the guidance of an Exciseman in gaging and fixing the utensils of a Distillery.

PROBLEM I.

To Gage and Inch a Still.

Lay a straight rule diametrically across the mouth of the collar, and slide a line furnished with a plummet, along the rule, till the lead hangs freely over the dint made by the copper-smith in the middle of the top of the crown. Then measure the perpendicular distance between the rule and the dint; from which subtract the height of the collar, and the remainder will be the internal depth of the Still from crown to collar. Next extend a thread or small cord diametrically across the Still, at the seam or joining of the globular part above, with the body of the Still. The perpendicular distance of this diameter, from the dint in the crown, is to be accurately measured, and subtracted from the internal depth from collar to crown: the remainder will evidently be the altitude of the globular frustum between the body of the Still and the collar.

Two cross diameters are now to be taken at the level of the crown-head, and half their sum is to be multiplied by ·707, (*see page* 135,) to obtain the side of the square inscribed in the Still at the level of the top of the crown. When the side of this square has been found, inscribe it in the Still, by marking its four angles with chalk: this is termed quartering the Still.

As soon as the Still has been quartered, let a lighted candle be set on the dint, and cause the Still to be darkened at the mouth. Then, holding a plumb-line between the candle and each of the angles of the square in succession, let the shadow of the line be traced with a piece of chalk all the way down from the seam of the globular part to the level of the crown, at each angle.

Consider, now, the depth of the Still internally, from frustum to crown, with a view of dividing it into some *even* number of divisions, each consisting of integral inches. The number of inches in a division needs not be less than 4, nor more than 10, according as the body of the Still is less or more cylindrical: that is, the divisions must be small when the curvature is sudden, but may be deeper if the curve is slow. Extra tenths on the depth (if any) must be placed in the bottom division, and the number of divisions must always be some multiple of 2; for if an *odd* number of divisions be made, one pair of cross diameters will fall in the middle of the body of the Still, where the diameter is the greatest, and consequently we should have an over-gage or excess in content.

The divisions being determined upon, let cross diameters at the middle of each division be taken, and let a *mean* diameter be deduced from each pair. Then find the respective areas, for an inch deep, in wine gallons, and multiply them severally by their depths. The products will be the contents of the different divisions; the sum of which, added to the quantity of liquor required to cover the crown, will give the content up to the beginning of the globular part above.

The content of the globular part is next to be calculated (*see page* 215) and added to the content under it.

The manner of inching the body of a still differs nothing from that explained and illustrated for a Brewer's Copper, and therefore needs not be repeated, as reference may easily be made to PROB. I. BREW-HOUSE. But the method of inching the frustum of a globe is more tedious, if not more difficult, in so far as finding the areas is concerned, which in some degree resembles the mode recommended for a Copper Pan in page 270.

METHOD OF DETERMINING THE DIAMETERS OF THE FRUSTUM OF A GLOBE AT EVERY INCH.

From the square of half the diameter of the greater base, subtract the square of the altitude, and the square of half the diameter of the less base; then divide the remainder by double the altitude, and the quotient will be the distance of the greater base from the centre of the sphere whereof the frustum is a part.

The square of this distance, added to the square of half the diameter of the greater base, will give the square of half the diameter of the sphere. Therefore, twice the square root of the sum of the two last mentioned squares will be THE DIAMETER OF THE SPHERE whereof the given frustum is a portion.

Consequently, if from half the diameter of the sphere we subtract the distance of the frustum's base from the centre, and then the altitude of the frustum, the last remainder will be the altitude of the spherical segment cut off at the less base of the frustum. Wherefore, by the continual addition of *one inch* to this altitude, diameters at every inch of the frustum will easily be obtained as directed for the Copper Pan in page 272.

It may here be observed that if the depth of the globular part of a Still consist not of integral inches, it will save much trouble to consider the odd tenths of this part as properly belonging to the highest division of the body of the Still; which, of course, it will be requisite to do before the cross diameters are taken, that the tenths on the whole depth may be in the lowermost division next the crown.

The content of the crown of the Still may be determined by the pen precisely in the same way as the crown of a Copper. But the most expeditious, and at the same time the surest method is, to ascertain by measure, the quantity of liquor necessary to cover the crown, because few crowns are true spherical segments.

To prevent the great trouble of calculating each area for an inch deep, in wine gallons, a Table similar to that given for ale gallons will be found of singular use; and this Table, carried to the same extent as the Table for Ale, we shall subjoin, before we submit an example of a Still; since, by reference to this Table, the operations will be better understood.

TABLE

OF

WINE GALLONS

(FOR ONE INCH DEEP)

In Cylinders from 1 inch to 210 inches in diameter,
with all the intermediate tenths ;

VERY USEFUL IN GAGING,

AND COMMONLY

CALLED THE TABLE OF CIRCULAR AREAS

FOR

WINE.

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
1	0·0034	0·0041	0·0049	0·0057	0·0067
2	0·0136	0·0150	0·0165	0·0180	0·0196
3	0·0306	0·0327	0·0348	0·0370	0·0393
4	0·0544	0·0572	0·0600	0·0629	0·0658
5	0·0850	0·0884	0·0919	0·0955	0·0991
6	0·1224	0·1265	0·1307	0·1349	0·1393
7	0·1666	0·1714	0·1763	0·1812	0·1862
8	0·2176	0·2231	0·2286	0·2342	0·2399
9	0·2754	0·2816	0·2878	0·2941	0·3004
10	0·3400	0·3468	0·3537	0·3607	0·3677
11	0·4114	0·4189	0·4265	0·4341	0·4419
12	0·4896	0·4978	0·5061	0·5144	0·5228
13	0·5746	0·5835	0·5924	0·6014	0·6105
14	0·6664	0·6760	0·6856	0·6953	0·7050
15	0·7650	0·7752	0·7855	0·7959	0·8063
16	0·8704	0·8813	0·8923	0·9033	0·9145
17	0·9826	0·9942	1·0059	1·0176	1·0294
18	1·1016	1·1139	1·1262	1·1386	1·1511
19	1·2274	1·2404	1·2534	1·2665	1·2796
20	1·3600	1·3736	1·3873	1·4011	1·4149
21	1·4994	1·5137	1·5281	1·5425	1·5570
22	1·6456	1·6606	1·6757	1·6908	1·7060
23	1·7986	1·8143	1·8300	1·8458	1·8617
24	1·9584	1·9748	1·9912	2·0077	2·0242
25	2·1250	2·1420	2·1591	2·1763	2·1935
26	2·2984	2·3161	2·3339	2·3517	2·3697
27	2·4786	2·4970	2·5155	2·5340	2·5526
28	2·6656	2·6847	2·7038	2·7230	2·7423
29	2·8594	2·8792	2·8990	2·9189	2·9388
30	3·0600	3·0804	3·1009	3·1215	3·1421
31	3·2674	3·2885	3·3097	3·3309	3·3523
32	3·4816	3·5034	3·5253	3·5472	3·5692
33	3·7026	3·7251	3·7476	3·7702	3·7929
34	3·9304	3·9536	3·9768	4·0001	4·0234
35	4·1650	4·1888	4·2127	4·2367	4·2607

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
1	0·0077	0·0087	0·0098	0·0110	0·0123
2	0·0213	0·0230	0·0248	0·0267	0·0286
3	0·0417	0·0441	0·0465	0·0491	0·0517
4	0·0689	0·0719	0·0751	0·0783	0·0816
5	0·1029	0·1066	0·1105	0·1144	0·1184
6	0·1437	0·1481	0·1526	0·1572	0·1619
7	0·1913	0·1964	0·2016	0·2069	0·2122
8	0·2457	0·2515	0·2573	0·2633	0·2693
9	0·3069	0·3133	0·3199	0·3265	0·3332
10	0·3749	0·3820	0·3893	0·3966	0·4040
11	0·4497	0·4575	0·4654	0·4734	0·4815
12	0·5313	0·5398	0·5484	0·5571	0·5658
13	0·6197	0·6289	0·6381	0·6475	0·6569
14	0·7149	0·7247	0·7347	0·7447	0·7548
15	0·8169	0·8274	0·8381	0·8488	0·8596
16	0·9257	0·9369	0·9482	0·9596	0·9711
17	1·0413	1·0532	1·0652	1·0773	1·0894
18	1·1637	1·1763	1·1889	1·2017	1·2145
19	1·2929	1·3061	1·3195	1·3329	1·3464
20	1·4289	1·4428	1·4569	1·4710	1·4852
21	1·5717	1·5863	1·6010	1·6158	1·6307
22	1·7213	1·7366	1·7520	1·7675	1·7830
23	1·8777	1·8937	1·9097	1·9259	1·9421
24	2·0409	2·0575	2·0743	2·0911	2·1080
25	2·2109	2·2282	2·2457	2·2632	2·2808
26	2·3877	2·4057	2·4238	2·4420	2·4603
27	2·5713	2·5900	2·6088	2·6277	2·6466
28	2·7617	2·7811	2·8005	2·8201	2·8396
29	2·9589	2·9789	2·9991	3·0193	3.0396
30	3·1629	3·1836	3·2045	3·2254	3·2464
31	3·3737	3·3951	3·4166	3 4382	3·4599
32	3·5913	3·6134	3·6356	3·6579	3·6802
33	3·8157	3·8385	3·8613	3·8843	3·9073
34	4·0469	4·0703	4·0939	4·1175	4·1412
35	4·2849	4·3090	4·3333	4·3576	4·3820

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
36	4·4064	4·4309	4·4555	4·4801	4·5049
37	4·6546	4·6798	4·7051	4·7304	4·7558
38	4·9096	4·9355	4·9614	4·9874	5·0135
39	5·1714	5·1980	5·2246	5·2513	5·2780
40	5·4400	5·4672	5·4945	5·5219	5·5493
41	5·7154	5·7433	5·7713	5·7993	5·8275
42	5·9976	6·0262	6·0549	6·0836	6·1124
43	6·2866	6·3159	6·3452	6·3746	6·4041
44	6·5824	6·6124	6·6424	6·6725	6·7026
45	6·8850	6·9156	6·9463	6·9771	7·0079
46	7·1944	7·2257	7·2571	7·2885	7·3201
47	7·5106	7·5426	7·5747	7·6068	7·6390
48	7·8336	7·8663	7·8990	7·9318	7·9647
49	8·1634	8·1968	8·2302	8·2637	8·2972
50	8·5000	8·5340	8·5681	8·6023	8·6365
51	8·8434	8·8781	8·9129	8·9477	8·9827
52	9·1936	9·2290	9·2645	9·3000	9·3356
53	9·5506	9·5867	9·6228	9·6590	9·6953
54	9.9144	9·9512	9·9879	10·0248	10·0618
55	10·2850	10·3224	10·3599	10·3975	10·4351
56	10·6624	10·7005	10·7387	10·7769	10·8153
57	11·0466	11·0854	11·1243	11·1632	11·2022
58	11·4376	11·4771	11·5166	11·5562	11·5959
59	11·8354	11·8756	11·9158	11·9561	11·9964
60	12·2400	12·2808	12·3217	12·3627	12·4037
61	12·6514	12·6929	12·7344	12·7761	12·8178
62	13·0696	13·1117	13·1540	13·1963	13·2388
63	13·4946	13·5374	13·5804	13·6234	13·6665
64	13·9264	13·9698	14·0136	14·0571	14·1010
65	14·3650	14·4092	14·4536	14·4979	14·5424
66	14·8104	14·8553	14·9002	14·9453	14·9904
67	15·2626	15·3081	15·3538	15·3995	15·4452
68	15·7216	15·7678	15·8142	15·8606	15·9072
69	16·1874	16·2343	16·2812	16·3284	16·3756
70	16·6600	16·7076	16·7552	16·8031	16·8508

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
36	4·5297	4·5545	4·5794	4·6044	4·6295
37	4·7813	4·8068	4·8324	4·8580	4·8838
38	5·0397	5·0659	5·0921	5·1185	5·1449
39	5·3049	5·3317	5·3587	5·3857	5·4128
40	5·5769	5·6044	5·6321	5·6598	5·6876
41	5·8557	5·8839	5·9122	5·9406	5·9691
42	6·1413	6·1702	6·1992	6·2283	6·2574
43	6·4337	6·4633	6·4929	6·5227	6·5525
44	6·7329	6·7631	6·7935	6·8239	6·8544
45	7·0389	7·0698	7·1009	7·1320	7·1632
46	7·3517	7·3833	7·4150	7·4468	7·4787
47	7·6713	7·7036	7·7359	7·7685	7·8010
48	7·9977	8·0307	8·0637	8·0969	8·1301
49	8·3309	8·3645	8·3983	8·4321	8·4660
50	8·6709	8·7052	8·7397	8·7742	8·8088
51	9·0177	9·0527	9·0878	9·1230	9·1583
52	9·3713	9·4070	9·4428	9·4787	9·5146
53	9·7317	9·7681	9·8045	9·8411	9·8777
54	10·0989	10·1359	10·1731	10·2103	10·2476
55	10·4729	10·5106	10·5485	10·5864	10·6244
56	10·8537	10·8921	10·9306	10·9692	11·0079
57	11·2413	11·2804	11·3196	11·3588	11·3982
58	11·6357	11·6755	11·7153	11·7553	11·7953
59	12·0369	12·0773	12·1179	12·1585	12·1992
60	12·4449	12·4860	12·5273	12·5686	12·6100
61	12·8595	12·9016	12·9436	12·9856	13·0277
62	13·2812	13·3236	13·3663	13·4090	13·4517
63	13·7096	13·7528	13·7961	13·8396	13·8829
64	14·1448	14·1888	14·2327	14·2768	14·3208
65	14·5868	14·6316	14·6760	14·7208	14·7655
66	15·0356	15·0808	15·1262	15·1716	15·2170
67	15·4912	15·5372	15·5831	15·6292	15·6753
68	15·9536	16·0004	16·0469	16·0936	16·1405
69	16·4228	16·4700	16·5175	16·5648	16·6124
70	16·8988	16·9468	16·9948	17·0428	17·0911

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
71	17·1394	17·1877	17·2360	17·2845	17·3330
72	17·6256	17·6745	17·7236	17·7727	17·8218
73	18·1186	18·1682	18·2180	18·2678	18·3176
74	18·6184	18·6687	18·7192	18·7696	18·8202
75	19·1250	19·1760	19·2272	19·2783	19·3295
76	19·6384	19·6901	19·7420	19·7937	19·8456
77	20·1586	20·2109	20·2634	20·3160	20·3684
78	20·6856	20·7386	20·7918	20·8450	20·8983
79	21·2194	21·2731	21·3268	21·3808	21·4348
80	21·7600	21·8144	21·8688	21·9235	21·9780
81	22·3074	22·3625	22·4176	22·4729	22·5282
82	22·8616	22·9173	22·9732	23·0291	23·0851
83	23·4226	23·4790	23·5356	23·5922	23·6488
84	23·9904	24·0475	24·1047	24·1620	24·2196
85	24·5650	24·6228	24·6807	24·7387	24·7968
86	25·1464	25·2049	25·2634	25·3221	25·3808
87	25·7346	25·7937	25·8530	25·9123	25·9716
88	26·3296	26·3894	26·4494	26·5094	26·5696
89	26·9314	26·9919	27·0524	27·1132	27·1740
90	27·5400	27·6012	27·6624	27·7239	27·7852
91	28·1552	28·2171	28·2790	28·3411	28·4032
92	28·7776	28·8401	28·9028	28·9655	29·0277
93	29·4064	29·4700	29·5329	29·5966	29·6596
94	30·0422	30·1061	30·1701	30·2342	30·2984
95	30·6848	30·7494	30·8141	30·8788	30·9437
96	31·3343	31·3995	31·4648	31·5303	31·5958
97	31·9903	32·0563	32·1224	32·1885	32·2547
98	32·6533	32·7200	32·7868	32·8536	32·9204
99	33·3231	33·3905	33·4579	33·5254	33·5930
100	33·9999	34·0680	34·1361	34·2042	34·2723
101	34·6831	34·7518	34·8206	34·8895	34·9584
102	35·3733	35·4427	35·5122	35·5817	35·6513
103	36·0703	36·1404	36·2105	36·2807	36·3510
104	36·7741	36·8449	36·9157	36·9866	37·0575
105	37·4847	37·5561	37·6278	37·6992	37·7709

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
71	17·3816	17·4303	17·4789	17·5280	17·5765
72	17·8712	17·9204	17·9697	18·0196	18·0689
73	18·3676	18·4176	18·4673	18·5180	18·5681
74	18·8708	18·9216	18·9723	19·0232	19·0740
75	19·3808	19·4322	19·4836	19·5353	19·5867
76	19·8976	19·9496	20·0018	20 0540	20·1062
77	20·4212	20·4740	20·5267	20·5796	20·6325
78	20·9515	21·0049	21·0584	21·1120	21·1655
79	21·4885	21·5428	21·5968	21·6512	21·7056
80	22·0326	22·0876	22·1424	22·1971	22·2523
81	22·5836	22·6392	22·6946	22·7504	22·8058
82	23·1412	23·1972	23·2535	23·3100	23·3661
83	23 7056	23·7624	23·8193	23·8764	23·9333
84	24·2767	24·3344	24·3918	24·4496	24·5070
85	24·8547	24·9132	24·9709	25·0296	25·0876
86	25·4393	25·4984	25·5569	25·6164	25·6749
87	26·0312	26·0908	26·1503	26·2100	26·2694
88	26·6296	26·6900	26·7503	26·8104	26·8710
89	27·2348	27·2956	27·3567	27·4176	27·4788
90	27·8467	27·9084	27·9700	28·0316	28·0935
91	28·4654	28·5276	28·5900	28·6524	28·7147
92	29·0909	29·1541	29·2168	29·2799	29·3434
93	29·7234	29·7868	29·8512	29·9139	29·9784
94	30·3627	30·4270	30·4915	30·5557	30·6202
95	31·0086	31·0736	31·1386	31·2037	31·2689
96	31·6613	31·7270	31·7928	31·8585	31·9244
97	32·3210	32·3873	32·4537	32·5202	32·5867
98	32·9874	33·0544	33·1215	33·1886	33·2558
99	33·6606	33·7283	33·7960	33·8639	33·9318
100	34·3406	34·4090	34·4774	34·5461	34·6145
101	35·0274	35·0964	35·1656	35·2347	35·3040
102	35·7210	35·7907	35·8605	35 9304	36·0003
103	36·4213	36·4918	36·5623	36·6328	36 7034
104	37·1285	37·1996	37·2708	37·3420	37·4133
105	37·8425	37·9143	37·9862	38·0581	38·1303

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
106	38·2024	38·2745	38·3466	38·4189	38·4912
107	38·9266	38·9993	39·0722	39·1451	39·2181
108	39·6576	39·7308	39·8043	39·8779	39·9516
109	40·3951	40·4692	40·5435	40·6177	40·6921
110	41·1400	41·2145	41·2894	41·3644	41·4394
111	41·8911	41·9666	42·0422	42·1178	42·1935
112	42·6493	42·7255	42·8017	42·8780	42·9544
113	43·4143	43·4911	43·5681	43·6451	43·7222
114	44·1860	44·2636	44·3412	44·4189	44·4967
115	44·9646	45·0429	45·1212	45·1995	45·2780
116	45·7500	45·8290	45·9079	45·9870	46·0661
117	46·5422	46·6218	46·7015	46·7812	46·8610
118	47·3412	47·4215	47·5018	47·5822	47·6627
119	48·1470	48·2280	48·3093	48·3901	48·4712
120	48·9596	49·0412	49·1229	49·2047	49·2866
121	49·7790	49·8613	49·9437	50·0261	50·1087
122	50·6052	50·6882	50·7713	50·8544	50·9376
123	51·4382	51·5219	51·6056	51·6894	51·7733
124	52·2780	52·3623	52·4467	52·5312	52·6158
125	53·1246	53·2096	53·2947	53·3800	53·4651
126	53·9780	54·0637	54·1495	54·2353	54·3212
127	54·8382	54·9246	55·0110	55·0976	55·1842
128	55·7052	55·7922	55·8794	55·9666	56·0539
129	56·5794	56·6667	56·7545	56·8424	56·9304
130	57·4595	57·5480	57·6365	57·7255	57·8137
131	58·3473	58·4360	58·5252	58·6145	58·7038
132	59·2416	59·3309	59·4209	59·5108	59·6009
133	60·1426	60·2326	60·3234	60·4139	60·5044
134	61·0500	61·1415	61·2323	61·3235	61·4154
135	61·9645	62·0563	62·1482	62·2402	62·3327
136	62·8864	62·9784	63·0710	63·1637	63·2564
137	63·8146	63·9073	64·0005	64·0939	64·1873
138	64·7494	64·8430	64·9369	65·0314	65·1255
139	65·6914	65·7859	65·8800	65·9747	66·0695
140	66·6399	66·7349	66·8305	66·9254	67·0213

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
106	38·5633	38·6358	38·7083	38·7809	38·8536
107	39·2909	39·3641	39·4373	39·5105	39·5839
108	40·0253	40·0991	40·1730	40·2470	40·3210
109	40·7665	40·8410	40·9156	40·9902	41·0649
110	41·5146	41·5897	41·6650	41·7403	41·8157
111	42·2693	42·3452	42·4211	42·4971	42·5731
112	43·0309	43·1074	43·1840	43·2607	43·3375
113	43·7993	43·8765	43·9540	44·0311	44·1086
114	44·5745	44·6524	44·7304	44·8084	44·8865
115	45·3565	45·4350	45·5137	45·5924	45·6712
116	46·1453	46·2245	46·3039	46·3832	46·4627
117	46·9409	47·0208	47·1008	47·1809	47·2610
118	47·7433	47·8239	47·9046	47·9853	48·0661
119	48·5525	48·6338	48·7151	48·7965	48·8780
120	49·3685	49·4504	49·5325	49·6146	49·6968
121	50·1912	50·2739	50·3566	50·4394	50·5223
122	51·0208	51·1042	51·1876	51·2710	51·3546
123	51·8572	51·9412	52·0253	52·1095	52·1937
124	52·7004	52·7851	52·8700	52·9547	53·0396
125	53·5504	53·6358	53·7212	53·8067	53·8923
126	54·4072	54·4932	54·5794	54·6655	54·7518
127	55·2708	55·3575	55·4443	55·5312	55·6181
128	56·1412	56·2286	56·3161	56·4036	56·4913
129	57·0184	57·1065	57·1946	57·2829	57·3712
130	57·9024	57·9911	58·0800	58·1699	58·2589
131	58·7932	58·8826	58·9722	59·0617	59·1514
132	59·6908	59·7809	59·8711	59·9614	60·0517
133	60·5952	60·6860	60·7768	60·8678	60·9588
134	61·5063	61·5978	61·6894	61·7814	61·8727
135	62·4243	62·5165	62·6088	62·7011	62·7934
136	63·3491	63·4420	63·5349	63·6279	63·7211
137	64·2807	64·3743	64·4679	64·5615	64·6553
138	65·2192	65·3133	65·4076	65·5019	65·5963
139	66·1643	66·2592	66·3542	66·4492	66·5443
140	67·1163	67·2119	67·3075	67·4032	67·4990

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
141	67·5954	67·6908	67·7868	67·8828	67·9789
142	68·5576	68·6536	68·7503	68·8470	68·9438
143	69·5266	69·6233	69·7207	69·8181	69·9155
144	70·5024	70·5998	70·6978	70·7959	70·8940
145	71·4849	71·5831	71·6818	71·7805	71·8794
146	72·4744	72·5731	72·6725	72·7720	72·8715
147	73·4706	73·5700	73·6701	73·7702	73·8704
148	74·4736	74·5737	74·6744	74·7752	74·8761
149	75·4834	75·5845	75·6856	75·7871	75·8886
150	76·4999	76·6018	76·7035	76·8057	76·9079
151	77·5234	77·6259	77·7283	77·8311	77·9340
152	78·5536	78·6564	78·7598	78·8634	78·9670
153	79·5906	79·6944	79·7982	79·9024	80·0067
154	80·6344	80·7385	80·8433	80·9482	81·0532
155	81·6849	81·7902	81·8953	82·0009	82·1065
156	82·7424	82·8481	82·9540	83·0603	83·1665
157	83·8066	83·9130	84·0196	84·1265	84·2335
158	84·8776	84·9849	85·0919	85·1995	85·3072
159	85·9554	86·0630	86·1714	86·2795	86·3882
160	87·0399	87·1482	87·2572	87·3663	87·4754
161	88·1314	88·2413	88·3510	88·4607	88·5698
162	89·2296	89·3395	89·4494	89·5597	89·6700
163	90·3346	90·4449	90·5557	90·6667	90·7778
164	91·4464	91·5577	91·6689	91·7805	91·8923
165	92·5649	92·6770	92·7888	92·9012	93·0136
166	93·6904	93·8029	93·9155	94·0286	94·1417
167	94·8226	94·9359	95·0496	95·1628	95·2766
168	95·9616	96·0751	96·1899	96·3038	96·4183
169	97·1074	97·2216	97·3369	97·4517	97·5668
170	98·2599	98·3752	98·4908	98·6063	98·7221
171	99·4194	99·5354	99·6517	99·7677	99·8842
172	100,5856	100·7023	100·8191	100·9360	101·0532
173	101·7586	101·8759	101·9931	102·1110	102·2289
174	102·9384	103·0564	103·1743	103·2928	103·4114
175	104·1249	104·2440	104·3625	104 4819	104·6007

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	5.	·6	·7	·8	·9
141	68·0751	68·1713	68·2677	68·3641	68·4605
142	69·0407	69·1376	69·2346	69.3017	69·4288
143	70·0131	70·1107	70·2084	70·3061	70·4039
144	70·9923	71·0906	71·1889	71·2874	71·3859
145	71·9783	72·0775	72·1763	72·2754	72·3746
146	72·9711	73·0707	73·1704	73·2702	73·3701
147	73·9707	74·0710	74·1714	74·2718	74·3724
148	74·9770	75·0781	75·1791	75·2803	75·3815
149	75·9902	76·0919	76·1937	76·2955	76·3974
150	77·0102	77·1126	77·2150	77·3175	77·4201
151	78·0370	78·1401	78·2432	78·3464	78·4496
152	79·0706	79·1744	79·2782	79·3820	79·4860
153	80·1110	80·2154	80·3199	80·4244	80·5291
154	81·1582	81·2633	81·3684	81·4737	81·5790
155	82·2122	82·3180	82·4238	82·5297	82·6357
156	83·2730	83·3794	83·4860	83·5026	83·6992
157	84·3406	84·4477	84·5549	84·6622	84·7695
158	85·4149	85·5227	85·6306	85·7386	85·8466
159	86·4965	86·6053	86·7135	86·8222	86·9315
160	87·5846	87·6939	87·8032	87·9126	88·0221
161	88·6794	88·7894	88·9000	89·0105	89·1201
162	89·7810	89·8913	90·0020	90·1121	90·2230
163	90·8889	91·0001	91·1114	91·2228	91·3342
164	92·0041	92·1160	92·2279	92·3340	92·4521
165	93·1261	93·2387	93·3513	93·4640	93·5768
166	94·2549	94·3681	94·4815	94·5948	94·7083
167	95·3905	95·5044	95·6184	95·7325	95·8466
168	96·5329	96·6475	96·7622	96·8769	96·9917
169	97·6821	97·7974	97·9127	98·0281	98·1436
170	98·8380	98·9540	99·0700	99·1861	99·3023
171	100·0008	100·1175	100·2342	100·3510	100·4678
172	101·1704	101·2878	101·4052	101·5226	101·6402
173	102·3468	102·4648	102·5829	102·7010	102·8193
174	103·5300	103·6487	103·7675	103·8863	104·0052
175	104·7200	104·8400	104·9588	105·0783	105·1979

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·0	·1	·2	·3	·4
176	105·3184	105·4381	105·5578	105·6777	105·7976
177	106·5186	106·6389	106·7594	106·8799	107·0005
178	107·7256	107·8466	107·9678	108·0890	108·2103
179	108·9394	109·0611	109·1829	109·3048	109·4268
180	110·1600	110·2824	110·4049	110·5275	110 6501
181	111·3874	111·5105	111·6336	111·7569	111·8802
182	112·6216	112·7453	112·8692	112·9931	113·1171
183	113·8626	113·9870	114·1116	114·2362	114·3609
184	115·1104	115·2355	115·3607	115·4860	115·6114
185	116·3650	116·4908	116·6167	116·7427	116·8687
186	117·6264	117·7529	117·8794	118·0061	118·1328
187	118·8946	119·0217	119·1490	119 2763	119·4037
188	120·1696	120·2974	120·4254	120·5534	120·6815
189	121·4514	121·5799	121·7085	121·8372	121·9660
190	122·7400	122·8692	122·9985	123·1279	123·2573
191	124·0354	124·1653	124·2952	124·4253	124·5554
192	125·3376	125·4681	125·5988	125·7295	125·8603
193	126·6466	126·7778	126·9092	127·0406	127·1721
194	127·9624	128·0943	128·2263	128·3584	128·4906
195	129·2850	129·4176	129·5503	129·6831	129·8159
196	130·6144	130·7477	130·8810	131·0145	131·1480
197	131·9506	132·0845	132·2186	132·3527	132·4869
198	133·2936	133·4282	133·5630	133·6978	133·8327
199	134·6434	134·7787	134·9141	135·0496	135·1852
200	136 0000	136·1360	136·2721	136·4083	136·5445
201	137·3634	137·5001	137·6368	137·7737	137·9106
202	138·7336	138·8709	139·0084	139·1459	139·2835
203	140·1106	140·2486	140·3868	140·5250	140·6633
204	141·4944	141·6331	141·7719	141·9108	142·0498
205	142·8850	143·0244	143·1639	143·3035	143·4431
206	144·2824	144·4225	144·5626	144·7029	144·8432
207	145.6866	145·8273	145·9682	146·1091	146·2501
208	147·0976	147·2390	147·3806	147·5222	147·6639
209	148·5154	148·6575	148·7997	148·9420	149·0844
210	149·9400	150·0828	150·2257	150·3687	150·5117

AREAS OF CIRCLES IN WINE GALLONS.

Diam. in Inches	·5	·6	·7	·8	·9
176	105·9168	106·0369	106·1570	106·2772	106·3974
177	107·1204	107·2411	107·3619	107·4828	107·6037
178	108·3308	108·4522	108·5737	108·6952	108·8171
179	109·5488	109·6701	109·7922	109·9145	110·0368
180	110·7720	110·8947	111·0176	111·1405	111·2635
181	112·0036	112·1271	112·2506	112·3742	112·4978
182	113·2412	113·3653	113·4895	113·6138	113·7381
183	114·4856	114·6104	114·7353	114·8602	114·9853
184	115·7368	115·8623	115·9879	116·1135	116·2392
185	116·9948	117·1210	117·2472	117·3735	117·4999
186	118·2596	118·3865	118·5134	118·6404	118·7674
187	119·5312	119·6587	119·7863	119·9140	120·0417
188	120·8096	120·9378	121·0661	121·1944	121·3229
189	122·0948	122·2237	122·3527	122·4817	122·6108
190	123·3868	123·5164	123·6460	123·7757	123·9055
191	124·6856	124·8159	124·9462	125·0766	125·2070
192	125·9912	126·1221	126·2531	126·3842	126·5153
193	127·3036	127·4352	127·5669	127·6986	127·8305
194	128·6228	128·7551	128·8875	129·0199	129·1524
195	129·9488	130·0818	130·2148	130·3479	130·4811
196	131·2816	131·4153	131·5490	131·6828	131·8166
197	132·6212	132·7555	132·8899	133·0244	133·1589
198	133·9676	134·1026	134·2377	134·3728	134·5081
199	135·3208	135·4565	135·5923	135·7281	135·8640
200	136·6808	136·8172	136·9536	137·0901	137·2267
201	138·0476	138·1847	138·3218	138·4590	138·5962
202	139·4212	139·5589	139·6967	139·8346	139·9725
203	140·8016	140·9400	141·0785	141·2170	141·3557
204	142·1888	142·3279	142·4671	142·6063	142·7456
205	143·5828	143·7226	143·8624	144·0023	144·1423
206	144·9836	145·1241	145·2646	145·4052	145·5458
207	146·3912	146·5323	146·6735	146·8148	146·9561
208	147·8056	147·9474	148·0893	148·2312	148·3733
209	149·2268	149·3693	149·5119	149·6545	149·7972
210	150·6548	150·7980	150·9412	151·0845	151·2279

USE OF THE TABLE.

When the diameter of a circle is given in inches and tenths, the content, in wine gallons, for an inch deep, may be found by inspection, as follows: look for the inches in the left-hand column of the Table, under the word *Diameter*, then in the same horizontal line, under the tenths, will be the content required.

EXAMPLE.

What is the content, for an inch deep, in wine gallons, of a circle 176·4 inches in diameter?

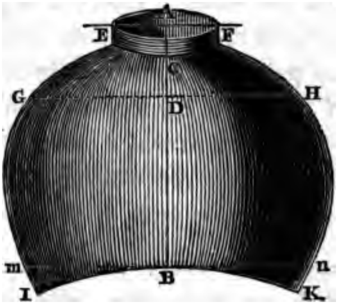
SOLUTION.

Looking in the column of diameters for 176 inches, we find this integral number to be the first in the sixth pair of pages of the Table: then proceeding horizontally along till we arrive under ·4, we discover the content in that angular space to be 105·7976 *wine gallons* required.

We shall now return to the method of gaging and inching a Still, by submitting an example in illustration of the Rule laid down in the beginning of this problem.

EXAMPLE OF A STILL.

Let the following diagram represent a Still, whereof EAFC is the collar: GH, the joining of the globular part to the body; IBK, the rise of the crown; and *mn* the level of the crown-head. It is required to gage and inch this Still.



METHOD OF PROCEEDING.

Lay a straight rule, as EF, diametrically across the mouth of the collar, and with a dimension-cane take the perpendicular AB, which let be 47·4 inches; and let the depth of the collar be 3·4 inches. Next extend a thread or small cord from G to H, diametrically across the Still at the seam of the body with the spherical frustum above, and measure the perpendicular DB, which we may suppose is 36 inches; the mean diameter of the Still at GH being 60 inches; and at EF, 25 inches.

Then CB, the internal depth of the Still from collar to crown, is 44 inches; and CD, the depth of the spherical frustum, is 8 inches.

If the depth of the body of the Still from the crown upwards, namely 36 inches, be now divided into six equal parts, each division of the body will be 6 inches deep. Wherefore at three inches above the crown, having first inscribed a square, and drawn four perpendicular lines on the sides as directed in page 330, we are to take a pair of cross diameters; and at six inches higher, another pair; and so on, at six-inch distances, till six pair of diameters have been taken.

Lastly, we are to admit water into the Still to cover the crown, the quantity of which, requisite for this purpose, we can suppose is 28 wine gallons.

A memorandum of every particular having been carefully noted, we make the following entry, in which the Dimensions are, we can imagine, those found by measurement, and faithfully copied from the memorandum.

DIMENSION BOOK.

A. B.'*s Wash Still, No.* 1, *Gaged,* 10 *May,* 1820.

Inches	Cross Diameters	Mean	Area	Content
8	25·0...25·0 60·1...59·9	25 60	....	58·62
6	61·2...61·4	61·3	12·77	76·62
6	62·4...62·8	62·6	13·32	79·92
6	63·8...64·0	63·9	13·88	83·28
6	64·5...64·5	64·5	14·14	84·84
6	63·6...63·4	63·5	13·71	82·26
6	62·6...62·8	62·7	13·36	80·16
44	To cover the crown			28·00
	Whole content of the Still			573·70

NOTE.—The content of the spherical frustum is found by PROB. XX. page 215, MENSURATION.

The next process is to tabulate or inch the Still all the way down from the collar to the crown. But before this can be done we are to ascertain by calculation, the diameters of the globular frustum at every inch as follows: [*See page* 331.]

2) 60 greater diameter
30 radius of the base
Multiply by 30 radius of the base
Product 900 square of half GH.
12·5 × 12·5 = 156·25 square of half EF.
Remainder 743·75
8 × 8 = 64·00 square of the altitude
8+8 = 16) 679·75 (42·48 inches.

Hence the distance of D from the centre of the sphere is 42·48 inches. Wherefore, to obtain the diameter of the sphere, it will be

42·48 × 42·48 = 1804·55
30 × 30 = 900·00
27|04|·55 (52 semi-diameter
25   2
102) 204   104 *Diameter.*
204
. . .

Again,

52 semi-diameter
Subtract 42·48 distance of D from the centre

Remainder 9·52
Subtract 8 altitude of the frustum

Remainder 1·52 *altitude of the segment cut off.*

Now the diameters of the frustum at the top and bottom being known, we need only calculate the *seven* following diameters: [*See page* 275.]

Inches	Diam. in Inches	Dry Inches
104—2·5=101·5 Multiply by 2·5 Product 253·75 whereof twice the sq. root is	31·8	1
104—3·5=100·5 Multiply by 3·5 Product 351·75 whereof twice the sq. root is	37·5	2
104—4·5 = 99·5 Multiply by 4·5 Product 447·75 whereof twice the sq. root is	42·3	3
104—5·5 = 98·5 Multiply by 5·5 Product 541·75 whereof twice the sq. root is	46·5	4

Inches	Diam. in Inches	Dry Inches
104—6·5 = 97·5		
Multiply by 65		
Product 633·75 whereof twice the sq. root is	50·3	5
104—7·5 = 96·5		
Multiply by 7·5		
Product 723·75 whereof twice the sq. root is	53·8	6
104—8·5 = 95·5		
Multiply by 8·5		
Product 811·75 whereof twice the sq. root is	56·9	7

All the diameters throughout the Still being now known, we may proceed to construct the Table, by taking half the sum of every pair of contiguous diameters in the globular part, for the mean diameter at the middle of every inch; and consequently the several means will be,

	Mean Diameter	Area
First inch	28·4 inches	2·74 gallons
Second	34·6 inches	4·07 gallons
Third	39·9 inches	5·41 gallons
Fourth	44·4 inches	6·70 gallons
Fifth	48·4 inches	7·96 gallons
Sixth	52·0 inches	9·19 gallons
Seventh	55·3 inches	10·39 gallons
Eighth	58·4 inches	11·59 gallons
	Sum	58·05 *gallons.*

With these *mean diameters* we have entered the Table of Circular Areas in Wine Gallons, to ascertain by inspection the content at every inch of the globular part.

It will here be seen that the whole content of the spherical frustum has lost about half a gallon in being tabled, but this trifling discrepancy is unavoidable, and may in a great degree be owing to the retention of so few decimal places in the operation.

A. B.'s WASH STILL TABLED.

Dry Inches	Content	Dry Inches	Content	Dry Inches	Content
Full	573·70 2·74	6	537·63 10·39	12	464 57 12·77
1	570·96 4·07	7	527·24 11·59	13	451·80 12·77
2	566·89 5·41	8 *	515·65 12·77	14 *	439·03 13·32
3	561·48 6·70	9	502·88 12·77	15	425·71 13·32
4	554·78 796	10	490·11 12·77	16	412·39 13·32
5	546·82 9·19	11	477·34 12·77	17	399 07 13·32
6	537·63	12	464·57	18	385·75

We have here carried the Table to 18 dry inches only, being convinced that this process is sufficiently understood by the learner, as it differs nothing from that employed for a Brewer's Copper, except in its being confined to gallons.

It is now proper to mention that the gage of the Wash is taken through a cylindrical tube in the breast of the Still, which tube rises some inches above the level of the base of the collar, where the Table begins: consequently, instead of commencing at FULL as in the Table, we must begin as many inches higher as the tube rises above the level of the top of the frustum. Conceive, for example, that the tube is elevated 3 inches above the surface of the globular part of the Still, and the Table must begin as follows:

TABLE-BOOK.

	Inches	Gallons	Inches	Gallons
Full	3	573·70	11	516·00
	4	571·	12	503·
	5	567·	13	490·
	6	561·	14	477·
	7	555·	15	465·
	8	547·	16	452·
	9	538·	17	439·
	10	527·	18	426·

In copying any Table into the Table Book, we omit all decimals of a gallon, as has been mentioned in treating of the Utensils of Common Brewers; to compensate, however, for this omission, any decimal greater than *five-tenths* is considered a gallon, as will appear on examining the above portion of a Table Book for the Still before us.

By what has been said and done it must seem no very difficult task to gage and inch a Still; but there are in the way impediments to which we have not yet alluded: for from the gross content of the Still, and from the several areas which they affect, we have to deduct for various incumbrances, such as *cross-bars*, *spindles*, *braces*, *hooks*, *chains*, *&c.* which are within the Still, and employed for rousing the Wash; because without this precaution, a sediment would be deposited at the bottom, and impart to the spirit a burnt, or otherwise disagreeable flavour. As no certain rule can be given for the different cases that will occur, we can only refer the practitioner to the THIRD SECTION OF MENSURATION, where he will find every sort of Solid, likely to be met with, in the course of his practice, treated at length both by *Pen* and *Rule*.

If he understands this fundamental part thoroughly, he will never be at a loss, whilst on the contrary if he is ill-versed in this, doubt and error will ever be his companions.

When a body, such as a hook or chain, is of so irregular a form, that none of the Problems in the Mensuration will apply to it, then the best practical mode of determining the content is to drop it into some vessel containing water, carefully observing the number of gallons it displaces.

We shall now exhibit the form of a Dimension Book, with deductions for incumbrances, supposed to be existing, and accurately measured.

DIMENSION BOOK.

A. B.'s *Wash Still, No.* 2, *Gaged,* 10 *May,* 1820.

Depth	Cross Diameters	Mean Diameter	Gross Area	Incumb.		Nett Area	Content
				Inch.	Area		
21	39·8.. 40·2 130·1..129·9	40·0 130·0	Glob. Part.	21	·02	.....	681·01
10	134·6..134·8	134·7	61·69	10	·03	61·66	616·60
10	140·0..139·6	139·8	66·45	10	·02	66·43	664·30
10	146·2..146·6	146·4	72·87	4	·02	72·85	291·40
				6	·84	72·03	432·18
10	152·8..152·4	152·6	79·17	10	·03	79·14	791·40
10	158·0..157·8	157·9	84·77	10	·03	84·74	847·40
10	154·4..154·2	154·3	80·95	3	·09	80·86	242·58
				7	·05	80·90	566·30
10	148·2..148·0	148·1	74·57	10	·02	74·55	745·50
10	140·4..141·0	140·7	67·30	10	·02	67·28	672·80
10	132·0..131·6	131·8	59·06	5	·04	59·02	295·10
				5	·00	59.06	295·30
114	Whole Depth.	Crown by Measure.					540·00
Whole Content of the Still in Wine Gallons.							7681·87

In the example above, it will be perceived that every part of the depth of the Still, except the 5 inches nearest the crown, is more or less affected with incumbrances, for which an allowance has been made.

The nett area having been thus obtained, the construction of a Table, for the Still before us, will be in principle the same as for the first Still.

Since, by Act of Parliament, Distillers are not at liberty to draw off any low wines before they have charged their Wash Stills with wash or wort, paying duty on three fourths of the whole content of each Still and Head; and besides, as the annual licences of Distillers as well as of Rectifiers, are regulated by these contents, it will be requisite in the next Problem to treat of the Head of a Still.

PROBLEM II.

To Gage a Still-Head.

The cylindrical portion of the head which is received into the collar of the Still, is always considered as part of the head, but the neck forms no part.

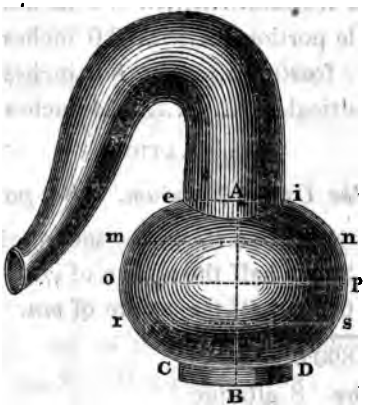
Hence the divisions in general to be made of a Still-Head are the four following:

First.........The cylindrical portion,
Second......The spherical frustum next the cylinder,
Third........The middle portion,
Fourth......The spherical frustum next the neck.

The middle portion of a Still-Head may, for the most part, be regarded as the middle frustum of a spheroid, but the shapes of Still-Heads are so various, that the practitioner must be guided by his own judgment, founded on a correct knowledge of the Problems treated of in MENSURATION. In short, the Head must be divided into such figures as can be gaged by the problems here alluded to.

EXAMPLE.

Let the following diagram represent the Head of the first Still in PROBLEM I., given to be gaged.



METHOD OF PROCEEDING.

Divide the Head into four portions, namely, *nmei*, *mrsn*, *r*CD*s*, and CBD; the first portion being the frustum of a sphere; the second, the middle frustum of an oblate spheroid; the third, the frustum of a sphere; and the fourth, a cylinder.

Then take cross diameters at *ei*, *mn*, *op*, *rs*, and CD; with the exact altitudes of each division.

Suppose the following to be the dimensions taken:

Mean diameter at *ei* ....20·0 inches
———— at *mn* ....36·0 inches
———— at *op* ....40·0 inches
———— at *rs* ....36·0 inches
———— at CD........24·8 inches

ALTITUDES.

Upper frustum............ 8·0 inches
Middle portion............18·0 inches
Lower frustum............ 8·0 inches
Cylindrical part.......... 3·4 inches

CALCULATION.

1. *For the Upper Frustum.* [*See page* 215.]

42·6 two thirds the square of the depth
200·0 half the square of *ei*.
648·0 half the square of *mn*.
890·6 sum
Multiply by 8 altitude
294·12) 7124·8 (24·22 wine gallons.

2. *For the Middle Portion.* [*See page* 219.]

1296 square of *mn*, or of *rs*.
3200 twice the square of *op*.
4496 sum
Multiply by 9 half the altitude
44118) 40464 (91·71 wine gallons.

3. *For the Lower Frustum.* [*See page* 215.]

42·6 two thirds the square of the depth
648·0 half the square of *rs.*
307·5 half the square of CD.

998·1 sum
Multiply by 8 altitude

294·12) 7984·8 (27·15 wine gallons.

4. *For the Cylinder.* [*See the Table for Wine.*]

24·8 diameter......... 2·09 area
Multiply by 3·4 depth

836
627

Product 7·106 wine gallons.

Now the sum of these four contents will be the content of the Still-Head as follows:

First division	24·22 gallons
Second	91·71 gallons
Third	27·15 gallons
Fourth	7·10 gallons
CONTENT OF THE STILL-HEAD	150·18 gallons

But,

	Gallons	
	573·70	content of the Still
	150·18	content of the Head
Sum	723·88	*content of both body and head.*
Multiply by	3	
4)	2171·64	product
Quotient	542·91	THREE-FOURTHS THE WHOLE CONTENT.

Having thus found *three-fourths* of the whole content, we are to enter the Table of the Still, and look for the number of gallons the nearest to this content, observing the dry inches opposite. In the present case, the number in the Table, nearest to 543, is 547, opposite to 5 inches dry: consequently, each charge of wash will be when the Still wants five inches of being full.

The Wash-Backs, Spirit-Vats, and other Utensils of a Distillery are gaged precisely in the same manner as the Utensils of a Brew-House, except that in the Distillery all areas and contents are calculated in wine gallons.

The method of gaging by Equidistant Ordinates has been fully explained and exemplified in the First and Second Sections of Mensuration, and consequently, needs not be repeated. We shall, therefore, only here remark, that when a Wash-Back or other Distillery utensil cannot be accurately measured by any other mode, recourse must be had to the method of Equidistant Ordinates. [*See pages* 111, *and* 161.]

OF MALT GAGING.

As the duty on Malt is charged by the Bushel, the content in Malt Gaging is always calculated in Bushels. For this reason it seems proper, before we proceed farther, to give some account of the present standard Bushel.

By act of parliament it has been enacted, *that every round Bushel, with a plane and even bottom, being eighteen inches and a half in diameter throughout, and* 8 *inches deep, shall be esteemed a legal* Winchester *bushel, according to the* STANDARD *in His Majesty's Exchequer*.

The content of any cylindrical vessel of these dimensions being 2150·42 cubic inches, that number of cubic inches, in whatever shape disposed, must be accounted a legal bushel.

The vessels employed for measuring bushels, half-bushels, pecks and gallons, are generally cylindrical, with plane bottoms, but their diameters and depths vary considerably from the diameter and depth mentioned in the Act, although in capacity they are all, or ought all, to be the same. That is, the bushel ought to contain 2150·42 cubic inches, the half-bushel 1075·21 cubic inches, the peck 537·6 cubic inches, and the gallon 268·8 cubic inches. The weight of water required to fill each of those measures, is as below:

Measures	Troy			Avoirdupois		
	lb.	oz.	dwt.	lb.	oz.	dr.
Bushel	94	6	6·88	77	12	8·60
Half-Bushel	47	3	3·44	32	14	4·30
Peck	27	3	11·72	19	7	2·15
Gallon	11	9	15·86	5	11	9·05

This Table will enable any one in possession of a good beam, and true weights, to try any of the above measures to a great nicety.

Half-gallon measures, quarts, and pints, may in the same manner be proved, as they are respectively the *half*, the *fourth*, and the *eighth-part* of a gallon.

The following Table will shew the depths to diameters varying from 17·5 to 19·5 inches, for cylinders answering to the bushel, and in proportion for the other measures.

A Table for finding, by inspection, whether any Bushel, Half-Bushel, Peck, or Gallon, be equal to the Standard.

	Bushel		Half Bushel		Peck		Gallon		
	Diam.	*Depth*	Diam.	*Depth*	Diam.	*Depth*	Diam.	*Depth*	
	17·5	8·94	13·7	7·30	10·7	5·98	8·3	4·97	
	17·6	8·84	13·8	7·20	10·8	5·87	8·4	4·85	
	17·7	8·74	13·9	7·09	10·9	5·76	8·5	4·74	
	17·8	8·64	14·0	6·98	11·0	5·66	8·6	4·63	
	17·9	8·55	14·1	6·89	11·1	5·56	8·7	4·52	
	18·0	8·45	14·2	6·79	11·2	5·46	8·8	4·42	
	18·1	8·36	14·3	6·69	11·3	5·36	8·9	4·32	
	18·2	8·27	14·4	6·60	11·4	5·27	8·0	4·23	
	18·3	8·18	14·5	6·51	11·5	5·18	9·1	4·13	
	18·4	8·09	14·6	6·42	11·6	5·09	9·2	4·04	
Stan	18·5	8·00	14·7	6·34	11·7	5·00	9·3	3·96	dard.
	18·6	7·91	14·8	6·25	11·8	4·92	9·4	3·87	
	18·7	7·83	14·9	6·17	11·9	4·83	9·5	3·79	
	18·8	7·75	15·0	6·08	12·0	4·75	9·6	3·71	
	18·9	7·67	15·1	6·00	12·1	4·68	9·7	3·64	
	19·0	7·58	15·2	5·93	12·2	4·60	9·8	3·56	
	19·1	7·50	15·3	5·85	12·3	4·52	9·9	3·49	
	19·2	7·42	15·4	5·77	12·4	4·45	10·0	3·42	
	19·3	7·35	15·5	5·72	12·5	4·38	10·1	3·36	
	19·4	7·27	15·6	5·63	12·6	4·31	10·2	3·29	
	19·5	7·20	15·7	5·56	12·7	4·24	10·3	3·23	

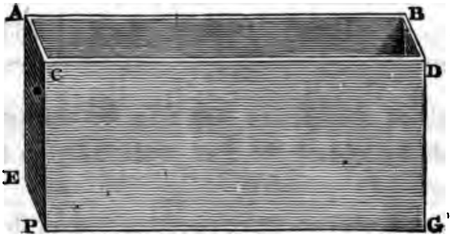
By help of the above Table any cylindrical vessel, according to the dimensions generally in use, and purporting to be either a Bushel, Half-Bushel, Peck, or dry Gallon, may be easily tried. For suppose it a Bushel 18 inches in diameter, it is only necessary to look in the column of bushel-diameters for 18 inches, and opposite in the column of depths will be found 8·45 inches: hence, if the depth be less than 8·45 inches, the measure is deficient; whereas if it be more, the measure is redundant, or more than a bushel. We come now to the several Problems in Malt Gaging.

PROBLEM I.

To Gage and Tabulate a Maltster's Cistern.

Maltsters' Cisterns are in general rectangular, though some are circular, and others oval. But whatever be the shape, the method of taking the dimensions of a Cistern is in all respects the same as that employed for the utensils of a Brew-House; that is, if the Cistern be rectangular, a mean length is to be deduced from several lengths in various parts, and at different depths of the Cistern; so, likewise, a mean breadth from the sum of several breadths. Or, if the Cistern be cylindrical, a mean diameter is to be found from several cross diameters. In like manner, if the Cistern be elliptical, with perpendicular sides, the mean transverse and the mean conjugate diameter must be ascertained. Lastly, if the sides of the Cistern be not perpendicular to the base, the area at different depths, as directed for similar vessels in the Brew-House, must be found in the manner there pointed out and elucidated. It will be quite sufficient for our present purpose to give one example of a Cistern, as follows:

Let ABCDEPG represent a Maltster's square Cistern, to be gaged and tabled.



METHOD OF PROCEEDING.

Take the length of the Cistern with a sliding-cane, in various parts and at different depths, also several breadths in the same manner. Then divide the sum of the lengths by the number taken, for the *mean length;* and the sum of the breadths, by the number of breadths, for the *mean breadth.*

The mean length, multiplied by the mean breadth, will be the area of the Cistern in inches, and these divided by 2150·42, the number of cubic inches in a malt bushel, will give the area of the base, for one inch deep, in malt bushels; this area multiplied by the depth will evidently be the whole content.

Having taken the dimensions, and found the area, an entry is to be made in the Dimension Book, which, in the case before us, will be of the form following.

DIMENSION BOOK.

A. B.'s *Square Cistern Gaged*, 10 *May*, 1820.

Depth	Length	Breadth	Area	Content
50	111·6	90·0	4·67	233.5

A Table of the Cistern may now be constructed to any depth, by adding the area continually to itself for wet inches.

A Table, however, more useful still than this would be, will arise, if after 10 inches from the bottom, we tenth the cistern; that is, if to the content at 10 inches deep, we continually add ·467, which is *one-tenth* the area in bushels. Thus,

4·67 area of the base
Multiply by 10 for ten inches deep

46·7 bushels at 10 inches deep
Add ·467 for one tenth of an inch

Sum 47·167 bushels for 10·1 inches deep
Add ·467

Sum 47·634 bushels for 10·2 inches deep.

By the continual addition of ·467 we obtain the depth for every tenth of an inch after 10 inches from the bottom.

Suppose we have carried the tenthing through 5 inches, and the Table will appear as follows :

A. B.'s Square Cistern Tabled.

Depth in Inches	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
10	46·7	47 2	47·6	48·1	48·6	49.0	49·5	50·0	50.4	50·9
11	51·4	51·8	52·3	52·8	53·2	53·7	54·2	54·6	55.1	55·6
12	56·0	56·5	57·0	57·4	57·9	58·4	58·8	59·3	59·8	60·2
13	60·7	61·2	61·6	62 1	62·6	63·0	63·5	64·0	64·4	64·9
14	65·4	65·8	66·3	66·8	67·2	67·7	68·2	68·6	69·1	69·6
15	70·0	...	...	...	...	...	...	...	...	...

As this method by the pen is tedious, we shall subjoin the shorter mode by the Sliding Rule.

1. *For the Area of the Base.*

Set 2150·42 on A, to the length on B, then against the breadth on A, will be the area on B, in malt bushels.

2. *For the Content at any Depth.*

Set the area on A, to 1 on B, then opposite the depth on B, will be the content on A.

3. *For the Content at one Operation.*

Set the length on B, to the depth on the Line MD, then against the breadth on A, will be the content on B, in malt bushels.

Hence, for the whole content of the Cistern before us, at one operation, it will be,

on B. on MD. on A. on B.
Set {111·6} to {50} against {90} are {233·5 M. B.}

It is found that barley steeped 40 hours and upwards in water, has increased in bulk *one-fourth* part its original size; that is, every 4 bushels of dry barley become 5 bushels in the Cistern, if covered with water forty hours.

According to this proportion, therefore, an allowance of 2 bushels in every 10 bushels of wet barley is to be made; in other words, *one-fifth* part the cistern-gage is to be deducted.

There are, of course, various ways of reducing cistern-bushels to dry bushels, but the following seems as good as any: Multiply the cistern-bushels by the decimal ·8, which is the equivalent of *four-fifths*, and the product will be the dry bushels.

EXAMPLE.

If a Cistern contain 233·5 bushels of barley steeped 40 hours and upwards, how many bushels must be charged?

SOLUTION.

233·5 wet bushels
Multiply by ·8 the equivalent of four-fifths
Product 186·8 dry bushels, *Answer.*

Cisterns in a variety of shapes will occur in practice, but the method of gaging them, as has already been said, is the same as for utensils of similar shape fully treated of in the Brew-House, and which to repeat would be only a waste of time. The manner of employing the Sliding Rule, we have amply discussed in the MENSURATION.

PROBLEM II.

To Gage a Couch of Malt.

By a *Couch of Malt* is understood any quantity of steeped barley, emptied either on a floor, or into a frame, but which has not been out of the Cistern *thirty hours.*

It is found by experience that barley in this condition is more bulky than when dry by 2 bushels in every 10 bushels, which we mentioned in treating of the CISTERN. Hence the same deduction is to be made from couch-gages, as from the cistern-gage; that is, *one-fifth* the whole is to be abated.

CASE 1.

To Gage a Couch of Malt in a Rectangular Frame.

With a tape, or dimension-cane, take the length of the Frame, and likewise the breadth. Then with a dipping-rod ascertain the depth of the malt in several places: and divide the sum of the depths, by their number, for a *mean depth.* It is always convenient that the number of depths taken be 10, since the mean depth in this case is obtained by merely shifting the decimal point in the sum, one place to the left.

To determine the content of the Couch, multiply the length by the breadth, and the product by the mean depth: after which, divide by 2150·42, the number of cubic inches in a malt bushel, and the quotient will be the number of wet bushels in the Couch. Lastly, these multiplied by ·8 will give the bushels to be charged.

By the Sliding Rule.

Set the length of the Frame on B, to the mean depth on MD, and against the breadth on A, will be the number of wet bushels on B.

EXAMPLE.

Required the content of a Couch of malt in an oblong Frame, the length of which is 120 inches, the breadth 109 inches, and mean depth 22 inches.

SOLUTION.

By the Pen.

109 breadth of the couch
Multiply by 120 length of the couch
Product 13080 area in square inches
Multiply by 22 mean depth
26160
26160
2150) 287760 (133·84 malt bushels, wet
·8 equivalent of *four-fifths*
Product 107·07 dry bushels, *Answer.*

By the Sliding Rule.

	on B.		on MD.		on A.		on B.
Set	{120}	to	{22}	against	{109}	are	{133·8 M.B.}

Multiply by ·8
Dry bushels 107·04 *Answer.*

CASE 2.

To Gage a Rectangular Couch of Malt not in a Frame.

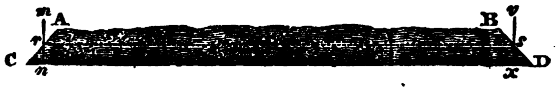
When a rectangular Couch is spread on the middle of a floor, its sides and ends are always slanting; but if one of the sides, or one of the ends of the Couch be against a perpendicular wall, as is often the case, then the opposite side or end will slant, whilst the side or end next the wall will be at right angles to the horizon.

METHOD OF PROCEEDING.

Take the length of the Couch with a tape, not at the bottom nor at the top, but at a line passing horizontally through the middle of the slant or slants; and in like manner take the breadth. Then from ten dips with a gaging rod and brass plate, deduce a mean depth, and with these dimensions proceed exactly as in Case 1.

EXAMPLE 1.

Let ABCD be the end of a Couch of malt with slanting sides, to be gaged.



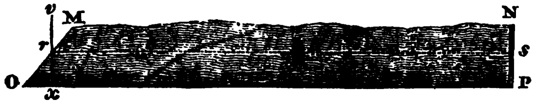
METHOD OF GAGING.

Half way between A and C let a rule, as *mn*, be set perpendicularly up, and another, as *vx*, perpendicularly, half way between B and D. Then extend a tape from *r* to *s*, or from *m* to *v* parallel to *rs*, regarding *rs* as the breadth of the Couch. In like manner if the ends slant, must the length be taken.

Lastly, from several dips with a gaging rod and brass plate, deduce a mean depth, and calculate the content as in Case 1.

EXAMPLE 2.

Let MNOP be the end of a Couch of malt to be gaged, having one side against a perpendicular wall at NP, but the other side slanting.



METHOD OF GAGING.

Set a rule, as *vx*, perpendicularly up, half way between M and O. Then extend a tape from *r* to *s*, considering *rs* as the breadth of the Couch.

Precisely in the same way if either of the ends slant, take the length. In all other respects proceed as in Case 1.

CASE 3.

To Gage a Couch of Malt in the form of the Frustum of a Cone.

Level the top of the frustum with your gaging-rod, and take the depth of the Couch in the middle.

Then in the middle of the slant side set up a rod perpendicular to the horizon, and having fixed one end of your tape at the centre of the top of the frustum, measure the horizontal distance of the rod from that centre. This distance doubled may be regarded as the mean diameter of the frustum considered as a cylinder, although this practice is by no means correct in principle.

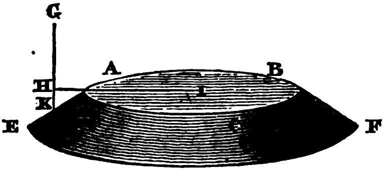
The mean diameter squared, and divided by 2738, the circular divisor for malt bushels, will give the area of the cylinder, and this multiplied by the depth will be the content.

By the Sliding Rule.

Set the circular gage-point, for malt bushels, on D, to the depth on C, and against the mean diameter on D, will be the content on C.

EXAMPLE.

Let ABCDEF be a Couch of malt in the form of the frustum of a cone, to be gaged.



METHOD OF PROCEEDING.

Level the top of the frustum with the gaging-rod, and take the perpendicular depth in the middle, which let be 10 inches. Then half way between A and E, set up a rule, as GHK, perpendicularly; and measure the horizontal distance of I, the centre of the surface of the malt, from H, the nearest part of the rule; which distance let be 35 inches, and the mean diameter will be 70 inches, the double of 35. Wherefore we shall have,

70 mean diameter
Multiply by 70 mean diameter

M.B. cir. divisor 2738) 4900 (1·79 malt bushels area
10 depth

Product 17·9 malt bushels wet
Multiply by ·8

Product 14·3 *malt bushels dry.*

By the Sliding Rule.

on D. on C. on D. on C.
M.C.G.P. {52·32} to {10} against {70} are {17·9 M.B.}
Multiply by ·8

Malt bushels dry 14·3 *Ans.*

PROBLEM III.

To Gage a Floor of Malt.

By a *Floor of Malt* is understood any quantity of steeped barley spread out for the purpose of *spritting*, and which has been out of the Cistern above thirty hours. It is found that barley in this state has doubled its bulk, and hence the gross bushels in a Floor are double the nett bushels of dry malt.

Floors in general are of a rectangular form, but of very uneven surface, and consequently a mean depth has to be deduced from several depths taken with a gaging-rod and brass-plate.

The method of taking the dimensions of a Floor, and also of calculating the gross content, both by *Pen* and *Rule*, is the same as that laid down for a Couch, and needs not be repeated. The gross bushels divided by 2, or multiplied by ·5, will evidently give the nett bushels, since the nett bushels must be in number equal to half the gross bushels. If the nett bushels be required without finding the gross bushels, they will be obtained by using the half instead of the whole of any *one* of the three dimensions of the Floor.

EXAMPLE 1.

How many bushels of malt, both gross and nett, are contained in a Floor 200 inches in length, 140 inches, in breadth, and 11 inches in mean depth?

SOLUTION.

By the Pen.

	140	breadth of the floor
Multiply by	200	length of the floor
Product	28000	area in square inches
Multiply by	11	mean depth
2160)	308000	(143·2 gross bushels
		·5 equivalent of *one-half.*
	Product	71·6 nett bushels, *Answer.*

By the Sliding Rule.

on MD. on B. on A. on B.
Set {11} to {200} against {140} are {143·2 M.B.}
Multiply by ·5

Nett malt bushels 71·6 *Answer.*

EXAMPLE 2.

How many nett bushels of malt are in a Floor 309 inches long, 180 inches broad, and 14 inches in mean depth?

SOLUTION.

By the Pen.

309 length of the floor
Multiply by 180 breadth of the floor

24720
309

Product 55620 area in square inches
7 half the depth

2150) 389340 (181·08 nett bushels, *Answer.*

By the Sliding Rule.

on MD. on B. on A. on B.
Set {7} to {309} against {180} are {181·0 M.B.}

Or,

on MD. on B. on A. on B.
Set {14} to {309} against {90} are {181·0 M.B.}

Having now offered every thing really useful in Malt Gaging, we come to one of the most difficult parts of an officer's duty, viz. *Cask Gaging*.

OF CASK GAGING.

THERE are four varieties of Casks, but no certain rule, applicable in practice, can be given for ascertaining exactly to which of them any particular cask belongs.

When the staves are much curved, the cask is deemed to be the middle frustum of a spheroid, and is said to be of the *First Variety*.

When the staves are curved in a less degree, the cask is regarded as the middle frustum of a parabolic spindle, and is termed a cask of the *Second Variety*.

When the staves are curved so little as to appear almost straight, the cask is supposed to be formed of the frusta of two equal parabolic conoids, joined together by their bases; which form constitutes the *Third Variety*.

And lastly, when the staves are quite straight from bung to head, the cask is considered as made up of the frusta of two equal cones, with co-incident bases; this shape of a cask being the *Fourth Variety.*

Casks of the third and fourth varieties are seldom or never met with, the generality of casks being either of the *First* or *Second Variety.*

To whatever variety a Cask belongs, the three following dimensions must be accurately taken.

1. The Bung Diameter;
2. The Head Diameter,
3. The Length of the Cask.

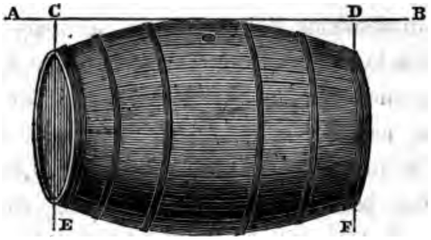
These dimensions being known, a *mean* diameter between the head and bung diameters of the Cask must be found, such that a cylinder having for diameter this *mean,* and for altitude the length of the Cask, shall be in capacity equal to the Cask. Such is the general and best method of determining the content with ease, but it will be first necessary to shew in what manner the dimensions are to be taken in different positions of the Cask.

PROBLEM I.

To Gage a Cask Lying.

Observe whether the bung hole is exactly half way between the ends of the Cask, and whether both ends are truly circular and equal: also, whether the stave opposite to the bung is even within, and of the same form as the other staves. Then measure the shortest distance between the inside of the chime and the outermost edge of the stave opposite, where it is sloped: this distance will be equal to the head diameter of the cask within, *very nearly*. Next, through the bung hole take the bung diameter of the Cask, allowing for the thickness of the stave at the bung, which thickness will be perceptible at the hole.

Lastly, take the length, which is most commodiously, as well as expeditiously, found by a pair of long callipers; but in the absence of these, it may be ascertained as follows.



Lay a straight rod, as AB, longitudinally along the Cask, so as to rest on the bulge, and at the same time be equidistant from each chime. Then lay a straight rule diametrically across each end of the Cask, in contact with the chimes, as CE, and DF : the distance between the rules, namely CD, measured on the rod AB, will be the gross length of the Cask.

Lastly, from the gross length, subtract the depth of both chimes, and the thickness of both heads; the remainder will be the internal or nett length of the Cask.

Suppose, now, the dimensions found by measure are the following:

Bung diameter............33 inches
Head diameter............26 inches
Gross length..............44 inches.

Deduct from the gross length 4 inches for the two chimes and two heads, and the nett length of the Cask will be 40 inches.

Now from these dimensions the content may be found by the rules laid down in Mensuration, for the particular solid according to the variety of the Cask, but the best practical method is that of reducing the Cask to a cylinder as we have just now mentioned. This is done with tolerable correctness in the manner following:

When the difference between the bung and head diameters does not exceed *six* inches, multiply it

$$\text{By}\left\{\begin{matrix}\cdot68\\ \cdot62\\ \cdot55\\ \cdot50\end{matrix}\right\}\text{in any Cask of the}\left\{\begin{matrix}\text{1st}\\ \text{2nd}\\ \text{3rd}\\ \text{4th}\end{matrix}\right\}\text{Variety.}$$

Then add the product to the head diameter, and the sum will be the diameter of a cylinder, which being in altitude equal to the length of the Cask, shall be of the same capacity as the Cask. But if the head diameter differs more than *six* inches from the bung diameter, then multiply the difference

$$\text{By}\left\{\begin{matrix}\cdot70\\ \cdot64\\ \cdot57\\ \cdot52\end{matrix}\right\}\text{in any Cask of the}\left\{\begin{matrix}\text{1st}\\ \text{2nd}\\ \text{3rd}\\ \text{4th}\end{matrix}\right\}\text{Variety.}$$

In which case the product is to be added to the head diameter, or subtracted from the bung diameter, for a *mean* diameter, as in the first case.

Having obtained the mean diameter, enter the Table of Circular Areas for Ale or for Wine, as may be required, and multiply the corresponding area by the length of the Cask for the content.

EXAMPLE.

Required the content of a Cask of the *first variety*, whereof the bung diameter is 31 inches, the head diameter 25 inches, and the length inside 40 inches.

SOLUTION.

	31	bung diameter
	25	head diameter
	6	difference
Multiply by	·68	according to rule
Product	4·08	to be added
	25·00	head diameter
Sum	29·08	*mean diameter.*

Now entering the Table of Circular Areas in Ale, with 29·1 as diameter, we find the corresponding area to be,

2·35 ale gallons
Multiply by 40 length of the cask
Product 94·00 *content in ale gallons.*

Hence the content of the cask in ale-measure is 2 barrels, 2 firkins, and 4 gallons.

Again, entering the Table of Circular Areas in Wine, with 29·1 as diameter, we find the corresponding area to be

2·879 wine gallons
Multiply by 40 length of the cask
Product 115·16 *content in wine gallons.*

To find the Content by the Sliding Rule.

From the bung diameter subtract the head diameter by the pen, and look for the difference on the inch-line of the Slide of *Varieties*; then to the head diameter add the number opposite to this difference on the line corresponding to the particular variety: the sum will be the mean diameter of the Cask considered as a cylinder.

Next, set the circular gage-point, for the proposed integer, on D, to the length of the Cask on C, and against the mean diameter on D, will be the content on C.

EXAMPLE.

The content of the last Cask in wine gallons is sought by the Sliding Rule.

SOLUTION.

Opposite to 6, the difference of the bung and head diameters on the first or inch-line of the Slide of *Varieties*, we discover 4·18 on the line marked spheroid. Consequently the *mean* diameter is 29·18, and therefore it will be

on D. on C. on D. on C.
C.G.P.{17·15}to{40}against{29·18}are{115·7W.G.{

The content found by the Sliding Rule is thus about half a gallon more than by the pen.

PROBLEM II.

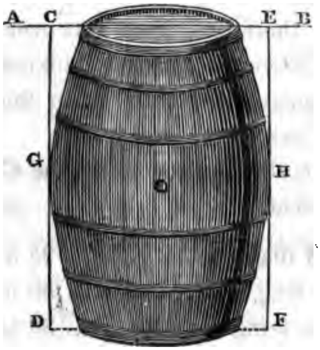
To Gage a Cask Standing.

Measure the head diameter as directed for the Cask lying, and if there be a hole in the head at the top, take the perpendicular depth of the Cask through the hole, to the outside at the top, afterwards abating for the thickness of the head, as seen at the hole.

Then lay a straight rod diametrically across the head, of length sufficient to project both ways beyond the bulge of the Cask. Two other straight rods are now to be applied longitudinally to the bulge on opposite sides, so as to intersect the first rod at right angles; for in this event they may be presumed parallel to the longer axis of the Cask. The distance between the upright rods will be the gross diameter of the Cask at the bung, and this diminished by the thickness of the two staves at the bulge, (which we must judge of by the size of the Cask) will give the internal diameter at the bung. The bung diameter, head diameter, and length of the Cask being now known, the content is to be calculated precisely in the same way as in the preceding Problem.

EXAMPLE.

Let the following figure represent a Cask of the *Second Variety* to be gaged standing.



METHOD OF TAKING THE DIMENSIONS.

Measure the shortest distance between the inside of the chime, and the outermost sloped edge of the opposite stave: and this distance will be the head diameter *very nearly*, inside. Then through the hole in the head take the perpendicular length of the Cask.

If there be no hole in the head, the gross length may be taken outside, and a deduction for the two chimes and two heads can be made according to size and appearances.

Next lay a straight rod, as AB, diametrically across the head, and place two upright rods, centrally opposed to each other, in contact with the bulge at G and H. These rods must intersect AB at right angles in the points C and E, which they will do when the four points C, D, F, E, are all at equal distances from the outside of the chimes. The distance of C from E is equal to the gross diameter of the Cask at the bung. If, therefore, from this gross diameter we abate a fair allowance for the thickness of the two staves at the bulge, we shall obtain the internal diameter of the Cask at the bung.

Suppose now the dimensions of the Cask before us, to be the following:

Head diameter..............	25 inches
Nett length....................	50 inches
Gross bung diameter.......	36 inches

Deduct from the gross diameter at the bung 3·5 inches for the thickness of the two staves, and there will remain 32·5 for the internal bung-diameter.

CALCULATION OF THE CONTENT IN ALE AND WINE.

32·5 bung diameter
25·0 head diameter
7·5 difference
Multiply by ·64 according to rule for *2nd Var.*
300
450
Product 4·800 to be added
Add 25·000 head diameter
Sum 29·800 *mean diameter.*

With this mean diameter entering the Table of Circular Areas in Ale, we find the corresponding area to be

2·47 ale gallons
Multiply by 50 length of the cask
Product 123·5 *content in ale gallons.*

Hence the content of the Cask in ale-measure is 1 butt, 1 firkin, and 6·5 gallons.

Again, with 29·8 inches as diameter, entering the Table of Circular Areas in Wine, we find the area corresponding to this diameter to be

3·019 wine gallons
Multiply by 50 length of the cask
Product 150·95 *content in wine gallons.*

PROBLEM III.

To find the Content of a Cask of the FIRST VARIETY, *by the Diagonal Rule.*

CASE I. If the Cask be lying, put the Rule into the bung-hole in such manner as that its extremity may meet the intersection of the head and stave on the opposite side of the Cask, then on the Rule at the centre of the bung-hole will be the content, in ale gallons and wine gallons respectively, on the diagonal lines of the Rule.

CASE II. If the Cask be standing, apply the diagonal lines to the head diameter, and twice the diagonal measuring the head will be the *content.*

When the diagonal of a Cask of the *First Variety* is known, the content may be found by the Sliding Rule as follows:

For Ale Gallons.

Set the diagonal on C, to 21·2 on D, then against the diagonal on D, will be the content on C.

For Wine Gallons.

Set the diagonal on C, to 19·2 on D, then against the diagonal on D, will be the content on C.

EXAMPLE.

Required the content of a Cask of the *First Variety,* in Ale and Wine, the diagonal being 29·3 inches.

SOLUTION.

on D. on C. on D. on C.

$$\text{Diag.P.}\left\{\begin{matrix}21\cdot2\\19\cdot2\end{matrix}\right\}\text{to}\left\{29\cdot3\right\}\text{against}\left\{29\cdot3\right\}\text{are}\left\{\begin{matrix}56\cdot2\text{ A.G.}\\69\cdot0\text{ W.G.}\end{matrix}\right\}$$

PROBLEM IV.

To determine the Content of a Cask by a Table of Quotients.

Divide the head diameter by the bung diameter to two decimal places in the quotient, and seek this quotient in the column of Quotients in the Table: then from the column answering to the proposed *Variety* and denomination, take out the decimal, which divide by 100; next multiply the quotient by the square of the bung diameter, and afterwards by the length of the Cask, for the Content.

TABLE OF QUOTIENTS FOR THE FOUR VARIETIES OF CASKS IN ALE AND WINE MEASURES.

FIRST VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·50	·20888	·25500	·61	·22021	·26883
·51	·20982	·25614	·62	·22136	.27023
·52	·21077	·25731	·63	·22252	·27164
·53	·21175	·25850	·64	·22370	·27308
·54	·21274	·25971	·65	·22489	·27454
·55	·21375	·26094	·66	·22611	·27603
·56	·21478	·26220	·67	·22734	·27754
·57	·21583	·26348	·68	·22860	·27907
·58	·21690	·26479	·69	·22987	·28062
·59	·21798	·26611	·70	·23116	·28219
·60	·21909	·26746	·71	·23247	·28379

TABLE OF QUOTIENTS FOR THE FOUR VARIETIES OF CASKS IN ALE AND WINE MEASURES.

FIRST VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·72	·23380	·28541	·87	·25594	·31244
·73	·23514	·28706	·88	·25756	·31443
·74	·23651	·28872	·89	·25921	·31643
·75	·23789	·29040	·90	·26087	·31846
·76	·23929	·29212	·91	·26255	·32052
·77	·24071	·29386	·92	·26425	·32259
·78	·24215	·29561	·93	·26596	·32468
·79	·24361	·29739	·94	·26770	·32680
·80	·24508	·29919	·95	·26945	·32894
·81	·24658	·31102	·96	·27123	·33111
·82	·24809	·30287	·97	·27302	·33330
·83	·24962	·30474	·98	·27483	·33551
·84	·25117	·30552	·99	·27666	·33774
·85	·25274	·30854	1·00	·27851	·33999
·86	·25433	·31047	1·01	·28042	·34223

SECOND VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·50	·19959	·24366	·59	·21175	·25850
·51	·20089	·24525	·60	·21315	·26021
·52	·20222	·24687	·61	·21457	·26194
·53	·20355	·24849	·62	·21602	·26371
·54	·20489	·25012	·63	·21744	·26544
·55	·20623	·25176	·64	·21889	·26721
·56	·20750	·25332	·65	·22034	·26899
·57	·20897	·25511	·66	·22182	·27079
·58	·21035	·25679	·67	·22331	·27261

TABLE OF QUOTIENTS FOR THE FOUR VARIETIES OF CASKS IN ALE AND WINE MEASURES.

SECOND VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·68	·22480	·27443	·85	·25191	·30752
·69	·22630	·27627	·86	·25361	·30960
·70	·22782	·27811	·87	·25532	·31168
·71	·22934	·27998	·88	·25703	·31378
·72	·23087	·28184	·89	·25876	·31589
·73	·23244	·28376	·90	·26050	·31801
·74	·23437	·28612	·91	·26225	·32015
·75	·23557	·28758	·92	·26401	·32230
·76	·23716	·28952	·93	·26579	·32447
·77	·23875	·29146	·94	·26757	·32664
·78	·24036	·29343	·95	·26936	·32883
·79	·24197	·29539	·96	·27117	·33104
·80	·24360	·29738	·97	·27299	·33326
·81	·24524	·29939	·98	.27482	·33549
·82	·24690	·30130	·99	·27665	·33773
·83	·24856	·30343	1·00	·27851	·33999
·84	·25023	·30547	1·01	·28035	·34223

THIRD VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·50	·17407	·21250	·61	·19108	·23325
·51	·17548	·21421	·62	·19279	·23534
·52	·11692	·21596	·63	·19453	·23747
·53	·17838	·21775	·64	·19630	·23963
·54	·17987	·21957	·65	·19809	·24182
·55	·18139	·22142	·66	·19992	·24388
·56	·18293	·22331	·67	·20177	·24631
.57	·18451	·22523	·68	·20365	·24800
·58	·18611	·22718	·69	·20556	·25093
·59	·18774	·22917	·70	·20749	·25329
·60	·18939	·23119	·71	·20946	·25569

TABLE OF QUOTIENTS FOR THE FOUR VARIETIES OF CASKS IN ALE AND WINE MEASURES.

THIRD VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·72	·21145	·25812	·87	·24466	·29867
·73	·21347	·26059	·88	·24710	·30164
·74	·21552	·26389	·89	·24956	·30465
·75	·21759	·26562	·90	·25206	·30769
·76	·21969	·26818	·91	·25459	·31077
·77	·22183	·27079	·92	·25713	·31388
·78	·22398	·27342	·93	·25970	·31703
·79	·22617	·27609	·94	·26231	·32020
·80	·22838	·27879	·95	·26494	·32342
·81	·23063	·28153	·96	·26760	·32666
·82	·23290	·28430	·97	·27029	·32995
·83	·23519	·28711	·98	·27300	·33326
·84	·23752	·28994	·99	·27575	·33661
·85	·23987	·29282	1·00	·27851	·33999
·86	·24225	·29572	1·01	·28126	·34337

FOURTH VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·50	·16247	·19833	·60	·18196	·22213
·51	·16433	·20061	·61	·18402	·22464
·52	·16622	·20291	·62	·18609	·22716
·53	·16812	·20523	·63	·18817	·22971
·54	·17047	·20758	·64	·19028	·23228
·55	·17198	·20995	·65	·19241	·23488
·56	·17394	·21234	·66	·19455	·23750
·57	·17592	·21475	·67	·19672	·24014
·58	·17792	·21719	·68	·19891	·24280
·59	·17993	·21965	·69	·20110	·24549

TABLE OF QUOTIENTS FOR THE FOUR VARIETIES OF CASKS IN ALE AND WINE MEASURES.

FOURTH VARIETY.

Quot.	Ale Gallons	Wine Gallons	Quot.	Ale Gallons	Wine Gallons
·70	·20332	·24820	·86	·24134	·29462
·71	·20555	·25092	·87	·24388	·29771
·72	·20779	·25368	·88	·24634	·30083
·73	·21007	·25646	·89	·24900	·30397
·74	·21236	·25926	·90	·25159	·30713
·75	·21467	·26208	·91	·25420	·31034
·76	·21700	·26493	·92	·25683	·31352
·77	·21935	·26779	·93	·25947	·31675
·78	·22172	·27068	·94	·26214	·32001
·79	·22411	·27360	·95	·26482	·32328
·80	·22653	·27653	·96	·26752	·32658
·81	·22895	·27949	·97	·27024	·32990
·82	·23139	·28247	·98	·27300	·33324
·83	·23385	·28547	·99	·27574	·33661
·84	·23633	·28850	1·00	·27851	·33999
·85	·23883	·29155	1·01	·28125	·34336

To illustrate the use of the Table we shall give two examples, which we think will be amply sufficient.

EXAMPLE 1.

Required the content of a Cask of the *2nd Variety*, in wine gallons, the length of the Cask being 50 inches, the head diameter 25 inches, and bung diameter 32·5 inches.

SOLUTION.

Bung Diameter	Head Diameter	Quotient
32·5)	25·0	(·77 *nearly.*

Now entering the Table with ·77, we find the decimal corresponding to this quotient in the *Second Variety*, under WINE GALLONS to be ·29146, wherefore we have

	·0029146	tabular decimal divided by 100
32·5 × 32·5 =	1056·25	square of the bung diameter
Product	3·078	area of a cylinder
Multiply by	50	length of the cask
Product	153·9	wine galls. *content required.*

The content thus found differs 3 gallons from the content of the same Cask found in PROBLEM II., but this is partly owing to ·77 in the quotient being too great.

EXAMPLE 2.

Required the content of a Cask of the *First Variety*, in ale gallons, the bung diameter being 40 inches, the head diameter 30 inches, and the length of the Cask 48 inches.

SOLUTION.

Bung Diameter	Head Diameter	Quotient
40)	30·00	(·75

Now entering the Table with ·75, we find the decimal corresponding to this quotient in the *First Variety* under ALE GALLONS to be ·23789, wherefore we have

	·0023789	tabular decimal divided by 100
40 × 40	1600	square of the bung diameter
	142734	
	23789	
Product	3·80624	area of a cylinder
Multiply by	48	length of the cask
	3044992	
	1522496	
Product	182·69952	ale gallons, *content required.*

PROBLEM V.

To determine the Content of a Cask of the FIRST VARIETY, *in Wine Gallons, by a Table of Diameters for the Head and Bung.*

With the head diameter in inches and tenths enter the Table, and take out the area corresponding to the *Head Diameter*: in like manner, with the bung diameter enter the Table, and take out the area corresponding to the *Bung Diameter;* then add the first area to twice the second, and multiply the sum by the length of the Cask: the product will be the content in wine gallons.

NOTE. If Wine Gallons be multiplied by 9, and divided by 11, the quotient will be the equivalent in Ale Gallons *very nearly.*

TABLE FOR HEAD AND BUNG DIAMETERS.

Diam. in Inches	·0	·1	·2	·3	·4
10	0·1133	0·1156	0·1179	0·1202	0·1226
11	0·1371	0·1396	0·1422	0·1447	0·1473
12	0·1632	0·1659	0·1687	0·1715	0·1743
13	0·1915	0·1945	0·1975	0·2005	0·2035
14	0·2221	0·2253	0·2285	0·2317	0·2350
15	0·2550	0·2584	0·2618	0·2653	0·2688
16	0·2901	0·2938	0·2974	0·3011	0·3048
17	0·3275	0·3314	0·3353	0·3392	0·3431
18	0·3672	0·3713	0·3754	0·3795	0·3837
19	0·4091	0·4134	0·4178	0·4221	0·4265
20	0·4533	0·4579	0·4624	0·4670	0·4716
21	0·4998	0·5046	0·5094	0·5142	0.5190
22	0·5485	0·5535	0·5585	0·5636	0·5687
23	0·5995	0·6047	0·6100	0·6153	0·6206
24	0·6528	0·6582	0·6637	0·6692	0·6747
25	0·7083	0·7140	0·7197	0·7254	0·7312
26	0·7661	0·7720	0·7780	0·7839	0·7899
27	0·8262	0·8323	0·8385	0·8447	0·8508
28	0·8885	0·8949	0·9013	0·9077	0·9141
29	0·9531	0·9597	0·9663	0·9729	0·9796
30	1·0200	1·0268	1·0336	1·0405	1·0474
31	1·0891	1·0962	1·1032	1·1103	1·1174
32	1·1605	1·1678	1·1751	1·1824	1·1897
33	1·2342	1·2417	1·2492	1·2567	1·2643
34	1·3101	1·3178	1·3256	1·3333	1·3411
35	1·3833	1·3963	1·4042	1·4122	1·4202

TABLE FOR HEAD AND BUNG DIAMETERS.

Diam. in Inches	5·	·6	·7	·8	·9
10	0·1249	0·1273	0·1297	0·1322	0·1346
11	0·1499	0·1525	0·1551	0·1578	0·1605
12	0·1771	0·1799	0·1828	0·1857	0·1886
13	0·2065	0·2096	0·2127	0·2158	0·2190
14	0·2383	0·2416	0·2449	0·2482	0·2516
15	0·2723	0·2758	0·2793	0·2829	0·2865
16	0·3085	0·3123	0·3161	0·3199	0·3237
17	0·3471	0·3511	0·3551	0·3591	0·3631
18	0·3879	0·3921	0·3963	0·4006	0·4048
19	0·4309	0·4354	0·4398	0·4443	0·4488
20	0·4763	0·4809	0·4856	0·4903	0·4950
21	0·5239	0·5288	0·5337	0·5386	0·5435
22	0·5737	0·5788	0·5840	0·5891	0·5943
23	0·6259	0·6312	0·6366	0·6420	0·6474
24	0·6803	0·5858	0·6914	0·6970	0·7027
25	0·7369	0·7427	0·7485	0·7544	0·7602
26	0·7959	0·8019	0·8079	0·8140	0·8201
27	0·8571	0·8633	0·8696	0·8759	0·8822
28	0·9205	0·9270	0·9335	0·9400	0·9466
29	0·9863	0·9930	0·9997	1·0064	1·0132
30	1·0543	1·0612	1·0681	1·0751	1·0821
31	1·1245	1·1317	1·1389	1·1461	1·1533
32	1·1971	1·2044	1·2118	1·2193	1·2267
33	1·2719	1·2795	1·2871	1·2948	1·3024
34	1·3489	1·3568	1·3646	1·3725	1·3804
35	1·4283	1·4363	1·4444	1·4525	1·4606

TABLE FOR HEAD AND BUNG DIAMETERS.

Diam. in Inches	·0	1·	·2	·3	·4
36	1·4688	1·4770	1·4852	1·4934	1·5016
37	1·5515	1·5599	1·5683	1·5768	1·5852
38	1·6365	1·6451	1·6538	1·6625	1·6712
39	1·7238	1·7326	1·7415	1·7504	1·7593
40	1·8133	1·8224	1·8315	1·8406	1·8498
41	1·9051	1·9144	1·9238	1·9331	1·9425
42	1·9992	2·0087	2·0183	2·0278	2·0374
43	2·0955	2·1053	2·1151	2·1249	2·1347
44	2·1941	2·2041	2·2141	2·2241	2·2342
45	2·2950	2·3052	2·3154	2·3257	2·3360
46	2·3981	2·4086	2·4190	2·4295	2·4400
47	2·5035	2·5142	2·5249	2·5356	2·5463
48	2·6112	2·6221	2·6330	2·6439	2·6549
49	2·7211	2·7322	2·7434	2·7545	2·7657
50	2·8333	2·8447	2·8560	2·8674	2·8788
51	2·9478	2·9594	2·9709	2·9826	2·9942
52	3·0645	3·0763	3·0881	3·1000	3·1119
53	3·1835	3·1955	3·2076	3·2197	3·2318
54	3·3048	3·3170	3·3293	3·3416	3·3539
55	3·4283	3·4408	3·4533	3·4658	3·4784
56	3·5541	3·5668	3·5795	3·5923	3·6051
57	3·6822	3·6951	3·7081	3·7210	3·7340
58	3·8125	3·8257	3·8389	3·8521	3·8653
59	3·9451	3·9585	3·9719	3·9853	3·9988
60	4·0800	....	....	....	....

TABLE FOR HEAD AND BUNG DIAMETERS.

Diam. in Inches	·5	·6	·7	·8	·9
36	1·5099	1.5182	1·5265	1·5348	1·5431
37	1·5937	1·6022	1·6108	1·6193	1·6279
38	1·6799	1·6886	1·6974	1·7062	1·7150
39	1·7683	1·7772	1·7862	1·7952	1·8043
40	1·8589	1·8681	1·8773	1·8866	1·8958
41	1·9519	1·9613	1·9707	1·9802	1·9897
42	2·0471	2·0567	2·0664	2·0761	2·0858
43	2·1445	2·1544	2·1643	2·1742	2·1842
44	2·2443	2·2544	2·2645	2·2746	2·2848
45	2·3463	2·3566	2·3669	2·3773	2·3877
46	2·4505	2·4611	2·4717	2·4823	2·4929
47	2·5571	2·5678	2·5786	2·5895	2·6003
48	2·6659	2·6769	2·6879	2·6989	2·7100
49	2·7769	2·7882	2·7994	2·8107	2·8220
50	2·8903	2·9017	2·9132	2·9247	2·9362
51	3·0059	3·0176	3·0293	3·0410	3·0527
52	3·1237	3·1356	3·1476	3·1595	3·1715
53	3·2439	3·2560	3·2682	3·2803	3·2926
54	3·3663	3·3786	3·3910	3·4034	3·4159
55	3·4909	3·5035	3·5161	3·5288	3·5414
56	3·6179	3·6307	3·6435	3·6564	3·6693
57	3·7471	3·7601	3·7732	3·7863	3·7994
58	3·8785	3·8918	3·9051	3·9184	3·9318
59	4·0123	4·0258	4·0393	4·0528	4·0664
60	....	....	....	....	....

EXAMPLE TO ILLUSTRATE THE USE OF THE TABLE.

Required the content of a Cask of the *First Variety*, in wine gallons, and in ale gallons, the head diameter being 30·3 inches, the bung diameter 40·5 inches, and the length of the Cask 50 inches.

SOLUTION.

Head diameter 30·3 inches... 1·0405 tabular number
Bung diameter 40·5 inches... 1·8589 tabular number
Repeat the last... 1·8589 tabular number

Sum 4·7583
Multiply by 50 length

Content in wine gallons 237·9150 *product.*

The content in ale gallons will be *nine-elevenths* of this content; that is,

237·9 wine gallons
Multiply by 9 according to rule

Divide by 11) 2141·1

Quotient 194·6 content in ale gallons.

OF ULLAGING.

When a Cask is partly filled, the vacuity or space unoccupied by the liquor is termed the DRY ULLAGE, and the content of liquor actually in the Cask is called the WET ULLAGE of the Cask.

By this definition it is evident that there will be two cases of Ullaging Casks; the *first* when the Cask is in a horizontal position, or lying on its bulge, and the *second* when the Cask stands on its head or end.

Of all the methods yet proposed for readily ascertaining the Ullage of a Cask in either of those positions, the best is by the lines of Segments on the Sliding Rule: all others are too intricate and tedious for practical use.

It ought here to be mentioned that a valuable set of Ullage Tables by BOYDELL are to be had, in which the Ullage of Casks may be discovered at once by inspection. It was our design to give a set of Tables of this description, though on a plan different from BOYDELL's, in the present work; the enormous extent, however, to which these Tables must have carried the work, would have rendered it so expensive and ponderous, as in a great measure to defeat its utility, and we have consequently with reluctance abandoned the intention: yet nothing essentially requisite in this branch of Cask Gaging shall be omitted.

PROBLEM I.

To find the Ullage of a Cask Lying.

In Ullaging any Cask it is necessary first to know the content of the Cask when full. This being known, measure the bung diameter with your gaging-rod, and observe the number of inches either wet or dry. Then set the bung-diameter on the segment-lying slide B, to 100 on the line SEGMENT LYING, and opposite to the number of wet inches on B, observe the number on the line SEGMENT LYING; which number reserve.

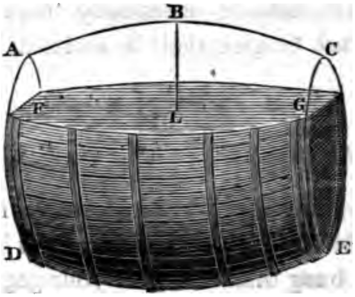
Again, set 100 on A, to the content of the Cask on B, and against the reserved number on A, will be the WET ULLAGE on B.

If the dry Ullage be sought, *dry* inches instead of wet, are in the first part of the operation to be taken on the slide B.

It is manifest that the dry and wet Ullages of any Cask must together be equal to the whole content of the Cask, and hence we have a sure means of proving the correctness of every Ullage.

EXAMPLE.

Let ABCDE be a Cask partly filled, whereof the whole content when full is 150 gallons, given to be ullaged on its bulge.



METHOD OF PROCEEDING.

With your gaging-rod measure the bung-diameter of the Cask, which let be 35 inches; and observe at the same time the wet inches on the rod, which let be 20: then BL, the dry inches, will be 15.

SOLUTION.

Set 35 on the Slide of SEG. LYING, to 100 on the Rule, and opposite to 20 on the Slide, will be seen 60 on the Rule, which reserve.

Then set 100 on A, to 150 on B, and opposite to 60 on A, will be found 90 on B, for the *wet ullage of the Cask.*

NOTE.

A Sliding Rule with two slides, every line on the Rule as well as on both slides being of double radius, is one of the best possible Tables for Ullaging. The first line is A, the first slide B; the second line is SL, the second slide, N; and the third and last line is SS. The line N of course slides between SL and SS, differing in its divisions nothing from the lines A and B, being a line of numbers in logarithmic ratio, like the lines A and B on all Sliding Rules.

PROOF OF THE LAST EXAMPLE.

Let us now prove the truth of the result of the Ullage in the last Cask, by setting 35 on the Slide of

Seg. Lying, to 100 on the Rule, and looking opposite to 15 on the Slide for 40 on the Rule, which we reserve.

Then let us set 100 on A, to 150 on B, and opposite to 40 on A, we shall find 60 on B for the *dry ullage of the Cask.*

But,
90 gallons wet
60 gallons dry
Sum 150 gallons, whole content.

Wherefore the operations have been rightly performed.

PROBLEM II.

To find the Ullage of a Cask Standing.

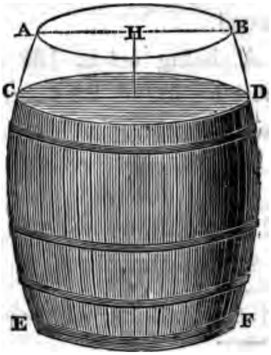
The whole content of the Cask being first known, ascertain the exact length, and also the wet inches, with your gaging-rod. Then set the length of the Cask on the segment-standing slide C, to 100 on the line Segment Standing, and opposite to the number of wet inches on C, observe the number on the line Segment Standing, which number reserve.

Again, set 100 on A, to the content of the Cask on B, and against the reserved number on A, will be the wet ullage on B.

This case evidently admits of the same method of proof as the first.

EXAMPLE.

Let ABCDEF be a Cask partly filled, whereof the whole content when full is 138 gallons, given to be ullaged standing.



METHOD OF PROCEEDING.

Measure the length of the Cask through the hole in the head, deducting for the thickness of the head, and let the nett length found be 48 inches: then observe on the rod the number of wet inches, which let be 36, and the dry inches will be 12.

SOLUTION.

Set 48 on the Slide of SS, to 100 on the Rule, and opposite to 36 on the Slide, you may perceive 83 on the Rule, which reserve.

Then set 100 on A, to 138 on B, and against 83 on A, will be found 114·6 on B, for the *wet ullage of the Cask.*

PROOF.

Set 48 on the Slide, to 100 on the line SS, and opposite to 12 on the Slide, may be seen 17·7 on the Rule, to be reserved.

Then 100 on A, being set to 138 on B, opposite to 17·7 on A, will be found 23·4 on B, for the *dry ullage of the Cask.*

But,

	114·6	gallons wet
	23·4	gallons dry
Sum	138·0	gallons, whole content.

By which it appears that the operations have been rightly performed.

PROBLEM III.

To Ullage a Cask Lying, by the Pen.

RULE.

Divide the number expressing the wet inches, by the bung diameter, and if the quotient be less than ·500, subtract from the quotient *one fourth* part the decimal number by which it falls short of ·500. Then multiply the *remainder* by the content of the cask, and the product will be the wet ullage.

If the quotient arising from the wet inches be greater than ·500, then *one fourth* part its excess above ·500, must be added to the quotient; and in this case the *sum* multiplied by the content of the Cask will be the wet ullage.

EXAMPLE.

Required the quantity of liquor in a Cask capable of holding 150 gallons, the bung diameter being 35 inches, and the wet dip on the bulge, 20 inches.

SOLUTION.

Bung Diam. Wet Inches Quotient
35) 20·00 (·571, which is more than ·500
Excess
Divide by 4) ·0710 (·018 one fourth the excess
Sum ·589
Multiply by 150 content of the cask
29450
589
Product 88·350 gallons, WET ULLAGE.

The data in this example being the same as in the example given in Prob. I., the same result may have been expected: but, by the pen, the result differs more than a gallon and a half from the ullage found by the Sliding Rule. We can here only remark that the results by the Pen and Rule seldom agree exactly in ullages.

If the dry ullage had been required, the operation would have been as follows:

FOR THE DRY ULLAGE.

Bung Diam. Dry Inches Quotient
35) 15·00 (·428, which is less than ·500
Defect
Divide by 4) ·0720 (·018 one fourth the defect
Remainder ·410
Multiply by 150 content of the cask
20500
410
61·500 gallons, DRY ULLAGE.

PROOF.

88·35 gallons wet
61·50 gallons dry
Sum 149·85 gallons, WHOLE CONTENT.

PROBLEM IV.

To Ullage a Cask Standing, by the Pen.

RULE.

Divide the number expressing the wet inches, by the length of the Cask, and if the quotient be less than ·500, subtract from the quotient *one-tenth* part the decimal number by which it falls short of ·500. Then multiply the *remainder* by the content of the Cask, and the product will be the wet ullage.

If the quotient arising from the wet inches be greater than ·500, then *one-tenth* part its excess above ·500 must be added to the quotient; and in this case the *sum* multiplied by the content of the Cask will be the wet ullage.

EXAMPLE.

A Cask, of which the length is 50 inches, capable of holding 120 gallons, being partly filled, and standing on its head, wets 20 inches of the dipping rod. The content of liquor in the Cask is required.

SOLUTION.

Length Wet Inches Quotient
50) 20·00 (·400, which is less than ·500
Defect
Divide by 4) ·1000 (·025 one fourth the defect
Remainder ·375
120
Product 45·000 gallons, WET ULLAGE.

If the dry ullage had been required, the operation would have been as follows:

FOR THE DRY ULLAGE.

Length Dry Inches Quotient
50) 30·00 (·600, which is more than ·500
Excess
Divide by 4) ·1000 (·025 one fourth the excess
Sum ·625
Multiply by 120 content of the cask
Product 75·000 gallons, DRY ULLAGE.

PROOF.

	45	gallons wet
	75	gallons dry
Sum	120	gallons, WHOLE CONTENT.

PROBLEM V.

To Inch a Standing Cask of the First Variety.

RULE.

From the area of the bung diameter subtract the area of the head diameter, and divide the remainder by three times the square of half the length of the Cask, reserving the quotient.

Then multiply the square of the proposed distance from the bung by the reserved quotient, and subtract the product from the area of the bung diameter. Lastly multiply this remainder by the distance from the bung, and the product will be the content of liquor in the Cask above the middle of the bung.

EXAMPLE.

The bung and head diameters of a Cask of the *First Variety* are respectively 30 inches, and 20 inches, the length of the Cask being 40 inches. The method of obtaining the content, in wine gallons, on every inch of the depth standing, is required.

SOLUTION.

	Inches		Area
Bung diameter	30	....	3·0600
Head diameter	20	....	1·3600

20 × 20 × 3 = 1200) 1·7000 (·001416 *quotient.*

Then,

For the first inch from the bung.

From the area of the bung diameter 3·060000
Subtract ·001416 × 1 × 1 = 0·001416

CONTENT of *one* inch from the bung 3·058584 W.G.

For the second inch from the bung.

From the area of the bung diameter 3·060000
Subtract ·001416 × 2 × 2 = 0·005664

Remainder 3·054336
Multiply by the distance from the bung 2

CONTENT of *two* inches from the bung 6·108672 W.G.

For the third inch from the bung.

From the area of the bung diameter 3·060000
Subtract ·001416 × 3 × 3 = 0·012744

Remainder 3·047256
Multiply by the distance from the bung 3

CONTENT of *three* inches from the bung 9·141768 W.G.

For the fourth inch from the bung.

From the area of the bung diameter	3·060000
Subtract ·001416×4×4 =	0·022656
Remainder	3·037344
Multiply by the distance from the bung	4
CONTENT of *four* inches from the bung	12·149376 W.G.

As soon as the content of the fourth inch has been found, the content of each of the remaining inches may be determined very easily by the method of differences; for if each content be subtracted from the content before it, the remainders will form a set of *First Differences:* and the differences of the first differences will be the *Second Differences:* in like manner, the differences of the second differences will be the *Third Differences.* But all the third differences will be equal; wherefore, by continually adding this difference to the first of the *second differences*, we shall have all the second differences: and by continually subtracting any second difference from the corresponding first difference, we shall obtain the next first difference. Lastly, by adding the first difference to the content before it, we get the next content, and in this manner we proceed through the upper half of the Cask. The content at every inch of this half being known, it follows that, by supposing the Cask reversed, we may obtain the content on every inch up to the bung by subtraction; and then by adding to this content the several contents first found, a true Table of the Cask for every wet inch of its depth will inevitably be the result; but the Cask may likewise be tabled without subtraction.

Table of Inches and Differences, for the Cask of the FIRST VARIETY *in the Example before us.*

Inche from the Bung	Contents	First Difference	Second Difference	Third Difference
1	3·0586	3·0501	·0170	·0085
2	6·1087	3·0331	·0255	·0085
3	9·1418	3·0076	·0340	·0085
4	12·1494	2·9730	·0425	·0085
5	15·1224	2·9305	·0510	·0085
6	18·0529	2·8795	·0595	·0085
7	20·9324	2·8200	·0680	·0085
8	23·7524	2·7520	·0765	·0085
9	26·5044	2·6755	·0850	·0085
10	29·1799	2·5905	·0935	·0085
11	31·7704	2·4970	·1020	·0085
12	34·2674	2·3950	·1105	·0085
13	36·6624	2·2855	·1190	·0085
14	38·9479	2·1665	·1275	·0085
15	41·1144	2·0390	·1360	·0085
16	43·1534	1·9030	·1445	·0085
17	45·0564	1·7585	·1530	·0085
18	46·8149	1·6055	·1615	....
19	48·4204	1·5440	....	....
20	49·9644	....	....	....

Having now found the content for every inch from the bung to the head, it will be easy to construct a Table of the Cask, by adding the first differences in succession up to the bung, and down again to the bottom, in the manner following.

A Cask of the First Variety Tabled.

Wet Inches	Content	Wet Inches	Content	Wet Inches	Content
1	1·5440 1·6055	14	31·9115 2·9305	27	70·8968 2·8200
2	3·1495 1·7585	15	34·8420 2·9730	28	73·7168 2·7520
3	4·9080 1·9030	16	37·8150 3·0076	29	76·4688 2·6755
4	6·8110 2·0390	17	40·8226 3·0331	30	79·1443 2·5905
5	8·8500 2·1665	18	43·8557 3·0501	31	81·7348 2·4970
6	11·0165 2·2855	19	46·9058 3·0586	32	84·2318 2·3950
7	13·3020 2·3950	20	49·9644 3·0586	33	86·6268 2·2855
8	15·6970 2·4970	21	53·0230 3·0501	34	88·9123 2·1665
9	18·1940 2·5905	22	56·0731 3·0331	35	91·0788 2·0390
10	20·7845 2·6755	23	59·1062 3·0076	36	93·1178 1·9030
11	23·4600 2·7520	24	62·1138 2·9730	37	95·0208 1·7585
12	26·2120 2·8200	25	65·0868 2·9305	38	96·7793 1·6055
13	29·0320 2·8795	26	68·0173 2·8795	39	98·3848 1·5440
14	31·9115	27	70·8968	40	99·9288

PROBLEM VI.

To Inch a Standing Cask of the Second Variety.

From the area of the bung diameter subtract the area of the head diameter, and multiply the remainder by ·3·5; then divide the product by 9 times the square of half the length, and reserve the quotient.

In all other respects proceed as directed for a Cask of the *First Variety*, given in the preceding Problem.

As we treated on a Cask of the *First Variety* at considerable length in the last Problem, and as the method of inching a Cask of the *Second Variety* differs nothing from that of the *First*, except in obtaining the reserved quotient or common multiplier, we do not think any further example necessary, and shall therefore conclude this part of the work with a Table of a common Beer Butt inched in round ale gallons.

OF A BEER BUTT.

A Beer Butt is a Cask of the *First Variety*, purposing to hold exactly 108 gallons, or 3 barrels of beer; and is, when properly made, 46 inches in length, 31 inches in diameter at the bung, and 24·7 inches in diameter at the head.

The content of a Cask of these dimensions, standing on its head, may, by the foregoing rules be determined for every wet inch of its depth. The content when reduced to the nearest whole gallon will be as follows, if the Butt hold exactly 3 barrels.

A BEER BUTT INCHED.

Wet Inches	Ale Gallons	Wet Inches	Ale Gallons	Wet Inches	Ale Gallons
1	2	17	38	33	80
2	4	18	41	34	83
3	5	19	43	35	85
4	6	20	46	36	87
5	9	21	49	37	90
6	12	22	51	38	92
7	14	23	54	39	94
8	16	24	57	40	96
9	18	25	59	41	99
10	21	26	62	42	101
11	23	27	65	43	103
12	25	28	67	44	105
13	28	29	70	45	106
14	30	30	73	46	108
15	33	31	75	...	......
16	36	32	78	...	......

USE OF THE CALLIPERS.

A complete set of inſtruments for taking the dimensions of any Cask correctly and with ease, are,

Two pair of Callipers, and
Two Dimension-Rods.

One pair of Callipers is for taking the bung diameter of a Cask, and being shorter than the pair employed for the length, is generally denominated the CROSS CALLIPERS, in contradistinction to the other pair which is termed the LONG CALLIPERS.

One of the dimension rods is called the BUNG-ROD, and is used in taking the bung diameter, as well as the wet inches of the cask lying.

The other rod is designated the HEAD ROD, and is used in ascertaining the diameter of the heads of Casks, and in computing the *content* and *ullage*.

METHOD OF USING THE INSTRUMENTS.

For a Cask Lying.

With the Cross Callipers take the horizontal bung diameter, allowing for the thickness of the staves, and with the bung-rod, take the perpendicular bung-diameter and wet inch.

Half the sum of the cross diameters must be considered the mean diameter of the Cask at the bung. This mean and the wet inch are now to be chalked on the Cask.

Then, with the head-rod take the diameters of the back and front head, and write half their sum also on the Cask with chalk, as the mean head diameter.

In using the head-rod the edge of the brass at the end of the rod is to be set within the chime of the Cask to be gaged; then, the upright brass on the slide is be moved to the middle of the chime diametrically opposite, and the diameter of the head of the Cask will be found on the lower line of the rod against the upright brass on the slide.

Next with the Long Callipers, allowing two inches for the thickness of the two heads, take the true length of the Cask externally.

Regarding, now, the form of the Cask, make such alterations in the dimensions found as shall appear to reduce the Cask to the middle frustum of a spheroid, if it be not of that form. This will best be done by adding a little to the length of the Cask according to circumstances, or by shortening the mean bung diameter by a few tenths. The dimensions thus altered (where necessary) are likewise to be chalked on the head of the Cask.

METHOD OF DETERMINING THE CONTENT.

To the mean head diameter on the stock, set the brass on the slide, and observe the number on the middle line of the slide, against the bung diameter on the stock, for opposite to the observed number on the lower line will stand the mean diameter of the Cask considered as a cylinder. Then set the brass pin or gage point at the end of the slide, to the length of the Cask on the stock, and look opposite to the mean diameter on the slide for the content of the Cask on the stock.

The Ullage, if required, must be found in the usual way, either by the lines of segments fully explained in PROBLEMS I. and II., of ULLAGING; or by the pen as explained in the Third and Fourth PROBLEMS of the same.

It is here proper to caution the young Gager against a species of deception practised in many, but especially in brandy Casks, namely, that of fixing a piece of wood an inch or two thick inside the Cask on the stave opposite to the bung, in order to render the bung diameter in appearance less than it really is. This wood is often so artfully fixed, as at first sight to deceive even officers of experience; but the deception may always be detected by taking the diameter of the Cask at the bung with the Callipers, and comparing the result with the internal diameter given by the bung-rod; or by piercing with a wood-gage the staves in various parts of the Cask, particularly opposite to the bung, and thus obtaining to great exactness the due thickness of the staves.

OF CASKS OF IRREGULAR FORM.

A Cask may be said to be of an irregular form when the cross bung-diameters differ considerably, or when owing either to a protuberance, or a flatness on one or more of the sides, the Cask appears out of shape. Irregular Casks may be divided into the four following Classes.

CLASS 1.

Casks exceedingly High-Quartered all Round.

A Cask is denominated high-quartered, when the diameter half way between the bung and head appears nearly equal to the bung diameter. In this case apply the Cross Callipers at the bung hole, or where the diameter of the Cask is the greatest; and from the Calliper gage, deduct two inches for the thickness of the staves. In like manner take another diameter of the Cask crossing the former at right angles. Then, if one of the bung-diameters exceeds the other, from 3 tenths of an inch to two inches and a half, add to the greater diameter from 5 to 12 or 14 tenths, according to the difference of the diameters by the Callipers; and call the greater diameter thus augmented the bung diameter of the Cask, which bung diameter must be chalked on the head: in like manner chalk on the head, the length of the Cask given by the Callipers without deduction.

Every Cask of irregular form should be rolled quite over, that its figure may be seen before it is gaged.

Sometimes the length of a Cask is greater on one side than on the other; that is, the length given by the Callipers at the top, differs considerably from the length at the bottom. In this event half the sum of the lengths must be considered the mean length, which will in general be averaging the distance of the centres of the heads.

Lastly, to the wet inches actually in the Cask, add the difference between the horizontal and perpendicular bung diameters, and from the sum subtract half an inch, or an inch' if the Cask be nearly full, remembering to chalk the remainder on the head for the corrected wet inches.

CLASS 2.

Casks Full-Quartered and Flattened.

A Cask is said to be full-quartered when the diameter half way between the bung and head is greater than it ought to be in the middle frustum of any spheroid, and to be flattened when the perpendicular bung diameter falls much short of the horizontal or cross diameter of the Cask at the middle. In a Cask of this description, take cross diameters at the bung, the one horizontal, the other perpendicular; and if the perpendicular diameter be less than the horizontal by an inch and upwards, subtract a few tenths from the horizontal diameter, according as the difference of the cross diameters is more or less, and call the remainder the bung diameter, which chalk on the head. Next find the wet inches, as directed for Casks of the first class, and let the length of the Cask without deduction, be likewise chalked on the head.

CLASS 3.

Casks Flattened and not sufficiently Curved.

When a Cask appears less curved from bung to head than the middle frustum of a spheroid ought to be, and is at the same time of less perpendicular diameter than horizontal diameter, take the dimensions as before directed for Casks of the first and second classes; and subtract from the horizontal diameter about one third its excess over the perpendicular diameter, and call the remainder the corrected bung diameter of the Cask, which chalk on the head. Also, from the length, by the Callipers, subtract half an inch, or three quarters of an inch, according to the flatness of the Cask, and other circumstances; always deducting more in long than in short Casks. In respect of the wet inches, they are to be corrected precisely as in the preceding cases.

CLASS 4.

Casks with High Quarters and Four Angles.

Brandy-Pieces and Puncheons are often found high-quartered, and at the same time flattened in two opposite directions, forming a kind of square at the bulge. In Casks of this description take with the Cross Callipers the greater diagonal of the two diagonals at the bulge, and add to it one or more tenths, according as the perpendicular diameter falls short of the horizontal diameter at the bung; then chalk the sum on the head of the Cask as the proper bung diameter. Chalk also the whole length given by the Callipers, without deduction.

In gaging Hogsheads and Pipes it is customary to take all the dimensions without abatement, as given by the instruments; but in every case an officer must be guided by his own judgment, as no fixed and determinate rule can be given so general as to apply to every particular instance that may occur. Cask Gaging, as we said before, is, of all other parts of an officer's duty, the most difficult, and to be learnt only by extensive and long practice.

OF STARCH GAGING.

Starch is generally made of the meal of corn, put into a Vat with a sufficient quantity of water, in which state fermentation is suffered to go on for seven or eight days, according to circumstances and the state of the weather. As soon as the fermentation is complete the finest part of the flour settles at the bottom of the Vat, from which it is removed in small quantities at a time, and placed in hair sieves, where it is washed to separate the Starch from the bran. In this process the fine matter runs through the sieves with the water into the Frames or Tubs, and the water thus impregnated with Starch is called a *Sour Water.*

In two or three days the Starch settles at the bottom of the Tub or Frame, and then the water is decanted off. The Starch is now washed with a fresh portion of water, and the washings containing the coarse slimy part of the meal or flour is put into another Frame, and denominated a *Slime.*

Clean water is now poured on the remaining pure sediment, and the whole is stirred and passed through a finer sieve into another Frame to settle, and in this state it is termed a *Green Water.*

When a *Green Water* has stood two or three days, the Starch deposits itself at the bottom of the Frame, where it is collected and boxed.

Sometimes the Starch which has settled at the bottom of a *Green Water* is again washed, when the water employed for this purpose is, when impregnated with Starch and decanted into a Tub or Frame, called a *White Water*.

As soon as the Starch at the bottom of a *Green Water* is thought sufficiently cleansed from impurities, it is taken out, and disposed of in Boxes with holes in the bottom to drain and harden; and when sufficiently hardened it is taken from the Boxes, and broken into pieces to be dried on bricks. After drying, it is put into a stove to crust, and the crusted part being afterwards scraped off, the pure part is papered, labelled, and stamped. In this state it is again placed in a stove to dry, and when dry is taken out and weighed in the presence of the officer appointed for that purpose.

The instruments used in Starch Gaging, are, besides a dimension-cane and tape, a spit inched and tenthed, a brass plate to screw to the end of the cane, and a spatula for examining the Starch in the utensils during the process of making.

PROBLEM I.

To Gage and Tabulate a Starch-Maker's Vat.

Take the dimensions precisely in the same way as directed for a Maltster's Cistern, and find the area in square inches. Then divide the area in inches, by 2828, the number of cubic inches into which a bushel of meal resolves itself on charging the Vat, and the product will be the area in bushels of the raw materials. The area in bushels, multiplied by the depth, will be the content in raw bushels. For a circular Vat, the square of the diameter divided by 3601 will give the area in bushels, without finding it in inches.

By experience it is ascertained, that every bushel of meal on an average gives 25 pounds of Green Starch, and consequently any number of bushels multiplied by 25, will give the pounds equivalent to that number of bushels.

EXAMPLE.

Given a rectangular Vat 200 inches in length, and 141·4 in breadth, to find the area in bushels, and in pounds.

SOLUTION.

141·4 breadth
200 length

2828) 28280·0 (10 *area in bushels.*
Multiply by 25 pounds in a bushel

Product 250 *area in pounds.*

This area multiplied by the depth of the Vat, whatever it may be, will give the whole content of the Vat in pounds of Green Starch.

TABLE OF DIVISORS AND GAGE POINTS FOR STARCH BUSHELS.

STATE OF THE MATERIALS.	BUSHEL DIVISORS.		BUSHEL GAGE POINTS	
	SQUARES	CIRCLES	SQUARES	CIRCLES
Before Fermentation	2828	3601	53·17	60·0
After Fermentation	2386	3038	48·8	55·1

A Starch-Maker's Vat is tabulated in the same manner as a Brewer's Copper, by continually subtracting the area from the whole content for dry inches, because the gage is always taken for the distance of the top of the settlement from the level of the mouth of the Vat.

It seems unnecessary here to give a specimen of a Dimension Book, or to enlarge further on this subject, as every officer, whose duty leads him to visit Starch-Makers, is furnished with ample instructions for his guidance in every particular.

PROBLEM II.

To Gage a Starch-Maker's Water Frame.

From several lengths in various parts of the Frame deduce a mean length; so in like manner a mean breadth, and a mean depth: then multiply the mean length by the mean breadth, and divide by the proper divisor, in the Table of Factors, Divisors, and Gage Points, for Green Starch pounds, given in page 142 of this work: the quotient will be the area of the Frame, and this multiplied by the depth will give the content.

A Table of the Frame may evidently be constructed by continually subtracting the area from the whole content of the *Frame;* but as the sediment in either a *Sour Water* or *Green Water* seldom reaches within 12 inches of the top of the Frame, it is customary to deduct 12 times the area from the content before the tabulation is begun. This deduction may be more conveniently made by taking away 12 inches from the depth of the Frame, for the area multiplied by the remainder will be the content of 12 dry inches.

To render this plainer, we shall suppose the length of a Water Frame to be 233·8 inches, the breadth 52 inches, and the depth 32 inches, the operation of finding the area and commencing the tabulation will be as under.

233·8 length of the frame
52

4676
11690

34·8) 12157·6 (349·3 *lb.* area in green starch
Depth 32 — 12 = 20

Product 6986 *lb.* content at 12 dry inches.

As the content at an inch deep is 349·3 lb. the content at half an inch will be 174·65 lb. Wherefore, a Table for every half inch beyond 12 inches may be constructed by continually subtracting 174·65 lb. from 5986lb. the content of the Frame at 12 dry inches. Thus,

A. B.'s Water Frame, No. 1, Tabled.

Dry Inches	Content in Green Starch Pounds
12·0	6986·00 174·65
12·5	6811·35 174·65
13·0	6636·70 174·65
13·5	6462·05 174·65
14·0	6287·40

In this manner may the tabulation be continued down to the bottom of the Frame.

PROBLEM III.

To Gage a Starch-Maker's Box with a Slider and Cloth.

Take several lengths of the box carefully with the Slider and Cloth in it; and from these deduce a mean length; then having removed the Slider and Cloth, take several lengths again, and find the mean length. The difference between the first and last mean length will be the allowance on the length to be made for the cloth; next, from several breadths, on each side of the Slider with the Cloth on, deduce a mean breadth, which double for the whole breadth of the Frame with the Slider and Cloth in: likewise ascertain the mean breadth without the Slider and Cloth. The difference of the *mean* breadths will be the allowance to be made for the Slider and Cloth. This allowance will in general be found to be 7 tenths of an inch; that is, *five tenths* for the Slider, and *two tenths* for the Cloth.

The area of the Box is now to be calculated in the usual way, both with and without the Slider, by multiplying the length by the breadth, and dividing the product by 34·8 the number of cubic inches in a pound of Green Starch. This area multiplied by the depth will evidently be the content of the Box in Green Starch pounds. Suppose the length of the Box to be 60·3 inches, and the breadth 11·8 inches with the Slider and Cloth, but 12·3 inches without the Slider, and the entry will be as follows:

A. B.'s STARCH BOX.

LENGTH.	BREADTH		AREA	
	With Slider and Cloth	Without Slider and Cloth	With Slider and Cloth	Without Slider and Cloth
60·3	11·8	12·3	20·44	21·30

A Table of the Box may be constructed by continually adding the area to itself for wet inches, as the mean depth of Starch in the Box is always deduced from the sum of several depths taken with the spit. The most useful Table, however, will be one similar to the Table of the Square Cistern in page 367, having the content for every tenth of an inch, but beginning in the Frame at 4 inches deep instead of at 10, as in the Cistern.

OF SPECIFIC GRAVITY.

By SPECIFIC GRAVITY is meant the relation which the weight of a given portion of any kind of body whatever bears to the weight of an equal bulk of any other body whatever.

Now to simplify the doctrine of Specific Gravity it was necessary to assume some body as the standard with which other bodies should be compared; and as soft water is in almost all places of nearly the same weight, it occurred that no other body could be a better or more universal standard than rain or soft water.

Hence, as the gravity or weight of any portion of water is always called 1, the specific gravity of a body twice as heavy as water will be 2, and the specific gravity of a body half as heavy as water will be ·5.

A knowledge of specific gravity is of great use in computing the weight of such bodies as by their size are too unwieldy to be weighed, or by their form too irregular to be measured.

METHOD OF FINDING THE SPECIFIC GRAVITIES OF BODIES.

Of a Solid heavier than Water.

Suspend the body from the arm of a balance by a fine thread, and weigh it accurately in air, then immersing the body in soft water, weigh it again by means of the same balance and thread, noting the weight lost by the immersion of the body in water. The weight in air, divided by the weight lost, will be the Specific Gravity of the Body immersed.

EXAMPLE.

A body weighs 9 drams in air, but when immersed in water it weighs only 8 drams: its specific gravity is required.

SOLUTION.

Weight lost 1 dr.) 9 dr. weight in air

Quotient 9 *specific gravity required.*

2. *Of a Solid lighter than Water.*

Affix to the light body something sufficiently ponderous to cause the whole to sink in water. Then weigh the compound mass and heavier body separately, both in and out of water, carefully noting the weight each loses by immersion. Next, from the greater loss subtract the less, and divide the weight of the light body in air, by the remainder: the quotient will be the Specific Gravity required.

EXAMPLE.

A piece of light substance weighing by itself 3 drams in air, weighs with a portion of metal attached to it, 10 drams in air, and 4·4 drams in water: also the metal by itself weighs 7 drams in air, and 6·2 drams in water. Required the Specific Gravity of the lighter body.

SOLUTION.

COMPOUND		HEAVIER BODY.	
In air	10·0 drs.	In air	7·0 drs.
In water	4·4 drs.	In water	6·2 drs.
Loss of the compound	5·6 drs.	Loss of the heavy body	·8 drs.

But 5·6 — ·8 = 4·8, difference of the losses.

And,

Difference 4·8)3(·625 specific gravity of the light body.

3. *For the Specific Gravity of a Fluid.*

Weigh any solid substance that will sink in the given Fluid, first in air, and then in the Fluid, noting the loss of weight in the body weighed. If this loss be divided by the weight of the heavy body in air, a quotient will arise, which multiplied by the Specific Gravity of the solid body, will be the Specific Gravity of the Fluid.

EXAMPLE.

Required the Specific Gravity of a Fluid in which a solid body (whereof the Specific Gravity is 4) weighs

4·6 drams, the same solid body weighing in air 5 drams.

SOLUTION.

Weight in air.........	5·0	drams
Weight in the fluid..	4·6	drams
Difference............	·4	drams

And

Weight in air 5) ·4 difference

Quotient ·08

Multiply by 4 specific gravity of the solid

Product ·32 *specific gravity required.*

It is a remarkable fact that a cubic foot of soft water weighs exactly 1000 ounces avoirdupois: and hence the Specific Gravity of any body multiplied by 1000 will be the weight of a cubic foot of that body.

And as 1728 cubic inches constitute a cubic foot, it follows that the 1728th part of the weight of a cubic foot of any body, must be the weight of a cubic inch of the same body. Wherefore 1000 ounces divided by 1728, will be the weight of a cubic inch of soft water.

We shall now subjoin a Table of the Specific Gravities of a few Solids as well as Fluids, and afterwards a Table of the weight of a cubic inch of various substances in ounces Troy and Avoirdupois.

TABLE OF SPECIFIC GRAVITIES.

SOLIDS	Specific Gravity	FLUIDS	Specific Gravity
Pure platina	23·0000	Quicksilver	14·0000
Fine gold	19·6400	Sulphuric acid	1·7000
Standard gold	18·8879	Honey	1·4500
Lead	11·3400	Aquafortis	1·3000
Fine silver	11·0920	Treacle	1·2900
Standard silver	10·5360	Milk	1·0310
Copper	9·0000	Sea water	1·0300
Fine brass	8·3500	Ale	1·0280
Steel	7·8500	Vinegar	1·0260
Bar iron	7·6440	Tar	1·0150
Pewter	7·4710	Rain water	1·0000
Cast iron	7·4250	Port wine	0·9900
Block tin	7·3200	Linseed oil	0·9320
Diamond	3·5170	Proof spirit	0·9270
Green glass	2·6000	Olive oil	0·9130
Box wood	1·0300	Spirit of wine	0·8660
Bees-wax	0·9550	Oil of turpentine	0·8100
Elm	0·6000	Ether	0·7260
Cork	0·2380	Common air	0·0012

TABLE OF THE WEIGHT OF A CUBIC INCH OF VARIOUS BODIES.

NAMES.	Weight in Oz.		NAMES.	Weight in Oz.	
	Troy	Avoird.		Troy	Avoird.
Fine Gold	10·3592	11·3656	Common Glass	1·3608	1·4930
Standard Gold	9·9626	10·9804	Dry Ivory	·9621	1·0555
Quicksilver	7·3844	8·1017	Boxwood	·5629	·5960
Lead	5·9840	6·5538	Milk	·5438	·5893
Fine Silver	5.8500	6·4183	Sea Water	·5427	·5949
Standard Silver	5·5567	6·0966	Claret	·5237	·5746
Rose Copper	4·7475	5·2083	Linseed Oil	·4916	·5393
Steel	4·1421	4·5445	Proof Spirit	·4890	·5368
Wrought Iron	4·0313	4·4229	Dry Oak	·4893	·5365
Block Tin	3·8615	4·2366	Olive Oil	·4816	·5283
Fine Marble	1·4294	1·5688	Turpentine Oil	·3837	·4158

The following examples will illustrate the use of these Tables.

EXAMPLE 1.

Required the weight of 10 cubic inches of wrought iron, in ounces Troy.

SOLUTION.

4·0313 weight of a cubic inch
Multiply by 10 number of inches
Product 40·313 oz. troy, *Answer.*

EXAMPLE 2.

The content of an iron chain, weighing 107 ounces avoirdupois, is required in cubic inches.

SOLUTION.

Tab.No.	Given Weight	Quotient
4·4229)	107·0000	(24·19 *cubic inches required.*

After the manner of the last example, the content in wine gallons of such incumbrances in a Distiller's Wash Still as cannot be exactly determined by the Problems in Mensuration, may at once be ascertained; but in treating of a Still, we gave another practical mode of finding the content to the greatest nicety. [*See page* 354.]

The easiest method of determining the Specific Gravity of any Fluid is by the Hydrometer, although some difficulty accompanies its use, owing to the change produced in the Specific Gravity of a Fluid by difference of temperature.

As the strength of spirituous liquors is known by their Specific Gravity, and as the Specific Gravity of all Fluids is more readily ascertained by an Hydrometer than by any other means, it has been established by Act of Parliament that the HYDROMETER shall be the instrument employed by the Officers of the Revenue in finding the *per-centage* over and under proof of all spirituous liquors whatever. SYKES's Hydrometer, with copious directions for its use, and printed Tables, shewing the *per-centage* by inspection, may be had of MR. BATE, in the Poultry, Mathematical Instrument Maker to the Board of Excise.

There is likewise to be had an Hydrometer improved by CLARKE, and which many prefer to SYKES's.

USE OF THE HYDROMETER.

It has already been remarked that Fluids vary a little in their Specific Gravities by a change of temperature, and hence, as a difference in the Specific Gravity argues a difference in the strength, it is always requisite to use a THERMOMETER with the HYDROMETER.

Also, in taking samples for trying the strength of spirits, it is necessary in Store-Pieces and large Vats, to make use of sinking vials, in order to get the sample from the middle of the liquor in the Piece or Vat, because a sample at the top may differ very considerably in strength from a sample at the bottom. In small vessels and Casks this difference is too trifling to deserve notice.

METHOD OF ASCERTAINING THE STRENGTH OF SPIRITUOUS LIQUORS.

Let a *Glass Tube* be nearly filled with the liquor to be tried, and into the Tube introduce a THERMOMETER, with which agitate the liquor till the mercury becomes stationary: hereupon observe the temperature indicated, always taking the nearest degree, when the mercury rests between two degrees.

Next, wet the HYDROMETER up to the Cipher on the stem, in the sample to be tried; then chusing one of the hydrometer-weights, place it on the spindle, so as to rest on the lower bulb of the instrument. Returning the Hydrometer now into the sample, taking care to remove all air bubbles, press it gently down till the cipher on the stem is level with the surface of the liquor; after this, suffer the instrument to rise gradually and to assume a state of rest.

When the Hydrometer appears stationary, observe the sub-division on the stem corresponding to the surface of the sample. If the surface is not exactly at any division or sub-division, the nearest mark on the stem under the surface must be considered the surface-mark.

The division corresponding with the surface of the sample is next to be added to the weight used, and the sum will be the *Indication.*

Then to find the strength by the Tables, look for the temperature given by the thermometer, and in the margin seek the *Indication.* Under the temperature, and in the same line as the *Indication*, will be the PER CENTAGE of strength, the numbers above the zig-zag being *Over Proof*, and those below it *Under Proof.*

PROBLEM.

To calculate the Quantity of Proof Spirit in Spirits of any other Strength.

CASE 1.

FOR OVER PROOF SPIRITS.

Multiply the content or actual quantity by the *percentage* Over Proof, and divide the product by 100; then add the quotient to the actual content gaged, and the sum will be the quantity of Proof Spirit.

EXAMPLE.

If a puncheon of rum contain by gage 126 gallons of strength 18·5 per cent Over Proof, what is the quantity of Proof Spirit in the Cask?

	126	gallons per gage
	18·5	strength over proof
	630	
	1008	
	126	
100)	2331·0	product
Quotient	23·31	quantity over proof
Add	126·00	gallons per gage
Sum	149·31	GALLONS PROOF, *Answer.*

CASE 2.

FOR UNDER PROOF SPIRITS.

Multiply the gage by the *per-centage* Under Proof, and divide the quotient by 100; then from the content by gage deduct the quotient, and the remainder will be the quantity of Proof Spirit sought.

EXAMPLE.

If the quantity of brandy in a Cask be by gage 60 gallons of strength 12 per cent Under Proof, what is the actual quantity of Proof Spirit in the Cask?

60 gallons per gage
12 strength under proof
100) 720 product
Quotient 7·2 to be subtracted
From 60·0 gallons per gage
Remainder 52·8 GALLONS PROOF, *Answer.*

As ample directions for the use of the Hydrometer are always given as an accompaniment to it, and as the instructions to the different Officers visiting Distillers and Rectifiers, explain at large every thing connected with their duty not mentioned in this book, we shall now conclude our work with Tables for moneying the charges on all Exciseable Commodities, referring the reader to Jonas's Excise Laws for abstracts of the several Acts of Parliament relating immediately to this subject.

TABLE

OF THE

CLEAR DUTIES OF EXCISE,

ON

STRONG BEER AND TABLE BEER,

Payable by the Common Brewers of London, and in the Country; with the Malt Allowances deducted from the original duties, as by the latest Act of Parliament for regulating the duty on beer.

Barrels	Strong			Table			Barrels	Strong			Table		
	£.	*s.*	*d.*	*£.*	*s.*	*d.*		*£.*	*s.*	*d.*	*£.*	*s.*	*d.*
1	0	9	2	0	1	10	26	11	18	4	2	7	8
2	0	18	4	0	3	8	27	12	7	6	2	9	6
3	1	7	6	0	5	6	28	12	16	8	2	11	4
4	1	16	8	0	7	4	29	13	5	10	2	13	2
5	2	5	10	0	9	2	30	13	15	0	2	15	0
6	2	15	0	0	11	0	31	14	4	2	2	16	10
7	3	4	2	0	12	10	32	14	13	4	2	18	8
8	3	13	4	0	14	8	33	15	2	6	3	0	6
9	4	2	6	0	16	6	34	15	11	8	3	2	4
10	4	11	8	0	18	4	35	16	0	10	3	4	2
11	5	0	10	1	0	2	36	16	10	0	3	6	0
12	5	10	0	1	2	0	37	16	19	2	3	7	10
13	5	19	2	1	3	10	38	17	8	4	3	9	8
14	6	8	4	1	5	8	39	17	17	6	3	11	6
15	6	17	6	1	7	6	40	18	6	8	3	13	4
16	7	6	8	1	9	4	41	18	15	10	3	15	2
17	7	15	10	1	11	2	42	19	5	0	3	17	0
18	8	5	0	1	13	0	43	19	14	2	3	18	10
19	8	14	2	1	14	10	44	20	3	4	4	0	8
20	9	3	4	1	16	8	45	20	12	6	4	2	6
21	9	12	6	1	18	6	46	21	1	8	4	4	4
22	10	1	8	2	0	4	47	21	10	10	4	6	2
23	10	10	10	2	2	2	48	22	0	0	4	8	0
24	11	0	0	2	4	0	49	22	9	2	4	9	10
25	11	9	2	2	5	10	50	22	18	4	4	11	8

TABLE OF THE CLEAR DUTIES ON STRONG AND TABLE BEER.

Barrels	Strong			Table			Barrels	Strong			Table		
	£.	s.	d.	£.	s.	d.		£.	s.	d.	£.	s.	d.
51	23	7	6	4	13	6	86	39	8	4	7	17	8
52	23	16	8	4	15	4	87	39	17	6	7	19	6
53	24	5	10	4	17	2	88	40	6	8	8	1	4
54	24	15	0	4	19	0	89	40	15	10	8	3	2
55	25	4	2	5	0	10	90	41	5	0	8	5	0
56	25	13	4	5	2	8	91	41	14	2	8	6	10
57	26	2	6	5	4	6	92	42	3	4	8	8	8
58	26	11	8	5	6	4	93	42	12	6	8	10	6
59	27	0	10	5	8	2	94	43	1	8	8	12	4
60	27	10	0	5	10	0	95	43	10	10	8	14	2
61	27	19	2	5	11	10	96	44	0	0	8	16	0
62	28	8	4	5	13	8	97	44	9	2	8	17	10
63	28	17	6	5	15	6	98	44	18	4	8	19	8
64	29	6	8	5	17	4	99	45	7	6	9	1	6
65	29	15	10	5	19	2	100	45	16	8	9	3	4
66	30	5	0	6	1	0	101	46	5	10	9	5	2
67	30	14	2	6	2	10	102	46	15	0	9	7	0
68	31	3	4	6	4	8	103	47	4	2	9	8	10
69	31	12	6	6	6	6	104	47	13	4	9	10	8
70	32	1	8	6	8	4	105	48	2	6	9	12	6
71	32	10	10	6	10	2	106	48	11	8	9	14	4
72	33	0	0	6	12	0	107	49	0	10	9	16	2
73	33	9	2	6	13	10	108	49	10	0	9	18	0
74	33	18	4	6	15	8	109	49	19	2	9	19	10
75	34	7	6	6	17	6	110	50	8	4	10	1	8
76	34	16	8	6	19	4	111	50	17	6	10	3	6
77	35	5	10	7	1	2	112	51	6	8	10	5	4
78	35	15	0	7	3	0	113	51	15	10	10	7	2
79	36	4	2	7	4	10	114	52	5	0	10	9	0
80	36	13	4	7	6	8	115	52	14	2	10	10	10
81	37	2	6	7	8	6	116	53	3	4	10	12	8
82	37	11	8	7	10	4	117	53	12	6	10	14	6
83	38	0	10	7	12	2	118	54	1	8	10	16	4
84	38	10	0	7	14	0	119	54	10	10	10	18	2
85	38	19	2	7	15	10	120	55	0	0	11	0	0

TABLE OF THE CLEAR DUTIES ON STRONG AND TABLE BEER.

Barrels	Strong			Table			Barrels	Strong			Table		
	£.	s.	d.	£.	s.	d.		£.	s.	d.	£.	s.	d.
121	55	9	2	11	1	10	156	71	10	0	14	6	0
122	55	18	4	11	3	8	157	71	19	2	14	7	10
123	56	7	6	11	5	6	158	72	8	4	14	9	8
124	56	16	8	11	7	4	159	72	17	6	14	11	6
125	57	5	10	11	9	2	160	73	6	8	14	13	4
126	57	15	0	11	11	0	161	73	15	10	14	15	2
127	58	4	2	11	12	10	162	74	5	0	14	17	0
128	58	13	4	11	14	8	163	74	14	2	14	18	10
129	59	2	6	11	16	6	164	75	3	4	15	0	8
130	59	11	8	11	18	4	165	75	12	6	15	2	6
131	60	0	10	12	0	2	166	76	1	8	15	4	4
132	60	10	0	12	2	0	167	76	10	10	15	6	2
133	60	19	2	12	3	10	168	77	0	0	15	8	0
134	61	8	4	12	5	8	169	77	9	2	15	9	10
135	61	17	6	12	7	6	170	77	18	4	15	11	8
136	62	6	8	12	9	4	171	78	7	6	15	13	6
137	62	15	10	12	11	2	172	78	16	8	15	15	4
138	63	5	0	12	13	0	173	79	5	10	15	17	2
139	63	14	2	12	14	10	174	79	15	0	15	19	0
140	64	3	4	12	16	8	175	80	4	2	16	0	10
141	64	12	6	12	18	6	176	80	13	4	16	2	8
142	65	1	8	13	0	4	177	81	2	6	16	4	6
143	65	10	10	13	2	2	178	81	11	8	16	6	4
144	66	0	0	13	4	0	179	82	0	10	16	8	2
145	66	9	2	13	5	10	180	82	10	0	16	10	0
146	66	18	4	13	7	8	181	82	19	2	16	11	10
147	67	7	6	13	9	6	182	83	8	4	16	13	8
148	67	16	8	13	11	4	183	83	17	6	16	15	6
149	68	5	10	13	13	2	184	84	6	8	16	17	4
150	68	15	0	13	15	0	185	84	15	10	16	19	2
151	69	4	2	13	16	10	186	85	5	0	17	1	0
152	69	13	4	13	18	8	187	85	14	2	17	2	10
153	70	2	6	14	0	6	188	86	3	4	17	4	8
154	70	11	8	14	2	4	189	86	12	6	17	6	6
155	71	0	10	14	4	2	190	87	1	8	17	8	4

TABLE OF THE CLEAR DUTIES ON STRONG AND TABLE BEER.

Barrels	Strong			Table			Barrels	Strong			Table		
	£.	s.	d.	£.	s.	d.		£.	s.	d.	£.	s.	d.
191	87	10	10	17	10	2	226	103	11	8	20	14	4
192	88	0	0	17	12	0	227	104	0	10	20	16	2
193	88	9	2	17	13	10	228	104	10	0	20	18	0
194	88	18	4	17	15	8	229	104	19	2	20	19	10
195	89	7	6	17	17	6	230	105	8	4	21	1	8
196	89	16	8	17	19	4	231	105	17	6	21	3	6
197	90	5	10	18	1	2	232	106	6	8	21	5	4
198	90	15	0	18	3	0	233	106	15	10	21	7	2
199	91	4	2	18	4	10	234	107	5	0	21	9	0
200	91	13	4	18	6	8	235	107	14	2	21	10	10
201	92	2	6	18	8	6	236	108	3	4	21	12	8
202	92	11	8	18	10	4	237	108	12	6	21	14	6
203	93	0	10	18	12	2	238	109	1	8	21	16	4
204	93	10	0	18	14	0	239	109	10	10	21	18	2
205	93	19	2	18	15	10	240	110	0	0	22	0	0
206	94	8	4	18	17	8	241	110	9	2	22	1	10
207	94	17	6	18	19	6	242	110	18	4	22	3	8
208	95	6	8	19	1	4	243	111	7	6	22	5	6
209	95	15	10	19	3	2	244	111	16	8	22	7	4
210	96	5	0	19	5	0	245	112	5	10	22	9	2
211	96	14	2	19	6	10	246	112	15	0	22	11	0
212	97	3	4	19	8	8	247	113	4	2	22	12	10
213	97	12	6	19	10	6	248	113	13	4	22	14	8
214	98	1	8	19	12	4	249	114	2	6	22	16	6
215	98	10	10	19	14	2	250	114	11	8	22	18	4
216	99	0	0	19	16	0	251	115	0	10	23	0	2
217	99	9	2	19	17	10	252	115	10	0	23	2	0
218	99	18	4	19	19	8	253	115	19	2	23	3	10
219	100	7	6	20	1	6	254	116	8	4	23	5	8
220	100	16	8	20	3	4	255	116	17	6	23	7	6
221	101	5	10	20	5	2	256	117	6	8	23	9	4
222	101	15	0	20	7	0	257	117	15	10	23	11	2
223	102	4	2	20	8	10	258	118	5	0	23	13	0
224	102	13	4	20	10	8	259	118	14	2	23	14	10
225	103	2	6	20	12	6	260	119	3	4	23	16	8

TABLE OF THE CLEAR DUTIES ON STRONG AND TABLE BEER.

Barrels	Strong			Table			Barrels	Strong			Table		
	£.	s.	d.	£.	s.	d.		£.	s.	d.	£.	s.	d.
261	119	12	6	23	18	6	294	134	15	0	26	19	0
262	120	1	8	24	0	4	295	135	4	2	27	0	10
263	120	10	10	24	2	2	296	135	13	4	27	2	8
264	121	0	0	24	4	0	297	136	2	6	27	4	6
265	121	9	2	24	5	10	298	136	11	8	27	6	4
266	121	18	4	24	7	8	299	137	0	10	27	8	2
267	122	7	6	24	9	6	300	137	10	0	27	10	0
268	122	16	8	24	11	4	400	183	6	8	36	13	4
269	123	5	10	24	13	2	500	229	3	4	45	16	8
270	123	15	0	24	15	0	600	275	0	0	55	0	0
271	124	4	2	24	16	10	700	320	16	8	64	3	4
272	124	13	4	24	18	8	800	366	13	4	72	6	8
273	125	2	6	25	0	6	900	412	10	0	82	10	0
274	125	11	8	25	2	4	1000	458	6	8	91	13	4
275	126	0	10	25	4	2	2000	916	13	4	183	6	8
276	126	10	0	25	6	0	3000	1375	0	0	275	0	0
277	126	19	2	25	7	10	4000	1833	6	8	366	13	4
278	127	8	4	25	9	8	5000	2291	13	4	458	6	8
279	127	17	6	25	11	6	6000	2750	0	0	550	0	0
280	128	6	8	25	13	4	7000	3208	6	8	641	13	4
281	128	15	10	25	15	2	8000	3666	13	4	733	6	8
282	129	5	0	25	17	0	9000	4125	0	0	825	0	0
283	129	14	2	25	18	10	10000	4583	6	8	916	13	4
284	130	3	4	26	0	8	20000	9166	13	4	1833	6	8
285	130	12	6	26	2	6	30000	13750	0	0	2750	0	0
286	131	1	8	26	4	4	40000	18333	6	8	3666	13	4
287	131	10	10	26	6	2	50000	22916	13	4	4583	6	8
288	132	0	0	26	8	0	60000	27500	0	0	5500	0	0
289	132	9	2	26	9	10	70000	32083	6	8	6416	13	4
290	132	18	4	26	11	8	80000	36666	13	4	7333	6	8
291	133	7	6	26	13	6	90000	41250	0	0	8250	0	0
292	133	16	8	26	15	4	100000	45833	6	8	9166	13	4
293	134	5	10	26	17	2	200000	91666	13	4	18333	6	8

TABLE

Of the Duty on Malt at 3*s*. 6*d*. a bushel, as settled by the latest ACT on the subject.

Bushels	£.	*s*.	*d*.	Bushels	£.	*s*.	*d*.
1	0	3	6	33	5	15	6
2	0	7	0	34	5	19	0
3	0	10	6	35	6	2	6
4	0	14	0	36	6	6	0
5	0	17	6	37	6	9	6
6	1	1	0	38	6	13	0
7	1	4	6	39	6	16	6
8	1	8	0	40	7	0	0
9	1	11	6	41	7	3	6
10	1	15	0	42	7	7	0
11	1	18	6	43	7	10	6
12	2	2	0	44	7	14	0
13	2	5	6	45	7	17	6
14	2	9	0	46	8	1	0
15	2	12	6	47	8	4	6
16	2	16	0	48	8	8	0
17	2	19	6	49	8	11	6
18	3	3	0	50	8	15	0
19	3	6	6	51	8	18	6
20	3	10	0	52	9	2	0
21	3	13	6	53	9	5	6
22	3	17	0	54	9	9	0
23	4	0	6	55	9	12	6
24	4	4	0	56	9	16	0
25	4	7	6	57	9	19	6
26	4	11	0	58	10	3	0
27	4	14	6	59	10	6	6
28	4	18	0	60	10	10	0
29	5	1	6	61	10	13	6
30	5	5	0	62	10	17	0
31	5	8	6	63	11	0	6
32	5	12	0	64	11	4	0

Of the Duty on Malt at 3*s.* 6*d.* a bushel, as settled by the latest Act on the subject.

Bushels	£.	*s.*	*d.*	Bushels	£.	*s.*	*d.*
65	11	7	6	97	16	19	6
66	11	11	0	98	17	3	0
67	11	14	6	99	17	6	6
68	11	18	0	100	17	10	0
69	12	1	6	101	17	13	6
70	12	5	0	102	17	17	0
71	12	8	6	103	18	0	6
72	12	12	0	104	18	4	0
73	12	15	6	105	18	7	6
74	12	19	0	106	18	11	0
75	13	2	6	107	18	14	6
76	13	6	0	108	18	18	0
77	13	9	6	109	19	1	6
78	13	13	0	110	19	5	0
79	13	16	6	111	19	8	6
80	14	0	0	112	19	12	0
81	14	3	6	113	19	15	6
82	14	7	0	114	19	19	0
83	14	10	6	115	20	2	6
84	14	14	0	116	20	6	0
85	14	17	6	117	20	9	6
86	15	1	0	118	20	13	0
87	15	4	6	119	20	16	6
88	15	8	0	120	21	0	0
89	15	11	6	121	21	3	6
90	15	15	0	122	21	7	0
91	15	18	6	123	21	10	6
92	16	2	0	124	21	14	0
93	16	5	6	125	21	17	6
94	16	9	0	126	22	1	0
95	16	12	6	127	22	4	6
96	16	16	0	128	22	8	0

TABLE

Of the Duty on Malt at 3*s.* 6*d.* a bushel, as settled by the latest ACT on the subject.

Bushels	£.	*s.*	*d.*	Bushels	£.	*s.*	*d.*
129	22	11	6	161	28	3	6
130	22	15	0	162	28	7	0
131	22	18	6	163	28	10	6
132	23	2	0	164	28	14	0
133	23	5	6	165	28	17	6
134	23	9	0	166	29	1	0
135	23	12	6	167	29	4	6
136	23	16	0	168	29	8	0
137	23	19	6	169	29	11	6
138	24	3	0	170	29	15	0
139	24	6	6	171	29	18	6
140	24	10	0	172	30	2	0
141	24	13	6	173	30	5	6
142	24	17	0	174	30	9	0
143	25	0	6	175	30	12	6
144	25	4	0	176	30	16	0
145	25	7	6	177	30	19	6
146	25	11	0	178	31	3	0
147	25	14	6	179	31	6	6
148	25	18	0	180	31	10	0
149	26	1	6	181	31	13	6
150	26	5	0	182	31	17	0
151	26	8	6	183	32	0	6
152	26	12	0	184	32	4	0
153	26	15	6	185	32	7	6
154	26	19	0	186	32	11	0
155	27	2	6	187	32	14	6
156	27	6	0	188	32	18	0
157	27	9	6	189	33	1	6
158	27	13	0	190	33	5	0
159	27	16	6	191	33	8	6
160	28	0	0	192	33	12	0

Of the Duty on Malt at 3*s*. 6*d*. a bushel, as settled by the latest ACT on the subject.

Bushels	£.	*s*.	*d*.	Bushels	£.	*s*.	*d*.
193	33	15	6	225	39	7	6
194	33	19	0	226	39	11	0
195	34	2	6	227	39	14	6
196	34	6	0	228	39	18	0
197	34	9	6	229	40	1	6
198	34	13	0	230	40	5	0
199	34	16	6	231	40	8	6
200	35	0	0	232	40	12	0
201	35	3	6	233	40	15	6
202	35	7	0	234	40	19	0
203	35	10	6	235	41	2	6
204	35	14	0	236	41	6	0
205	35	17	6	237	41	9	6
206	36	1	0	238	41	13	0
207	36	4	6	239	41	16	6
208	36	8	0	240	42	0	0
209	36	11	6	241	42	3	6
210	36	15	0	242	42	7	0
211	36	18	6	243	42	10	6
212	37	2	0	244	42	14	0
213	37	5	6	245	42	17	6
214	37	9	0	246	43	1	0
215	37	12	6	247	43	4	6
216	37	16	0	248	43	8	0
217	37	19	6	249	43	11	6
218	38	3	0	250	43	15	0
219	38	6	6	251	43	18	6
220	38	10	0	252	44	2	0
221	38	13	6	253	44	5	6
222	38	17	0	254	44	9	0
223	39	0	6	255	44	12	6
224	39	4	0	256	44	16	0

TABLE

Of the Duty on Malt at 3*s*. 6*d*. a bushel, as settled by the latest ACT on the subject.

Bushels	£.	*s*.	*d*.	Bushels	£.	*s*.	*d*.
257	44	19	6	289	50	11	6
258	45	3	0	290	50	15	0
259	45	6	6	291	50	18	6
260	45	10	0	292	51	2	0
261	45	13	6	293	51	5	6
262	45	17	0	294	51	9	0
263	46	0	6	295	51	12	6
264	46	4	0	296	51	16	0
265	46	7	6	297	51	19	6
266	46	11	0	298	52	3	0
267	46	14	6	299	52	6	6
268	46	18	0	300	52	10	0
269	47	1	6	310	54	5	0
270	47	5	0	320	56	0	0
271	47	8	6	330	57	15	0
272	47	12	0	340	59	10	0
273	47	15	6	350	61	5	0
274	47	19	0	360	63	0	0
275	48	2	6	370	64	15	0
276	48	6	0	380	66	10	0
277	48	9	6	390	68	5	0
278	48	13	0	400	70	0	0
279	48	16	6	500	87	10	0
280	49	0	0	600	105	0	0
281	49	3	6	700	122	10	0
282	49	7	0	800	140	0	0
283	49	10	6	900	157	10	0
284	49	14	0	1000	175	0	0
285	49	17	6	2000	350	0	0
286	50	1	0	3000	525	0	0
287	50	4	6	4000	700	0	0
288	50	8	0	5000	875	0	0

Of the Square Yards and Duty at 1¾*d.* per yard, for any number of Pieces of Paper, from 1 to 3000, each Piece being 12 yards in length, and ·58 yards broad.

No. of Pieces	Square Yards	Duty for whole Yards			No. of Pieces	Square Yards	Duty for whole Yards		
		£.	*s.*	*d.*			£.	*s.*	*d.*
1	6·96	0	0	10½	29	201·84	1	9	3¾
2	13·92	0	1	10¾	30	208·80	1	10	4
3	20·88	0	2	11	31	215·76	1	11	4¼
4	27·84	0	3	11¼	32	222·72	1	12	4½
5	34·80	0	4	11½	33	229·68	1	13	4¾
6	41·76	0	5	11¾	34	236·64	1	14	5
7	48·72	0	7	0	35	243·60	1	15	5¼
8	55·68	0	8	0¼	36	250·56	1	16	5½
9	62·64	0	9	0½	37	257·52	1	17	5¾
10	69·60	0	10	0¾	38	264·48	1	18	6
11	76·56	0	11	1	39	271·44	1	19	6¼
12	83·52	0	12	1¼	40	278·40	2	0	6½
13	90·48	0	13	1½	41	285·36	2	1	6¾
14	97·44	0	14	1¾	42	292·32	2	2	7
15	104·40	0	15	2	43	299·28	2	3	7¼
16	111·36	0	16	2¼	44	306·24	2	4	7½
17	118·32	0	17	2½	45	313·20	2	5	7¾
18	125·28	0	18	2¾	46	320·16	2	6	8
19	132·24	0	19	3	47	327·12	2	7	8¼
20	139·20	1	0	3¼	48	334·08	2	8	8½
21	146·16	1	1	3½	49	341·04	2	9	8¾
22	153·12	1	2	3¾	50	348·00	2	10	9
23	160·08	1	3	4	51	354·96	2	11	7½
24	167·04	1	4	4¼	52	361·92	2	12	7¾
25	174·00	1	5	4½	53	368·88	2	13	8
26	180·96	1	6	3	54	375·84	2	14	8¼
27	187·92	1	7	3¼	55	382·80	2	15	8½
28	194·88	1	8	3½	56	389·76	2	16	8¾

Of the Square Yards and Duty at 1¾*d.* per yard, for any number of Pieces of Paper, from 1 to 3000, each Piece being 12 yards in length, and ·58 yards broad.

No. of Pieces	Square Yards	Duty for whole Yards			No. of Pieces	Square Yards	Duty for whole Yards		
		£.	*s.*	*d.*			£.	*s.*	*d.*
57	396·72	2	17	9	86	598·56	4	7	2½
58	403·68	2	18	9¼	87	605·52	4	8	2¾
59	410·64	2	19	9½	88	612·48	4	9	3
60	417·60	3	0	9¾	89	619·44	4	10	3¼
61	424·56	3	1	10	90	626·40	4	11	3½
62	431·52	3	2	10¼	91	633·36	4	12	3¾
63	438·48	3	3	10½	92	640·32	4	13	4
64	445·44	3	4	10¾	93	647·28	4	14	4¼
65	452·40	3	5	11	94	654·24	4	15	4½
66	459·36	3	6	11¼	95	661·20	4	16	4¾
67	466·32	3	7	11½	96	668·16	4	17	5
68	473·28	3	8	11¾	97	675·12	4	18	5¼
69	480·24	3	10	0	98	682·08	4	19	5½
70	487·20	3	11	0¼	99	689·04	5	0	5¾
71	494·16	3	12	0½	100	696·00	5	1	6
72	501·12	3	13	0¾	101	702·96	5	2	4½
73	508·08	3	14	1	102	709·92	5	3	4¾
74	515·04	3	15	1¼	103	716·88	5	4	5
75	522·00	3	16	1½	104	723·84	5	5	5¼
76	528·96	3	17	0	105	730·80	5	6	5½
77	535·92	3	18	0¼	106	737·76	5	7	5¾
78	542·88	3	19	0½	107	744·72	5	8	6
79	549·84	4	0	0¾	108	751·68	5	9	6¼
80	556·80	4	1	1	109	758·64	5	10	6½
81	563·76	4	2	1¼	110	765·50	5	11	6¾
82	570·72	4	3	1½	111	772·56	5	12	7
83	577·68	4	4	1¾	112	779·52	5	13	7¼
84	584·64	4	5	2	113	786·48	5	14	7½
85	591·60	4	6	2¼	114	793·44	5	15	7¾

Of the Square Yards and Duty at 1¾*d.* per yard, for any number of Pieces of Paper, from 1 to 3000, each Piece being 12 yards in length, and ·58 yards broad.

No. of Pieces	Square Yards	Duty for whole Yards			No. of Pieces	Square Yards	Duty for whole Yards		
		£.	*s.*	*d.*			£.	*s.*	*d.*
115	800·40	5	16	8	144	1002·24	7	6	1½
116	806·36	5	17	8¼	145	1009·20	7	7	1¾
117	813·32	5	18	8½	146	1016·16	7	8	2
118	820·28	5	19	8¾	147	1023·12	7	9	2¼
119	827·24	6	0	9	148	1030·08	7	10	2½
120	834·20	6	1	9¼	149	1037·04	7	11	2¾
121	841·16	6	2	9½	150	1044·00	7	12	3
122	848·12	6	3	9¾	151	1050·96	7	13	1½
123	855·08	6	4	10	152	1057·92	7	14	1¾
124	862·04	6	5	10¼	153	1064·88	7	15	2
125	869·00	6	6	10½	154	1071·84	7	16	2¼
126	875·96	6	7	9	155	1078·80	7	17	2½
127	882·92	6	8	9¼	156	1085·76	7	18	2¾
128	889·88	6	9	9½	157	1092·72	7	19	3
129	896·84	6	10	9¾	158	1099·68	8	0	3¼
130	903·80	6	11	10	159	1106·64	8	1	3½
131	910·76	6	12	10¼	160	1113·60	8	2	3¾
132	918·72	6	13	10½	161	1120·56	8	3	4
133	925·68	6	14	10¾	162	1127·52	8	4	4¼
134	932·64	6	15	11½	163	1134·48	8	5	4½
135	939·60	6	16	11¼	164	1141·44	8	6	4¾
136	946·56	6	17	11½	165	1148·40	8	7	5
137	953·52	6	18	11¾	166	1155·36	8	8	5¼
138	960·48	7	0	0	167	1162·32	8	9	5½
139	967·44	7	1	0¼	168	1169·28	8	10	5¾
140	974·40	7	2	0½	169	1176·24	8	11	6
141	981·36	7	3	0¾	170	1183·20	8	12	6¼
142	988·32	7	4	1	171	1190·16	8	13	6½
143	995·28	7	5	1¼	172	1197·12	8	14	6¾

Of the Square Yards and Duty at 1¾*d.* per yard, for any number of Pieces of Paper, from 1 to 3000, each Piece being 12 yards in length, and ·58 yards, broad.

No. of Pieces	Square Yards	Duty for whole Yards			No. of Pieces	Square Yards	Duty for whole Yards		
		£.	*s.*	*d.*			£.	*s.*	*d.*
173	1204·08	8	15	7	202	1405·92	10	4	10¾
174	1211·04	8	16	7¼	203	1412·88	10	5	11
175	1218·00	8	17	7½	204	1419·84	10	6	11¼
176	1224·96	8	18	6	205	1426·80	10	7	11½
177	1231·92	8	19	6¼	206	1433·76	10	8	11¾
178	1238·88	9	0	6½	207	1440·72	10	10	0
179	1245·84	9	1	6¾	208	1447·68	10	11	0¼
180	1252·80	9	2	7	209	1654·64	10	12	0½
181	1259·76	9	3	7¼	210	1461·60	10	13	0¾
182	1266·72	9	4	7½	211	1468·56	10	14	1¼
183	1273·68	9	5	7¾	212	1475·52	10	15	1½
184	1280·64	9	6	8	213	1482·48	10	16	1¾
185	1287·60	9	7	8¼	214	1489·44	10	17	1¼
186	1294·56	9	8	8½	215	1496·40	10	18	2½
187	1301·52	9	9	8¾	216	1503·36	10	19	2¾
188	1308·48	9	10	9	217	1510·32	11	0	2¼
189	1315·44	9	11	9¼	218	1517·28	11	1	2
190	1322·40	9	12	9½	219	1524·24	11	2	3
191	1329·36	9	13	9¾	220	1531·20	11	3	3
192	1336·32	9	10	10	300	2088·00	15	4	6
193	1343·28	9	15	10¼	400	2784·00	20	6	0
194	1350·24	9	16	10½	500	3480·00	25	7	0
195	1357·20	9	17	10¾	600	4176·00	30	9	0
196	1364·16	9	18	11	700	4872·00	35	10	6
197	1371·12	9	19	11¼	800	5568·00	40	12	0
198	1378·08	10	0	11½	900	6264·00	45	13	6
199	1385·04	10	1	11¾	1000	6960·00	50	15	0
200	1392·00	10	3	0	2000	13920·00	101	10	0
201	1398·96	10	3	10½	3000	20880·00	152	5	0

For charging the Duty on various Exciseable Commodities.

No.	At 1*d.*			At 1¼*d.*			At 1½*d.*			At 1¾*d.*			At 2*d.*		
	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1	0	0	1	0	0	1¼	0	0	1½	0	0	1¾	0	0	2
2	0	0	2	0	0	2½	0	0	3	0	0	3½	0	0	4
3	0	0	3	0	0	3¾	0	0	4½	0	0	5¼	0	0	6
4	0	0	4	0	0	5	0	0	6	0	0	7	0	0	8
5	0	0	5	0	0	6¼	0	0	7½	0	0	8¾	0	0	10
6	0	0	6	0	0	7½	0	0	9	0	0	10½	0	1	0
7	0	0	7	0	0	8¾	0	0	10½	0	1	0¼	0	1	2
8	0	0	8	0	0	10	0	1	0	0	1	2	0	1	4
9	0	0	9	0	0	11¼	0	1	1½	0	1	3¾	0	1	6
10	0	0	10	0	1	0½	0	1	3	0	1	5½	0	1	8
11	0	0	11	0	1	1¾	0	1	4½	0	1	7¼	0	1	10
12	0	1	0	0	1	3	0	1	6	0	1	9	0	2	0
13	0	1	1	0	1	4¼	0	1	7½	0	1	10¾	0	2	2
14	0	1	2	0	1	5½	0	1	9	0	2	0½	0	2	4
15	0	1	3	0	1	6¾	0	1	10½	0	2	2¼	0	2	6
16	0	1	4	0	1	8	0	2	0	0	2	4	0	2	8
17	0	1	5	0	1	9¼	0	2	1½	0	2	5¾	0	2	10
18	0	1	6	0	1	10½	0	2	3	0	2	7½	0	3	0
19	0	1	7	0	1	11¾	0	2	4½	0	3	9¼	0	3	2
20	0	1	8	0	2	1	0	2	6	0	2	11	0	3	4
30	0	2	6	0	3	1½	0	3	9	0	4	4½	0	5	0
40	0	3	4	0	4	2	0	5	0	0	5	10	0	6	8
50	0	4	2	0	5	2½	0	6	3	0	7	3½	0	8	4
60	0	5	0	0	6	3	0	7	6	0	8	9	0	10	0
70	0	5	10	0	7	3½	0	8	9	0	10	2½	0	11	8
80	0	6	8	0	8	4	0	10	0	0	11	8	0	13	4
90	0	7	6	0	9	4½	0	11	3	0	13	1½	0	15	0
100	0	8	4	0	10	5	0	12	6	0	14	7	0	16	8
200	0	16	8	1	0	10	1	5	0	1	9	2	1	13	4
300	1	5	0	1	11	3	1	17	6	2	3	9	2	10	0
400	1	13	4	2	1	8	2	10	0	2	18	4	3	6	8
500	2	1	8	2	12	1	3	2	6	3	12	11	4	3	4
600	2	10	0	3	2	6	3	15	0	4	7	6	5	9	0
700	2	18	4	3	12	11	4	7	6	5	2	1	5	16	8
800	3	6	8	4	3	4	5	0	0	5	16	8	6	13	4
900	3	15	0	4	13	9	5	12	6	6	11	3	7	10	0
1000	4	3	4	5	4	2	6	5	0	7	5	10	8	6	8

CASH TABLE

For charging the Duty on various Exciseable Commodities.

At 3*d*.			At 6*d*.			At 9¾*d*.			At 10*d*.			At 11*d*.		
£.	*s*.	*d*.	£.	*s*.	*d*.	£.	*s*.	*d*.	£.	*s*.	*d*.	£.	*s*.	*d*.
0	0	3	0	0	6	0	0	9¾	0	0	10	0	0	11
0	0	6	0	1	0	0	1	7½	0	1	8	0	1	10
0	0	9	0	1	6	0	2	5¼	0	2	6	0	2	9
0	1	0	0	2	0	0	3	3	0	3	4	0	3	8
0	1	3	0	2	6	0	4	0¾	0	4	2	0	4	7
0	1	6	0	3	0	0	4	10½	0	5	0	0	5	6
0	1	9	0	3	6	0	5	8¼	0	5	10	0	6	5
0	2	0	0	4	0	0	6	6	0	6	8	0	7	4
0	2	3	0	4	6	0	7	3¾	0	7	6	0	8	3
0	2	6	0	5	0	0	8	1½	0	8	4	0	9	2
0	2	9	0	5	6	0	8	11¼	0	9	2	0	10	1
0	3	0	0	6	0	0	9	9	0	10	0	0	11	0
0	3	3	0	6	6	0	10	6¾	0	10	10	0	11	11
0	3	6	0	7	0	0	11	4½	0	11	8	0	12	10
0	3	9	0	7	6	0	12	2¼	0	12	6	0	13	9
0	4	0	0	8	0	0	13	0	0	13	4	0	14	8
0	4	3	0	8	6	0	13	9¾	0	14	2	0	15	7
0	4	6	0	9	0	0	14	7½	0	15	0	0	16	6
0	4	9	0	9	6	0	15	5¼	0	15	10	0	17	5
0	5	0	0	10	0	0	16	3	0	16	8	0	18	4
0	7	6	0	15	0	1	4	4½	1	5	0	1	7	6
0	10	0	1	0	0	1	12	6	1	13	4	1	16	8
0	12	6	1	5	0	2	0	7½	2	1	8	2	5	10
0	15	0	1	10	0	2	8	9	2	10	0	2	15	0
0	17	6	1	15	0	2	16	10½	2	18	4	3	4	2
1	0	0	2	0	0	3	5	0	3	6	8	3	13	4
1	2	6	2	5	0	3	13	1½	3	15	0	4	2	6
1	5	0	2	10	0	4	1	3	4	3	4	4	11	8
2	10	0	5	0	0	8	2	6	8	6	8	9	3	4
3	15	0	7	10	0	12	3	9	12	10	0	13	15	0
5	0	0	10	0	0	16	5	0	16	13	4	18	6	8
6	5	0	12	10	0	20	6	3	20	16	8	22	18	4
7	10	0	15	0	0	24	7	6	25	0	0	27	10	0
8	15	0	17	10	0	28	8	9	29	3	4	32	1	8
10	0	0	20	0	0	32	10	0	33	6	8	36	13	4
11	5	0	22	10	0	36	11	3	37	10	0	41	5	0
12	10	0	25	0	0	40	12	6	41	13	4	45	16	8

For charging the Duty on various Exciseable Commodities.

No.	At 1*s.* 0¼*d.*			At 1*s.* 4¼*d.*			At 1*s.* 10*d.*			At 2*s.*		At 4*s.*	
	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	£	*s.*
1	0	1	0¼	0	1	4¼	0	1	10	0	2	0	4
2	0	2	0½	0	2	8½	0	3	8	0	4	0	8
3	0	3	0¾	0	4	0¾	0	5	6	0	6	0	12
4	0	4	1	0	5	5	0	7	4	0	8	0	16
5	0	5	1¼	0	6	9¼	0	9	2	0	10	1	0
6	0	6	1½	0	8	1½	0	11	0	0	12	1	4
7	0	7	1¾	0	9	5¾	0	12	10	0	14	1	8
8	0	8	2	0	10	10	0	14	8	0	16	1	12
9	0	9	2¼	0	12	2¼	0	16	6	0	18	1	16
10	0	10	2½	0	13	6½	0	18	4	1	0	2	0
11	0	11	2¾	0	14	10¾	1	0	2	1	2	2	4
12	0	12	3	0	16	3	1	2	0	1	4	2	8
13	0	13	3¼	0	17	7¼	1	3	10	1	6	2	12
14	0	14	3½	0	18	11½	1	5	8	1	8	2	16
15	0	15	3¾	1	0	3¾	1	7	6	1	10	3	0
16	0	16	4	1	1	8	1	9	4	1	12	3	4
17	0	17	4¼	1	3	0¼	1	11	2	1	14	3	8
18	0	18	4½	1	4	4½	1	13	0	1	16	3	12
19	0	19	4¾	1	5	8¾	1	14	10	1	18	3	16
20	1	0	5	1	7	1	1	16	8	2	0	4	0
30	1	10	7½	2	0	7½	2	15	0	3	0	6	0
40	2	0	10	2	14	2	2	13	4	4	0	8	0
50	2	11	0½	3	7	8½	4	11	8	5	0	10	0
60	3	1	3	4	1	3	5	10	0	6	0	12	0
70	3	11	5½	4	14	9½	6	8	4	7	0	14	0
80	4	1	8	5	8	4	7	6	8	8	0	16	0
90	4	11	10½	6	1	10½	8	5	0	9	0	18	0
100	5	2	1	6	15	5	9	3	4	10	0	20	0
200	10	4	2	13	10	10	18	6	8	20	0	40	0
300	15	6	3	20	6	3	27	10	0	30	0	60	0
400	20	8	4	27	1	8	36	13	4	40	0	80	0
500	25	10	5	33	17	1	45	16	8	50	0	100	0
600	30	12	6	40	12	6	55	0	0	60	0	120	0
700	35	14	7	47	7	11	64	3	4	70	0	140	0
800	40	16	8	54	3	4	73	6	8	80	0	160	0
900	45	18	1	60	18	9	82	10	0	90	0	180	0
1000	51	10	0	67	14	2	91	13	4	100	0	200	0

For charging the Duty on various Exciseable Commodities.

At 4*s.*		At 4*s.* 10*d.*			At 6*s.* 11*d.*			At 7*s.* 6*d.*			At 8*s.* 2*d.*		
£.	*s.*	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*
0	4	0	4	10	0	6	11	0	7	6	0	8	2
0	8	0	9	8	0	13	10	0	15	0	0	16	4
0	12	0	14	6	1	0	9	1	2	6	1	4	6
0	16	0	19	4	1	7	8	1	10	0	1	12	8
1	0	1	4	2	1	14	7	1	17	6	2	0	10
1	4	1	9	0	2	1	6	2	5	0	2	9	0
1	8	1	13	10	2	8	5	2	12	6	2	17	2
1	12	1	18	8	2	15	4	3	0	0	3	5	4
1	16	2	3	6	3	2	3	3	7	6	3	13	6
2	0	2	8	4	3	9	2	3	15	0	4	1	8
2	4	2	13	2	3	16	1	4	2	6	4	9	10
2	8	2	18	0	4	3	0	4	10	0	4	18	0
2	12	3	2	10	4	9	11	4	17	6	5	6	2
2	16	3	7	8	4	16	10	5	5	0	5	14	4
3	0	3	12	6	5	3	9	5	12	6	6	2	6
3	4	3	17	4	5	10	8	6	0	0	6	10	8
3	8	4	2	2	5	17	7	6	7	6	6	18	10
3	12	4	7	0	6	4	6	6	15	0	7	7	0
3	16	4	11	10	6	11	5	7	2	6	7	15	2
4	0	4	16	8	6	18	4	7	10	0	8	3	4
6	0	7	5	0	10	7	6	11	5	0	12	5	0
8	0	9	13	4	13	16	8	15	0	0	16	6	8
10	0	12	1	8	17	5	10	18	15	0	20	8	4
12	0	14	10	0	20	15	0	22	10	0	24	10	0
14	0	16	18	4	24	4	2	26	5	0	28	11	8
16	0	19	6	8	27	13	4	30	0	0	32	13	4
18	0	21	15	0	31	2	6	33	15	0	36	15	0
20	0	24	3	4	34	11	8	37	10	0	40	16	8
40	0	48	6	8	69	3	4	75	0	0	81	13	4
60	0	72	10	0	103	15	0	112	10	0	122	10	0
80	0	96	13	4	138	6	8	150	0	0	163	6	8
100	0	120	16	8	172	18	4	187	10	0	204	3	4
120	0	145	0	0	207	10	0	225	0	0	245	0	0
140	0	169	3	4	242	1	8	262	10	3	285	16	8
160	0	193	6	8	276	13	4	300	0	0	326	13	4
180	0	217	10	0	311	5	0	337	10	0	367	10	0
200	0	241	13	4	345	16	8	375	0	0	408	6	8

CASH TABLE

For charging the Duty on various Exciseable Commodities.

No.	At 8*s.* 5*d.*			At 13*s.* 4*d.*			At 14*s.* 6*d.*			At 16*s.* 5*d.*		
	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*	£.	*s.*	*d.*
1	0	8	5	0	13	4	0	14	6	0	16	5
2	0	16	10	1	6	8	1	9	0	1	12	10
3	1	5	0	2	0	0	2	3	6	2	9	3
4	1	13	8	2	13	4	2	18	0	3	5	8
5	2	2	1	3	6	8	3	12	6	4	2	1
6	2	10	8	4	0	0	4	7	0	4	18	6
7	2	18	11	4	13	4	5	1	6	5	14	11
8	3	7	4	5	6	8	5	16	0	6	11	4
9	3	15	9	6	0	0	6	10	6	7	7	9
10	4	4	2	6	13	4	7	5	0	8	4	2
11	4	12	7	7	6	8	7	19	6	9	0	7
12	5	1	0	8	0	0	8	14	0	9	17	0
13	5	9	5	8	13	4	9	8	6	10	13	5
14	5	17	10	9	6	8	10	3	0	11	9	10
15	6	6	3	10	0	0	10	17	6	12	6	3
16	6	14	8	10	13	4	11	12	0	13	2	8
17	7	3	1	11	6	8	12	6	6	13	19	1
18	7	11	6	12	0	0	13	1	0	14	15	6
19	7	19	11	12	13	4	13	15	6	15	11	11
20	8	8	4	13	6	8	14	10	0	16	8	4
30	12	12	6	20	0	0	21	15	0	24	12	6
40	16	16	8	26	13	4	29	0	0	32	16	8
50	21	0	10	33	6	8	36	5	0	41	0	10
60	25	5	0	40	0	0	43	10	0	49	5	0
70	29	9	2	46	13	4	50	15	0	57	9	2
30	33	13	4	53	6	8	58	0	0	65	13	4
90	37	17	6	60	0	0	65	5	0	73	17	6
100	42	1	8	66	13	4	72	10	0	82	1	8
200	84	3	4	133	6	8	145	0	0	164	3	4
300	126	5	0	200	0	0	217	10	0	246	5	0
400	168	6	8	266	13	4	290	0	0	328	6	8
500	210	8	4	333	6	8	362	10	0	410	8	4
600	252	10	0	400	0	0	435	0	0	492	10	0
700	294	11	8	466	13	4	507	10	0	574	11	8
800	336	13	4	533	6	8	580	0	0	656	13	4
900	378	15	0	600	0	0	652	10	0	738	15	0
1000	420	16	8	666	13	4	725	0	0	820	16	8

TABLE

Of the Duty on Sweets at 2*l.* 2*s.* per barrel of 31½ gallons.

Gallons	£.	*s.*	*d.*	Barrels	£.	*s.*	*d.*
¼	0	0	4	1	2	2	0
½	0	0	8	2	4	4	0
¾	0	1	0	3	6	6	0
1	0	1	4	4	8	8	0
2	0	2	8	5	10	10	0
3	0	4	0	6	12	12	0
4	0	5	4	7	14	14	0
5	0	6	8	8	16	16	0
6	0	8	0	9	18	18	0
7	0	9	4	10	21	0	0
8	0	10	8	11	23	2	0
9	0	12	0	12	25	4	0
10	0	13	4	13	27	6	0
11	0	14	8	14	29	8	0
12	0	16	0	15	31	10	0
13	0	17	4	16	33	12	0
14	0	18	8	17	35	14	0
15	1	0	0	18	37	16	0
16	1	1	4	19	39	18	0
17	1	2	8	20	42	0	0
18	1	4	0	30	63	0	0
19	1	5	4	40	84	0	0
20	1	6	8	50	105	0	0
21	1	8	0	60	126	0	0
22	1	9	4	70	147	0	0
23	1	10	8	80	168	0	0
24	1	12	0	90	189	0	0
25	1	13	4	100	210	0	0
26	1	14	8	200	420	0	0
27	1	16	0	300	630	0	0
28	1	17	4	400	840	0	0
29	1	18	8	500	1050	0	0
30	2	0	0	600	1260	0	0
31	2	1	4	700	1470	0	0
31½	2	2	0	800	1680	0	0
				900	1890	0	0
				1000	2100	0	0

TABLE

Of the Duty on Sweets, at 7*s.* per barrel of 31½ gallons.

Gallons	£.	*s.*	*d.*	Barrels	£.	*s.*	*d.*
¼	0	0	0½ ⅔	1	0	7	0
½	0	0	1¼ ⅓	2	0	14	0
¾	0	0	2	3	1	1	0
1	0	0	2½ ⅔	4	1	8	0
2	0	0	5¼ ⅓	5	1	15	0
3	0	0	8	6	2	2	0
4	0	0	10½ ⅔	7	2	9	0
5	0	1	1¼ ⅓	8	2	16	0
6	0	1	4	9	3	3	0
7	0	1	6½ ⅔	10	3	10	0
8	0	1	9½ ⅓	11	3	17	0
9	0	2	0	12	4	4	0
10	0	2	2½ ⅔	13	4	11	0
11	0	2	5¼ ⅓	14	4	18	0
12	0	2	8	15	5	5	0
13	0	2	10½ ⅔	16	5	12	0
14	0	3	1¼ ⅓	17	5	19	0
15	0	3	4	18	6	6	0
16	0	3	6½ ⅔	19	6	13	0
17	0	3	9¼ ⅓	20	7	0	0
18	0	4	0	30	10	10	0
19	0	4	2½ ⅔	40	14	0	0
20	0	4	5¼ ⅓	50	17	10	0
21	0	4	8	60	21	0	0
22	0	4	10½ ⅔	70	24	10	0
23	0	5	1¼ ⅓	80	28	0	0
24	0	5	4	90	31	10	0
25	0	5	6½ ⅔	100	35	0	0
26	0	5	9¼ ⅓	200	70	0	0
27	0	6	0	300	105	0	0
28	0	6	2½ ⅔	400	140	0	0
29	0	6	5¼ ⅓	500	175	0	0
30	0	6	8	600	210	0	0
31	0	6	10½ ⅔	700	245	0	0
31½	0	7	0	800	280	0	0
				900	315	0	0
				1000	350	0	0

TABLE

Of the Duty on Cider in Gallons at £1 per Hogshead, as settled by the latest Act of Parliament.

Galls.	£.	s.	d.	Rem.	Galls.	£.	s.	d.	Rem.
1	0	0	3¾	15	33	0	10	5½	54
2	0	0	7½	30	34	0	10	9½	6
3	0	0	11¼	45	35	0	11	1¼	21
4	0	1	3	60	36	0	11	5	36
5	0	1	7	12	37	0	11	8¾	51
6	0	1	10¾	27	38	0	12	0¾	3
7	0	2	2½	42	39	0	12	4½	18
8	0	2	6¼	57	40	0	12	8¼	33
9	0	2	10¼	9	41	0	13	0	48
10	0	3	2	24	42	0	13	4	0
11	0	3	5¾	39	43	0	13	7¾	15
12	0	3	9½	54	44	0	13	11½	30
13	0	4	1½	6	45	0	14	3¼	45
14	0	4	5¼	21	46	0	14	7	60
15	0	4	9	36	47	0	14	11	12
16	0	5	0¾	51	48	0	15	2¾	27
17	0	5	4¾	3	49	0	15	6½	42
18	0	5	8½	18	50	0	15	10¼	57
19	0	6	0¼	33	51	0	16	2¼	9
20	0	6	4	48	52	0	16	6	24
21	0	6	8	0	53	0	16	9¾	39
22	0	6	11¾	15	54	0	17	1½	54
23	0	7	3½	30	55	0	17	5½	6
24	0	7	7¼	45	56	0	17	9¼	21
25	0	7	11	60	57	0	18	1	36
26	0	8	3	12	58	0	18	4¾	51
27	0	8	6¾	27	59	0	18	8¾	3
28	0	8	10½	42	60	0	19	0½	18
29	0	9	2¼	57	61	0	19	4¼	38
30	0	9	6¼	9	62	0	19	8	48
31	0	9	10	24	63	1	0	0	
32	0	10	1¾	39					

USE OF THE TABLES.

Look for the number of things to be charged in the left hand column of the proper Table, and in the particular column for the proper charge; then in the same horizontal line with the number first found, will be the charge. When the number of things to be charged cannot be found in the first column on the left, take the charge for such numbers as will, by their sum, give the exact number to be charged.

EXAMPLE 1.

Required the nett duty on 149 barrels of strong beer.

SOLUTION.

In the 3rd page of the Table for Beer, we look for the number 149 under barrels, and opposite to it under " Strong" the charge is £68. *5s.* 10*d.*

EXAMPLE 2.

Required the number of square yards in 207 pieces of Paper, and the duty at 1¾*d.* per square yard.

SOLUTION.

In the Table for paper, opposite to 207 in the column of pieces, we find 1440·72 for the number of yards, and £10. 10*s.* for the duty.

It may here be noticed, that the charge on Paper is always made on the integral number of Square Yards, the decimal being called the odds to be carried to the next addition of yards.

EXAMPLE 3.

Required the duty on 55 gallons of Cider, at £1 per hogshead.

SOLUTION.

In the Table for Cider, opposite to 55 gallons, may be seen 17*s.* 5½*d.*, with 6 for remainder, signifying that the duty on 55 gallons of Cider is 17*s.* 5½*d.* and $\frac{6}{63}$rds of a farthing. This remainder being carried forward, will augment other remainders, and when the sum of the remainders amounts to 63, a farthing must evidently be charged for them.

Thus have we offered all that is really useful in Gaging, as far as the limits we had assigned ourselves will allow; and we trust that upon the whole, the New Practical Gager will be found to be, what we have endeavoured to make it, the most systematical and most intelligible Treatise extant on the valuable art of Gaging.

THE END.

W. SHACKELL, Printer,
Johnson's-court, Fleet-street, London.